External Flow – Cylinders and Spheres
Sections 7.4-7.5

Reminders…

• No school next week

• Project outline due today
  – Can turn in to basket in ChE office as late as 4:30 pm
  – See Oct 3 lecture slides for suggestion...

• Homework #6 due Friday by 4:00 pm
  – Help session today at 4:30 pm in MEB 2325

• Homework #7 due Friday Oct. 24
  – Help session Wednesday, Oct. 22 4:30 pm

• Heat exchanger lab day Friday Oct. 24

• Bethany teaches Mon and Wed after fall break
Flow Correlation Summary (Table 7.7 in 7th edition of book)

<table>
<thead>
<tr>
<th>Correlation</th>
<th>Geometry</th>
<th>Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta = 5xRe_x^{-1/2} )</td>
<td>(7.17)</td>
<td>Flat plate, Laminar, ( T_f )</td>
</tr>
<tr>
<td>( C_{tx} = 0.664Re_x^{1/2} )</td>
<td>(7.18)</td>
<td>Flat plate, Laminar, local, ( T_f )</td>
</tr>
<tr>
<td>( Nu_x = 0.332Re_x^{1/2}Pr^{1/3} )</td>
<td>(7.21)</td>
<td>Flat plate, Laminar, local, ( T_f, Pr \geq 0.6 )</td>
</tr>
<tr>
<td>( \delta = 3Pr^{-1/3} )</td>
<td>(7.22)</td>
<td>Flat plate, Laminar, ( T_f )</td>
</tr>
<tr>
<td>( C_{tx} = 1.328Re_x^{1/2} )</td>
<td>(7.24)</td>
<td>Flat plate, Laminar, average, ( T_f )</td>
</tr>
<tr>
<td>( Nu_x = 0.664Re_x^{1/2}Pr^{1/3} )</td>
<td>(7.25)</td>
<td>Flat plate, Laminar, average, ( T_f, Pr \geq 0.6 )</td>
</tr>
<tr>
<td>( Nu_x = 0.565PrRe_x^{1/2} )</td>
<td>(7.26)</td>
<td>Flat plate, Laminar, local, ( T_f, Pr \leq 0.05, Pr_x \geq 100 )</td>
</tr>
<tr>
<td>( C_{tx} = 0.0592Re_x^{-1/5} )</td>
<td>(7.28)</td>
<td>Flat plate, Turbulent, local, ( T_f, Re_x \leq 10^8 )</td>
</tr>
<tr>
<td>( \delta = 0.37Re_x^{-1/3} )</td>
<td>(7.29)</td>
<td>Flat plate, Turbulent, ( T_f, Re_x \leq 10^8 )</td>
</tr>
<tr>
<td>( Nu_x = 0.0296Re_x^{0.3}Pr^{1/3} )</td>
<td>(7.30)</td>
<td>Flat plate, Turbulent, local, ( T_f, Re_x \leq 10^8, 0.6 \leq Pr \leq 60 )</td>
</tr>
<tr>
<td>( C_{tx} = 0.074Re_x^{-1.15} - 1742Re_x^{-1} )</td>
<td>(7.33)</td>
<td>Flat plate, Mixed, average, ( T_f, Re_x,c = 5 \times 10^5, Re_x \leq 10^8 )</td>
</tr>
<tr>
<td>( Nu_L = (0.037Re_x^{0.4} - 871)Pr^{1/3} )</td>
<td>(7.31)</td>
<td>Flat plate, Mixed, average, ( T_f, Re_x,c = 5 \times 10^5, Re_x \leq 10^8, 0.6 \leq Pr \leq 60 )</td>
</tr>
</tbody>
</table>
Example – Book Problem 7.13

Consider a flat plate subject to parallel flow (top and bottom) characterized by $u_{\infty} = 5 \text{ m/s}$ and $T_{\infty} = 20^\circ\text{C}$. Determine the average convective heat transfer coefficient and convective heat transfer rate associated with a $L = 2 \text{ m}$ long, $w = 2 \text{ m}$ plate for surface temperatures of 50 and 80$^\circ\text{C}$.

Example – Book Problem 7.43

A flat solar collector is at 15$^\circ\text{C}$, while ambient air at 10$^\circ\text{C}$ is in parallel flow over the plate with $u_{\infty} = 2 \text{ m/s}$.

(a) What is the rate of convective heat loss from the plate?
(b) Same as (a), but the plate is 2 m from the leading edge.
Figure 7.5 Boundary layer formation and separation on a circular cylinder in cross flow.

Figure 7.6 Velocity profile associated with separation on a circular cylinder in cross flow.
Laminar vs. Turbulent Flow

Figure 7.7 The effect of turbulence on separation.

Convective Heat Transfer

Figure 7.9 Local Nusselt number for airflow normal to a circular cylinder.
Heat Transfer Correlations for Cylinders

Kays

\[ \text{Nu}_{D(0=0)} = 1.15 \text{Re}_{D}^{1/2} \text{Pr}^{1/3} \]  
(Eq. 7.51)

→ Local heat transfer at stagnation point ONLY
→ Properties evaluated at \( T_{\text{film}} \)

Hilpert

\[ \text{Nu}_{D} = \frac{\bar{h}D}{k} = C \text{Re}_{D}^{m} \text{Pr}^{1/3} \]  
(Eq. 7.52)

→ Properties evaluated at \( T_{\text{film}} \)
→ \( C \) and \( m \) from Table 7.2
→ Also can be used for non-circular cylinders using Table 7.3

<table>
<thead>
<tr>
<th>( Re_{D} )</th>
<th>( C )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4–4</td>
<td>0.989</td>
<td>0.330</td>
</tr>
<tr>
<td>4–40</td>
<td>0.911</td>
<td>0.385</td>
</tr>
<tr>
<td>40–4000</td>
<td>0.683</td>
<td>0.466</td>
</tr>
<tr>
<td>4000–40,000</td>
<td>0.193</td>
<td>0.618</td>
</tr>
<tr>
<td>40,000–400,000</td>
<td>0.027</td>
<td>0.805</td>
</tr>
</tbody>
</table>
More Correlations for Cylinders…

Zukauskas:

\[
\overline{\text{Nu}}_D = \frac{\overline{h}D}{k} = C \text{Re}_D^m \text{Pr}^n \left( \frac{\text{Pr}}{\text{Pr}_s} \right)^{1/4}
\]

(Eq. 7.53)

- Properties evaluated at \( T_\infty \) except \( \text{Pr}_s \)
- \( C \) and \( m \) from Table 7.4
- \( n = 0.37 \) if \( \text{Pr} < 10 \), otherwise \( n = 0.36 \)

<table>
<thead>
<tr>
<th>( Re_D )</th>
<th>( C )</th>
<th>( m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–40</td>
<td>0.75</td>
<td>0.4</td>
</tr>
<tr>
<td>40–1000</td>
<td>0.51</td>
<td>0.5</td>
</tr>
<tr>
<td>( 10^3 \times 10^5 )</td>
<td>0.26</td>
<td>0.6</td>
</tr>
<tr>
<td>( 2 \times 10^5 \times 10^6 )</td>
<td>0.076</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Churchill and Bernstein:

\[
\overline{\text{Nu}}_D = 0.3 + \frac{0.62 \text{Re}_D^{1/2} \text{Pr}^{1/3} \left[ 1 + \left( \frac{0.4}{\text{Pr}} \right)^{2/3} \right]^{7/4}}{1 + \left( \frac{\text{Re}_D}{282,000} \right)^{5/8}}^{4/5}
\]

(Eq. 7.54)

- Properties evaluated at \( T_{\text{film}} \)
- Does not require looking up in any table

A Few Notes…

- All these expressions are empirical. No single expression is "correct"
- Best accuracy is ~20%
- Book focuses on 7.53 and 7.54
- For all expressions, check the ranges of \( \text{Pr}, \text{Re}, \text{etc.} \) for which the expression is valid
Heat Transfer Correlations for Spheres

Whitaker

\[ \overline{\text{Nu}}_D = 2 + \left(0.4 \text{Re}^{1/2} + 0.06 \text{Re}^{2/3}_D \right) \text{Pr}^{0.4} \left( \frac{\mu}{\mu_s} \right)^{1/4} \]  \hspace{1cm} (Eq. 7.56)

\[ \rightarrow \text{Properties evaluated at } T_\infty \text{ except for } \mu_s \]

Ranz and Marshall (for falling liquid droplets)

\[ \overline{\text{Nu}}_D = 2 + 0.6 \text{Re}_D^{1/2} \text{ Pr}^{1/3} \]  \hspace{1cm} (Eq. 7.57)

\[ \rightarrow \text{For falling liquid droplets ONLY} \]

\[ \rightarrow \text{Properties evaluated at } T_{\text{film}} \]