Review: Convection

Reminders…

• Final exam Wednesday Dec. 17, 8-10 am
  – This room
  – Two parts
    • 10 multiple choice-type review questions
    • 3 radiation calculation problems
  – You can bring three 8.5 x 11 sheets of notes (both sides)
  – Don’t forget your calculator, scratch paper, pencil

• Check one last time that scores are correct
  – Scores have been updated
CHAPTER 6: Introduction to Convection

- Boundary layers
  - Velocity
  - Thermal
- Boundary layer equations

Three Boundary Layers

**Figure 6.9** Development of the velocity, thermal, and concentration boundary layers for an arbitrary surface.
**Figure 6.6** Velocity boundary layer development on a flat plate.

**Figure 6.8** Variation of velocity boundary layer thickness $\delta$ and the local heat transfer coefficient $h$ for flow over an isothermal flat plate.
Six Steps to Solving...

1. Determine flow geometry
2. Determine appropriate fluid temperature (e.g., $T_{film}$) and evaluate fluid properties
3. Consider fluid B (Applies only to mass transfer problems.)
4. Calculate Reynolds number to determine if laminar or turbulent flow
5. Decide whether a local or average coefficient is required
6. Select appropriate correlation

Dimensionless Groups

- **Reynolds number** ($Re_D$): $\frac{VL}{\nu}$
  - Ratio of the inertia and viscous forces.

- **Prandtl number** ($Pr$): $\frac{\nu \mu}{k} = \frac{\nu}{\alpha}$
  - Ratio of the momentum and thermal diffusivities.

- **Nusselt number** ($Nu_D$): $\frac{hL}{k_f}$
  - Ratio of convection to pure conduction heat transfer.

- **Sherwood number** ($Sh_L$): $\frac{h_mL}{D_{AB}}$
  - Dimensionless concentration gradient at the surface.

- **Schmidt number** ($Sc$): $\frac{\nu}{D_{AB}}$
  - Ratio of the momentum and mass diffusivities.

- **Coefficient of friction** ($C_f$): $\frac{\tau_s}{\rho V^2/2}$
  - Dimensionless surface shear stress.
CHAPTER 7: External Flow

• Geometries
  – Flat plate
  – Cylinder in cross flow
  – Sphere
  – Banks of tubes

• Heat transfer
  – Constant surface temperature
  – Constant heat flux

Flat Plate

• Critical Reynolds number: $5 \times 10^5$
  \[ \text{Re}_L = \frac{\rho u_x L}{\mu} = \frac{u_x L}{v} \]

• Local Nusselt number (use $T_{\text{film}}$):
  – Laminar
    \[ Nu_x = \frac{h_x x}{k} = 0.332 \ \text{Re}_x^{1/2} \text{Pr}^{1/3} \]
  – Turbulent
    \[ Nu_x = 0.0296 \ \text{Re}_x^{4/5} \text{Pr}^{1/3} \]
Flat Plate

- Average Nusselt number
  - Laminar
    \[ \overline{Nu_x} = 0.664 \, Re_x^{1/2} \, Pr^{1/3} \]
  - Turbulent (tripped at leading edge)
    \[ \overline{Nu_L} = 0.037 \, Re_L^{4/5} \, Pr^{1/3} \]
  - Transition
    \[ \overline{Nu_L} = \left( 0.037 \, Re_L^{4/5} - 871 \right) Pr^{1/3} \]
- Unheated starting length

Flow Correlation Summary

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Geometry</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta = 5x , Re_x^{-1/2} )</td>
<td>Flat plate</td>
<td>Laminar, ( T_f )</td>
</tr>
<tr>
<td>( C_{f_s} = 0.664 , Re_x^{-1/2} )</td>
<td>Flat plate</td>
<td>Laminar, local, ( T_f )</td>
</tr>
<tr>
<td>( Nu_x = 0.332 , Re_x^{1/3} , Pr^{1/3} )</td>
<td>Flat plate</td>
<td>Laminar, local, ( T_f ), ( Pr \gtrsim 0.6 )</td>
</tr>
<tr>
<td>( \delta_s = \delta , Pr^{-1/3} )</td>
<td>Flat plate</td>
<td>Laminar, ( T_f )</td>
</tr>
<tr>
<td>( C_{f_s} = 1.328 , Re_x^{-1/2} )</td>
<td>Flat plate</td>
<td>Laminar, average, ( T_f )</td>
</tr>
<tr>
<td>( \overline{Nu}_h = 0.664 , Re_x^{1/2} , Pr^{1/3} )</td>
<td>Flat plate</td>
<td>Laminar, average, ( T_f ), ( Pr \gtrsim 0.6 )</td>
</tr>
<tr>
<td>( Nu_x = 0.565 , Pr_x^{1/2} )</td>
<td>Flat plate</td>
<td>Laminar, local, ( T_f ), ( Pr \lesssim 0.05 ), ( Pe_x \gtrsim 100 )</td>
</tr>
<tr>
<td>( C_{f_s} = 0.059 , Re_x^{-1/3} )</td>
<td>Flat plate</td>
<td>Turbulent, local, ( T_f ), ( Re_x \lesssim 10^8 )</td>
</tr>
<tr>
<td>( \delta = 0.37 , Re_x^{1/3} )</td>
<td>Flat plate</td>
<td>Turbulent, ( T_f ), ( Re_x \lesssim 10^8 )</td>
</tr>
<tr>
<td>( Nu_x = 0.0296 , Re_x^{1/3} , Pr^{1/3} )</td>
<td>Flat plate</td>
<td>Turbulent, local, ( T_f ), ( Re_x \lesssim 10^8 ), ( 0.6 \lesssim Pr \lesssim 60 )</td>
</tr>
<tr>
<td>( \overline{C}_{f_s} = 0.074 , Re_x^{-1/3} - 1742 , Re_x^{-1} )</td>
<td>Flat plate</td>
<td>Mixed, average, ( T_f ), ( Re_{w-e} = 5 \times 10^5 ), ( Re_x \lesssim 10^8 )</td>
</tr>
<tr>
<td>( \overline{Nu}_h = (0.037 , Re_x^{1/3} - 871) , Pr^{1/3} )</td>
<td>Flat plate</td>
<td>Mixed, average, ( T_f ), ( Re_{w-e} = 5 \times 10^5 ), ( Re_x \lesssim 10^8 ), ( 0.6 \lesssim Pr \lesssim 60 )</td>
</tr>
</tbody>
</table>
Cylinder in Cross Flow

- Average Nusselt number

$$\overline{Nu_D} = 0.3 + \frac{0.62 \text{Re}_D^{1/2} \text{Pr}^{1/3}}{1 + (0.4 / \text{Pr})^{2/3}} \left[ 1 + \left( \frac{\text{Re}_D}{282,000} \right)^{5/8} \right]^{4/5}$$

Flow over a Sphere

- Average Nusselt number

$$\overline{Nu_D} = 2 + \left( 0.4 \text{Re}_D^{1/2} + 0.06 \text{Re}_D^{2/3} \right) \text{Pr}^{0.4} \left( \mu / \mu_s \right)^{1/4}$$
Flow Across Banks of Tubes

Aligned: \[ V_{\text{max}} = \frac{S_T}{S_T - D} V \]

Staggered: \[ V_{\text{max}} = \frac{S_T}{S_T - D} V \text{ if } 2(S_D - D) \geq (S_T - D) \]

or, \[ V_{\text{max}} = \frac{S_T}{2(S_D - D)} V \text{ if } 2(S_D - D) \leq (S_T - D) \]

Bakes of Tubes (cont.)

- Average Nusselt Number for an Isothermal Array:
  \[ \overline{Nu}_D = C_2 \left[ C \cdot Re_{D,\text{max}}^m \cdot Pr^{0.36} \left( \frac{Pr}{Pr_s} \right)^{1/4} \right] \]

  \[ C, m \rightarrow \text{Table 7.7} \]
  \[ C_2 \rightarrow \text{Table 7.8} \]

  All properties are evaluated at \((T_i + T_o)/2\) except for \(Pr_s\).
CHAPTER 8: Internal Flow

• Basics
  – Velocity profiles
  – Mean velocity, mean temperature

• Geometries
  – Circular tubes
  – Non-circular tubes
  – Concentric annulus

• Heat transfer
  – Constant surface temperature
  – Constant heat flux
Internal Flow Calculations

- Reynolds number (critical = 2300)
  \[ \text{Re}_D = \frac{\rho u_m D_h}{\mu} \]

- Hydraulic diameter for non-circular tubes
  \[ D_h = \frac{4 A_c}{P} \]

- For uniform surface temperature
  \[ \frac{\Delta T_o}{\Delta T_i} = \frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp \left( -\frac{P L}{m c_p} \frac{1}{h} \right) = \exp \left( -\frac{\bar{h} A_s}{m c_p} \right) \]
Laminar Flow: Non-Circular Tube

Turbulent Flow

- Circular or noncircular tube with small temperature diffs (Dittus-Boelter)

\[ \text{Nu}_D = 0.023 \text{Re}^{4/5}_D \text{Pr}^n \quad \begin{cases} 
  n = 0.3 & (T_s < T_m) \\
  n = 0.4 & (T_s > T_m) 
\end{cases} \]
CHAPTER 9: Free (Natural) Convection

- Fundamentals
  - Buoyancy
  - Boundary layer development
  - Transition from laminar to turbulent at $Ra_L = 10^9$

- Geometries
  - Vertical surface
  - Horizontal plates
  - Long horizontal cylinder
  - Spheres
  - Vertical parallel plate channels
  - Enclosures/cavities

Types of Free Convection

- Plume
- Buoyant Jet
- Boundary Layer
Empirical Correlations

• Similar to approach used previously for forced convection
  – Correlation for Nusselt number $Nu$
  – Instead of $Re$, we use $Ra$
  – Careful on which expression to use

• Fluid properties determined at film temp.
  \[ T_{film} = \frac{(T_s + T_\infty)}{2} \]

• Thermal expansion coefficient $\beta$
  – Gases: $\beta = 1/T(K)$
  – Liquids: Look up in appendix

Flow over Vertical Plate

• Rayleigh Number:
  \[ Ra_L = Gr_L Pr = \frac{g \beta (T_s - T_\infty) L^3}{\nu \alpha} \] (9.25)

• Laminar Flow ($Ra_L < 10^9$):
  \[ \bar{Nu}_L = 0.68 + \frac{0.670 \ Ra_L^{1/4}}{1 + (0.492 / Pr)^{9/16} \ }^{4/9} \] (9.27)

• All Conditions:
  \[ \bar{Nu}_L = \left\{ 0.825 + \frac{0.387 \ Ra_L^{1/6}}{1 + (0.492 / Pr)^{9/16} \ }^{8/27} \right\}^2 \] (9.26)
CHAPTER 11: Heat Exchangers

• Basics
  – Overall heat transfer coefficient
  – Phase change heat transfer one one side

• Heat exchanger configurations
  – Parallel flow
  – Counterflow
    • Multiple shells
    • Multiple tube passes
  – Cross-flow

• Analysis approaches
  – Log-mean temperature difference (design)
  – Effectiveness-NTU (performance and design)
  – Modified LMTD (design - fancy configurations)

Overall Heat Transfer Coefficient

• Contributing factors
  – Convection between the two fluids and solid
  – Conduction of the solid separator
  – Potential use of fins in one or both sides
  – Time-dependent surface fouling

• General expression \((c\text{ and } h = \text{ cold and hot})\)

\[
\frac{1}{UA} = \frac{1}{U_cA_c} + \frac{R''_f,c}{(\eta_o h A)_c} + R_w + \frac{R''_f,h}{(\eta_o h A)_h} + \frac{1}{(\eta_o h A)_h}
\]
Log-Mean Temperature Difference

\[ q = UA \Delta T_{lm} \]

\[ \Delta T_{lm} = \frac{\Delta T_2 - \Delta T_1}{\ln(\Delta T_2 / \Delta T_1)} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} \]

Special Operating Conditions

- \( C_h \gg C_c \) or a condensing vapor (\( C_h \to \infty \))
- \( C_h \ll C_c \) or an evaporating liquid (\( C_c \to \infty \))
- \( C_c = C_h \)

\[ C = \dot{m} c_p = "heat capacity rate" \]
Effectiveness-NTU Approach

- Effectiveness:

\[ \varepsilon = \frac{q}{q_{\text{max}}} \]

\[ q_{\text{max}} = C_{\text{mn}} (T_{h,i} - T_{c,i}) \]

\[ C_{\text{min}} = \begin{cases} C_h & \text{if } C_h < C_c \\ \text{or} & \\ C_c & \text{if } C_c < C_h \end{cases} \]

- NTU (Number of Transfer Units):

\[ NTU = \frac{UA}{C_{\text{mn}}} \]

<table>
<thead>
<tr>
<th>TABLE 11.3 Heat Exchanger Effectiveness Relations [5]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flow Arrangement</td>
</tr>
<tr>
<td>-------------------------------------------------------</td>
</tr>
<tr>
<td>Concentric tube</td>
</tr>
<tr>
<td>Parallel flow</td>
</tr>
<tr>
<td>Counterflow</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Shell-and-tube</td>
</tr>
<tr>
<td>One shell pass (2, 4, ... tube passes)</td>
</tr>
<tr>
<td>( n ) Shell passes (2n, 4n, ... tube passes)</td>
</tr>
<tr>
<td>Cross-flow (single pass)</td>
</tr>
<tr>
<td>Both fluids unmixed</td>
</tr>
<tr>
<td>( C_{\text{mix}} ) (mixed), ( C_{\text{mix}} ) (unmixed)</td>
</tr>
<tr>
<td>( C_{\text{mix}} ) (mixed), ( C_{\text{mix}} ) (unmixed)</td>
</tr>
<tr>
<td>All exchangers ( C_r = 0 )</td>
</tr>
</tbody>
</table>
### Table 11.4  Heat Exchanger NTU Relations

<table>
<thead>
<tr>
<th>Flow Arrangement</th>
<th>Relation</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Concentric tube</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Parallel flow</td>
<td>$NTU = -\frac{\ln [1 - \varepsilon(1 + C_r)]}{1 + C_r}$</td>
<td>(11.28b)</td>
</tr>
<tr>
<td>Counterflow</td>
<td>$NTU = \frac{1}{C_r-1} \ln \left( \frac{\varepsilon - 1}{\varepsilon C_r - 1} \right)$ ($C_r &lt; 1$)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$NTU = \frac{\varepsilon}{1 - \varepsilon}$ ($C_r = 1$)</td>
<td>(11.29b)</td>
</tr>
<tr>
<td><strong>Shell-and-tube</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One shell pass (2, 4, ... tube passes)</td>
<td>$(NTU)_h = -\frac{(1 + C_r^2)^{1/2}}{\varepsilon} \ln \left( \frac{F - 1}{F + 1} \right)$</td>
<td>(11.30b)</td>
</tr>
<tr>
<td></td>
<td>$E = \frac{2e_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$</td>
<td>(11.30c)</td>
</tr>
<tr>
<td>$n$ Shell passes (2n, 4n, ... tube passes)</td>
<td>Use Equations 11.30b and 11.30c with $e_i = \frac{F - 1}{F - C_r}$ [ F = \left( \frac{\varepsilon C_r - 1}{\varepsilon - 1} \right)^{1/n} ] $NTU = n(NTU)_h$ (11.31b, c, d)</td>
<td></td>
</tr>
<tr>
<td><strong>Cross-flow (single pass)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_{\text{min}}$ (mixed), $C_{\text{max}}$ (unmixed)</td>
<td>$NTU = -\ln \left[ 1 + \left( \frac{1}{C_r} \right) \ln(1 - \varepsilon C_r) \right]$</td>
<td>(11.33b)</td>
</tr>
<tr>
<td>$C_{\text{min}}$ (mixed), $C_{\text{max}}$ (unmixed)</td>
<td>$NTU = -\left( \frac{1}{C_r} \right) \ln[C_r \ln(1 - \varepsilon) + 1]$</td>
<td>(11.34b)</td>
</tr>
<tr>
<td>All exchangers ($C_r = 0$)</td>
<td>$NTU = -\ln(1 - \varepsilon)$</td>
<td>(11.35b)</td>
</tr>
</tbody>
</table>

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**Figure 11.10** Effectiveness of a parallel-flow heat exchanger (Equation 11.23).
Counterflow Heat Exchanger

Figure 11.11 Effectiveness of a counterflow heat exchanger (Equation 11.29).

Shell-and-Tube Heat Exchanger
(One Shell)

Figure 11.12 Effectiveness of a shell-and-tube heat exchanger with one shell and any multiple of two tube passes (two, four, etc. tube passes) (Equation 11.30).

NOTE!! You cannot use LMTD for this type of heat exchanger!
Shell-and-Tube Heat Exchanger
(Multiple Shells)

NOTE!! You cannot use LMTD for this type of heat exchanger!

Figure 11.13 Effectiveness of a shell-and-tube heat exchanger with two shell passes and any multiple of four tube passes (four, eight, etc. tube passes) (Equation 11.31 with \( n = 2 \)).

Cross-Flow Heat Exchanger
(Unmixed-Unmixed)

NOTE!! You cannot use LMTD for this type of heat exchanger!

Figure 11.14 Effectiveness of a single-pass, cross-flow heat exchanger with both fluids unmixed (Equation 11.32).
REVIEW: Radiation

Radiation Spectrum

Figure 12.3 Spectrum of electromagnetic radiation.
Intensity vs. Wavelength and Direction

**Figure 12.4** Radiation emitted by a surface. (a) Spectral distribution. (b) Directional distribution.

The Solid Angle
Solid Angle Geometry

\[ \omega = \frac{A_2 \cos \theta_2}{r^2} \]

Projected Area

\[ dA_1 \cos \theta \]
Radiation Heat Transfer

- Energy transfer between two elements $A_1$ and $A_2$

\[ q_{1-j} = I \times A_1 \cos \theta_1 \times \omega_{j-1} \]
\[ = \frac{I \times A_1 A_2 \cos \theta_1 \cos \theta_2}{r^2} \]

From Example 12.1...

\[ \omega_{3-1} = \omega_{4-1} = \frac{A_3}{r^2} = \frac{10^{-3} m^2}{(0.5\text{ m})^2} = 4.00 \times 10^{-4}\text{sr} \]

\[ \omega_{2-1} = \frac{A_2 \cos \theta_2}{r^2} = \frac{10^{-3} m^2 \times \cos 30^\circ}{(0.5\text{ m})^2} = 3.46 \times 10^{-3}\text{sr} \]
Blackbody

• Hypothetical perfect radiative surface

• Absorbs all incident radiation, regardless of wavelength and direction

• Emits maximum theoretical energy

• Diffuse emitter – Radiation emitted evenly in all directions

The Planck Distribution

• Emissive power of a blackbody depends on temperature and wavelength

• Planck figured out this relation

\[ E_{\lambda,b}(\lambda, T) = \pi I_{\lambda,b}(\lambda, T) = \frac{C_1}{\lambda^5 \left[ \exp\left( \frac{C_2}{\lambda T} \right) - 1 \right]} \]

First radiation constant: \( C_1 = 3.742 \times 10^8 \text{ W} \cdot \mu\text{m}^4 / \text{m}^2 \)
Second radiation constant: \( C_2 = 1.439 \times 10^4 \mu\text{m} \cdot \text{K} \)

• Plot of \( E \) vs. \( \lambda \) looks like this:
Wien’s Displacement Law

- For a given temperature, spectral emission goes through a maximum at a given wavelength.
- Wien figured this one out:

\[ \lambda_{max} T = C_3 = 2898 \ \mu m \cdot K \]

- This maximum is indicated by the dashed line in Figure 12.12
Stefan-Boltzmann Law

- If one were to integrate any of the curves shown in Figure 12.12 over the entire range of wavelengths, one would get the total emissive power for a blackbody:

\[
E_b = \int_0^\infty \frac{C_1}{\lambda^5 \left[ \exp\left(\frac{C_2}{\lambda T}\right) - 1 \right]} d\lambda
\]

\[
= \sigma T^4
\]

- The Stefan-Boltzmann constant \( \sigma \) is:

\[
\sigma = 5.670 \times 10^{-8} \text{ W/m}^2\text{K}^4
\]

Band Emission

- Amount of total emitted radiation depends on range of wavelengths of emission
- Effective emissivity determined by integrating over wavelengths
- Table 12.1, column “F” provides fraction of total integrated area to a given wavelength
The spectral, hemispherical emissivity of tungsten may be approximated by the distribution given below. What is the total hemispherical emissivity when the filament temperature is 2900 K.
Radiation Transfer Types

• Emission \( (E) \)
  – Associated with energy transfer due to surface temperature

• Irradiation \( (G) \)
  – Radiation incident onto a surface
  – Irradiation can have three fates:
    • Absorption by the surface
      \[ \alpha = \text{absorptivity} = \text{fraction of } G \text{ absorbed} \]
    • Reflection by the surface
      \[ \rho = \text{reflectivity} = \text{fraction of } G \text{ reflected} \]
    • Transmission through the material
      \[ \tau = \text{transmissivity} = \text{fraction transmitted} \]

Irradiation onto a Surface

• Irradiation can have three fates:
  – Absorption by the surface
    \[ \alpha = \text{absorptivity} = \text{fraction of } G \text{ absorbed} \]
  – Reflection by the surface
    \[ \rho = \text{reflectivity} = \text{fraction of } G \text{ reflected} \]
  – Transmission through the material
    \[ \tau = \text{transmissivity} = \text{fraction transmitted} \]

• Sum of \( \alpha + \rho + \tau = 1 \)
Radiosity ($J$)

- Total radiation leaving a surface.
- Sum of emission plus reflected portion of irradiation.

View Factors

- Fraction of radiation from surface $i$ that is captured by surface $j$

  - Summation rule: $\sum_{j=1}^{N} F_{ij} = 1$

  - Reciprocity: $A_i F_{ij} = A_j F_{ji}$
### Table 13.1  View Factors for Two-Dimensional Geometries [1]

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel Plates with Midlines Connected by Perpendicular</td>
<td>( F_g = \left[ (W_1 + W_2)^2 + 4\right]^{1/2} - \left[ (W_1 - W_2)^2 + 4\right]^{1/2} )</td>
</tr>
<tr>
<td></td>
<td>( W_1 = w_1/L, \ W_2 = w_2/L )</td>
</tr>
<tr>
<td>Inclined Parallel Plates of Equal Width and a Common Edge</td>
<td>( F_g = 1 - \sin \left( \frac{\alpha}{2} \right) )</td>
</tr>
<tr>
<td>Perpendicular Plates with a Common Edge</td>
<td>( F_g = \frac{1 + \left( w_1/w_2 \right)^2 - \left[ 1 + \left( w_1/w_2 \right)^2 \right]^{1/2}}{2} )</td>
</tr>
<tr>
<td>Three-Sided Enclosure</td>
<td>( F_g = \frac{w_1 + w_2 - w_3}{2w_1} )</td>
</tr>
</tbody>
</table>

### Table 13.1  Continued

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel Cylinders of Different Radii</td>
<td>( F_g = \frac{1}{2\pi} \left[ 1 + \left( C^2 - (R + 1)^2 \right)^{1/2} \right. )</td>
</tr>
<tr>
<td></td>
<td>\left. - \left( C^2 - (R - 1)^2 \right)^{1/2} + (R - 1) \cos^{-1} \left( \frac{R}{C} \right) - \left( \frac{1}{C} \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>\left. - (R + 1) \cos^{-1} \left( \frac{R}{C} + \left( \frac{1}{C} \right) \right) \right] )</td>
</tr>
<tr>
<td></td>
<td>( R = r/r_o, \ S = s/s_o )</td>
</tr>
<tr>
<td></td>
<td>( C = 1 + R + S )</td>
</tr>
<tr>
<td>Cylinder and Parallel Rectangle</td>
<td>( F_g = \frac{r}{2} - \frac{r}{2} - \left[ \tan^{-1} \frac{h_1}{L} - \tan^{-1} \frac{h_2}{L} \right] )</td>
</tr>
<tr>
<td>Infinite Plane and Row of Cylinders</td>
<td>( F_g = 1 - \left[ 1 - \left( \frac{D}{D_o} \right)^2 \right]^{1/2} )</td>
</tr>
<tr>
<td></td>
<td>( + \frac{D}{2} \tan^{-1} \left( \frac{L^2 - (D_o^2 / 4)}{L^2} \right) )</td>
</tr>
</tbody>
</table>
### Table 13.2 View Factors for Three-Dimensional Geometries [4]

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Relation</th>
</tr>
</thead>
</table>
| **Aligned Parallel Rectangles**  
(Figure 13.4) | \[ X = x/L, \overline{Y} = y/L \]  
\[
F_j = \frac{2}{\pi \overline{X} \overline{Y}} \left[ \ln \left( \frac{(1 + \overline{X}^2)(1 + \overline{Y}^2)}{1 + \overline{X}^2 + \overline{Y}^2} \right) \right]^{1/2} \\
+ \overline{X} (1 + \overline{Y}^2)^{1/2} \tan^{-1} \left( \overline{Y} \left( 1 + \overline{X}^2 \right)^{1/2} \right) \\
+ \overline{Y} (1 + \overline{X}^2)^{1/2} \tan^{-1} \left( \overline{X} \left( 1 + \overline{Y}^2 \right)^{1/2} \right) \] |
| **Coaxial Parallel Disks**  
(Figure 13.5) | \[ R_i = r_i/L, R_j = r_j/L \]  
\[
S = 1 + \frac{R_i^2}{R_j^2} \\
F_d = \frac{1}{2} \left( S - [S^2 - 4r_i^2 r_j^2]^{1/2} \right) \] |
| **Perpendicular Rectangles**  
with a Common Edge  
(Figure 13.6) | \[ H = Zx, W = Yx \]  
\[
F_d = \frac{1}{\pi W} \left( \frac{W \tan^{-1} \frac{1}{W} + H \tan^{-1} \frac{1}{H}}{H^2 + W^2} \right) \\
- \left( \frac{H^2 + W^2}{H^2 + W^2} \right)^{1/2} \left( 1 + W^2 \right) \left( 1 + H^2 \right) \] \\
+ \frac{1}{4} \left( \left( 1 + W^2 \right) \left( 1 + H^2 \right) \right) \left( 1 + W^2 + H^2 \right) \left( 1 + W^2 \right) \left( 1 + H^2 \right) \left( 1 + W^2 \right) \left( 1 + H^2 \right) \] \\
\[
\times \left( \frac{H^2 + H^2 + W^2}{H^2 + W^2} \right)^{1/2} \left( 1 + W^2 \right) \left( 1 + H^2 \right) \left( 1 + W^2 \right) \left( 1 + H^2 \right) \left( 1 + W^2 \right) \] |

---

**Figure 13.4** View factor for aligned parallel rectangles.
Figure 13.5  View factor for coaxial parallel disks.

Figure 13.6  View factor for perpendicular rectangles with a common edge.
Review: Radiation between Surfaces

Review: Two-Surface Enclosure
**Table 13.3** Special Diffuse, Gray, Two-Surface Enclosures

<table>
<thead>
<tr>
<th>Type</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Large (Infinite) Parallel Planes</td>
<td>[ q_{12} = \frac{A_1}{A_2} \left( T_1^4 - T_2^4 \right) ]</td>
</tr>
<tr>
<td>Long (Infinite) Concentric Cylinders</td>
<td>[ q_{12} = \frac{A_1}{A_2} \left( T_1^4 - T_2^4 \right) ]</td>
</tr>
<tr>
<td>Concentric Spheres</td>
<td>[ q_{12} = \frac{A_1}{A_2} \left( T_1^4 - T_2^4 \right) ]</td>
</tr>
<tr>
<td>Small Convex Object in a Large Cavity</td>
<td>[ q_{12} = \frac{A_1}{A_2} \left( T_1^4 - T_2^4 \right) ]</td>
</tr>
</tbody>
</table>

---

**Radiation Shield**

Diagram of radiation shield setup with equations:

\[ q_1 = \frac{1 - \varepsilon_1}{\varepsilon_1 A_1} \]
\[ q_{13} = \frac{1}{A_1 F_{13}} \]
\[ q_{32} = \frac{1 - \varepsilon_{3,1} A_3}{\varepsilon_{3,1} A_3} \]
\[ q_2 = \frac{1}{A_2 F_{32}} \]
\[ q_{32} = \frac{1 - \varepsilon_{3,2} A_3}{\varepsilon_{3,2} A_3} \]
Reradiating Surface

"Direct Method" for Solving Networks

- Useful for systems with >2 surfaces
- Balance radiant energy around each surface node $i$:

$$\frac{E_{bi} - J_i}{(1 - \varepsilon_i) / \varepsilon_i A_i} = \sum_{j=1}^{N} \frac{J_j - J_i}{A_i F_{ij}}^{-1}$$

- Solve system of equations

**Figure 13.12** A three-surface enclosure with one surface reradiating. (a) Schematic. (b) Network representation.
Multimode Heat Transfer

Radiation with Participating Media
(Gaseous Emission and Absorption)

- Gas radiation
  - Nonpolar gases (O₂, N₂) neither emit nor absorb radiation
  - Polar gases (CO₂, H₂O, hydrocarbons) do

- In most cases, contribution of gas to radiation can be safely neglected

- Exception:
Emissivity of Water Vapor

**Figure 13.15** Emissivity of water vapor in a mixture with nonradiating gases at 1-atm total pressure and of hemispherical shape [13]. Used with permission.

Emissivity of Carbon Dioxide

**Figure 13.17** Emissivity of carbon dioxide in a mixture with nonradiating gases at 1-atm total pressure and of hemispherical shape [13]. Used with permission.
**Pressure Correction**

**H₂O**

\[
\frac{P}{P + \frac{p}{2}} (\text{atm})
\]

**CO₂**

\[
\frac{P}{P + \frac{p}{2}} (\text{atm})
\]

**Figure 13.18** Correction factor for obtaining carbon dioxide emissivities at pressures other than 1 atm \((P_{\text{atm}} = \epsilon_{g_{\text{atm}}})\) [13]. Used with permission.

**H₂O + CO₂ Correction**

\[
\epsilon_g = \epsilon_w + \epsilon_c - \Delta \epsilon
\]

**Figure 13.19** Correction factor associated with mixtures of water vapor and carbon dioxide [13]. Used with permission.
## Gas Radiation - Geometries

### Table 13.4 Mean Beam Lengths $L_e$ for Various Gas Geometries

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Characteristic Length</th>
<th>$L_e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere (radiation to surface)</td>
<td>Diameter ($D$)</td>
<td>$0.65D$</td>
</tr>
<tr>
<td>Infinite circular cylinder (radiation to curved surface)</td>
<td>Diameter ($D$)</td>
<td>$0.95D$</td>
</tr>
<tr>
<td>Semi-infinite circular cylinder (radiation to base)</td>
<td>Diameter ($D$)</td>
<td>$0.65D$</td>
</tr>
<tr>
<td>Circular cylinder of equal height and diameter (radiation to entire surface)</td>
<td>Diameter ($D$)</td>
<td>$0.60D$</td>
</tr>
<tr>
<td>Infinite parallel planes (radiation to planes)</td>
<td>Spacing between planes ($L$)</td>
<td>$1.80L$</td>
</tr>
<tr>
<td>Cube (radiation to any surface)</td>
<td>Side ($L$)</td>
<td>$0.66L$</td>
</tr>
<tr>
<td>Arbitrary shape of volume $V$ (radiation to surface of area $A$)</td>
<td>Volume to area ratio ($V/A$)</td>
<td>$3.6V/A$</td>
</tr>
</tbody>
</table>