CH EN 3453 – Heat Transfer

The Flat Plate External Flow

Sections 7.1 to 7.3

Reminders…

- Homework #6 due Friday
  - Error in problem #1: "convection" should be "concentration"
- Help session Wednesday, Oct. 8 at 4:30 pm MEB 2325
- Project report outline due this Wed, Oct. 5 at 4:00 pm
  - Five main headings (e.g.: 1. Introduction)
  - Subheadings (e.g.: 1.1 Purpose of Heat Exchangers)
  - At least 1 bullet point beneath each subheading indicating what will be included in that section
  - Number the headings
- Fall break next week

Boundary Layer Similarity

- Similarity parameters allow results for a surface experiencing one set of conditions to be applied to geometrically similar surfaces experiencing different conditions
- Independent variables:

Geometric:	Size (*L*), Location (*x*,*y*)
Hydrodynamic:	Velocity (*V*)
Fluid properties:	ρ, μ, c_p, k

- Dependent boundary layer variables:

$u = f(x,y,L,V,\rho,\mu)$

$\tau_s = f(x,L,V,\rho,\mu)$

$T = f(x,y,L,V,\rho,\mu,c_p,k)$

$h = f(x,L,V,\rho,\mu,c_p,k)$

Boundary Layer Similarity, cont.

- Similarity parameters can be applied by non-dimensionalizing the momentum and energy equations
- Dimensionless forms of the independent variables:

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L} \qquad T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

$$u^* \equiv \frac{u}{V} \qquad v^* \equiv \frac{v}{V}$$

- Normalized momentum and energy equations (Table 6.1)

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{\mathrm{Re}_L \, \mathrm{Pr}} \frac{\partial^2 T^*}{\partial y^{*2}}$$

Momentum and Energy Relations

Local velocity: $u^* = f(x^*, y^*, \mathrm{Re}_L)$

Coefficient of friction: $C_f \equiv \dfrac{\tau_s}{\rho V^2/2} = \dfrac{2}{\mathrm{Re}_L} f(x^*, \mathrm{Re}_L)$

Local temperature: $T^* = f(x^*, y^*, \mathrm{Re}_L, \mathrm{Pr})$

Local convection coefficient: $Nu \equiv \dfrac{hL}{k_f} = f(x^*, \mathrm{Re}_L, \mathrm{Pr})$

→ The specific relations between these are *empirical*, based on experimental observations

Six Steps to Solving…

1. Determine flow geometry
2. Determine appropriate fluid temperature (e.g., T_{film}) and evaluate fluid properties
3. Consider fluid B (Applies only to mass transfer problems.)
4. Calculate Reynolds number to determine if laminar or turbulent flow
5. Decide whether a local or average coefficient is required
6. Select appropriate correlation

Dimensionless Groups

Reynolds number (Re_L)	$\frac{VL}{\nu}$	Ratio of the inertia and viscous forces.
Prandtl number (Pr)	$\frac{c_p\mu}{k} = \frac{\nu}{\alpha}$	Ratio of the momentum and thermal diffusivities.
Nusselt number (Nu_L)	$\frac{hL}{k_f}$	Ratio of convection to pure conduction heat transfer.
Sherwood number (Sh_L)	$\frac{h_m L}{D_{AB}}$	Dimensionless concentration gradient at the surface.
Schmidt number (Sc)	$\frac{\nu}{D_{AB}}$	Ratio of the momentum and mass diffusivities.
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho V^2/2}$	Dimensionless surface shear stress.

Prandtl number (Pr) $\frac{c_p\mu}{k} = \frac{\nu}{\alpha}$ Ratio of the momentum and thermal diffusivities.

Unheated Starting Length

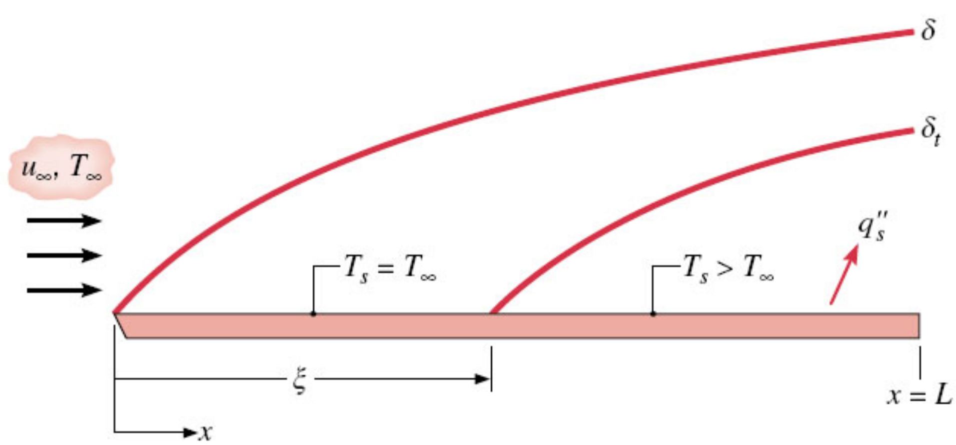

FIGURE 7.4 Flat plate in parallel flow with unheated starting length.

Flow Correlation Summary

Table 7.9 Summary of convection heat transfer correlations for external flow[a]

Correlation		Geometry	Conditions[b]
$\delta = 5x\, Re_x^{-1/2}$	(7.17)	Flat plate	Laminar, T_f
$C_{f,x} = 0.664 Re_x^{-1/2}$	(7.18)	Flat plate	Laminar, local, T_f
$Nu_x = 0.332 Re_x^{1/2}\, Pr^{1/3}$	(7.21)	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$
$\delta_t = \delta Pr^{-1/3}$	(7.22)	Flat plate	Laminar, T_f
$\overline{C}_{f,x} = 1.328 Re_x^{-1/2}$	(7.24)	Flat plate	Laminar, average, T_f
$\overline{Nu}_x = 0.664 Re_x^{1/2}\, Pr^{1/3}$	(7.25)	Flat plate	Laminar, average, T_f, $Pr \gtrsim 0.6$
$Nu_x = 0.565 Pe_x^{1/2}$	(7.26)	Flat plate	Laminar, local, T_f, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$
$C_{f,x} = 0.0592 Re_x^{-1/5}$	(7.28)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\, Re_x^{-1/5}$	(7.29)	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296 Re_x^{4/5}\, Pr^{1/3}$	(7.30)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{f,L} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$	(7.33)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$	(7.31)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$

Example – Book Problem 7.2

Engine oil at 100°C and a velocity of 0.1 m/s flows over both surfaces of a 1 meter long flat plate maintained at 20°C. Determine:

(a) The thermal boundary layer thickness at the trailing edge

(b) The local heat flux at the trailing edge

(c) Heat transfer per unit width of the plane

Example – Book Problem 7.11

Consider a flat plate subject to parallel flow (top and bottom) characterized by u_∞ = 5 m/s and T_∞ = 20°C. Determine the average convective heat transfer coefficient and convective heat transfer rate associated with a L = 2 m long, w = 2 m plate for surface temperatures of 50 and 80°C.

Example – Book Problem 7.37

A flat solar collector is at 15°C, while ambient air at 10°C is in parallel flow over the plate with u_∞ = 2 m/s.

(a) What is the rate of convective heat loss from the plate?

(b) Same as (a), but the plate is 2 m from the leading edge