CH EN 3453 – Heat Transfer

Shape Factors

Section 4.3

Reminders…

- Homework #4 due Friday
  - Help session Wednesday 4:30 pm
- Engineering Career Fair September 23
  - Internship (and job) opportunities!
  - ca. 220 days until end of spring term finals week
  - Most of you have about 600 days until graduation

Fin Efficiencies

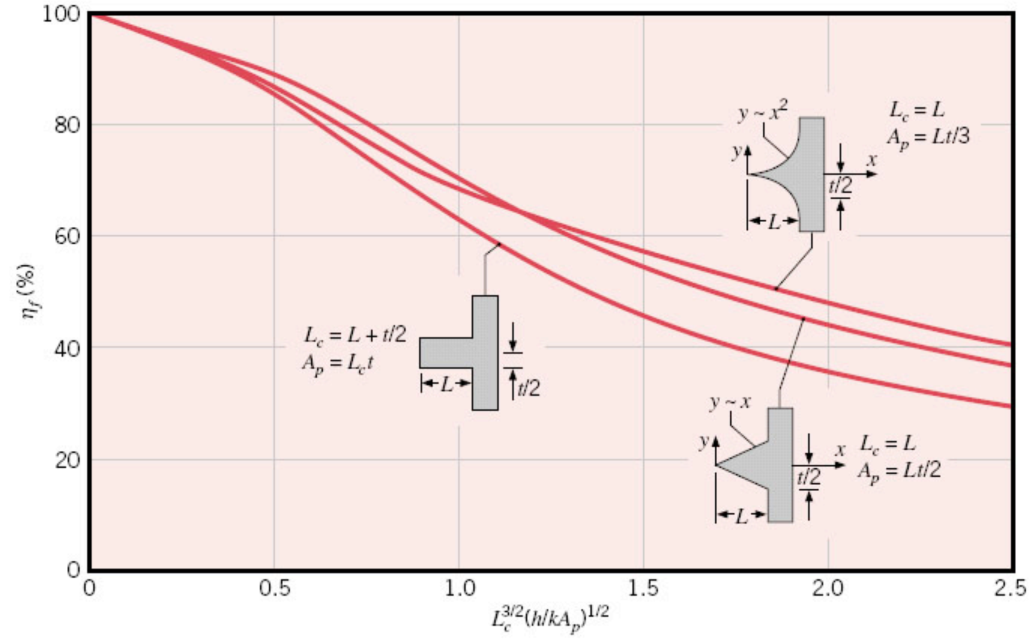


FIGURE 3.18 Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

Fin Efficiencies

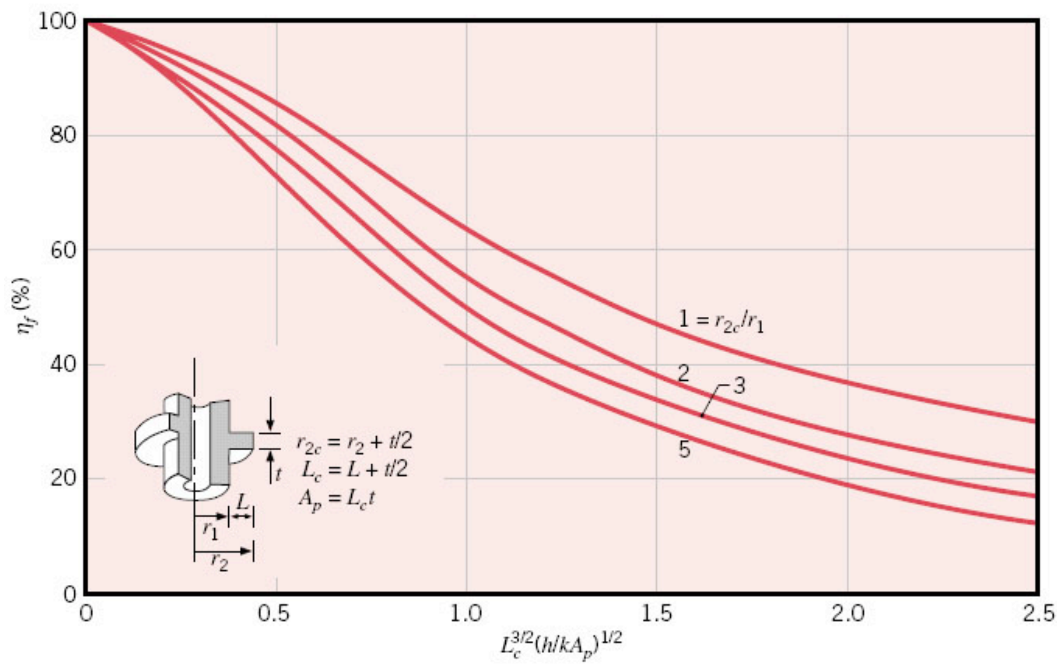
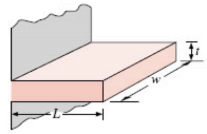
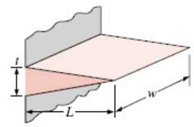
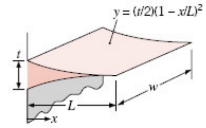


FIGURE 3.19 Efficiency of annular fins of rectangular profile.

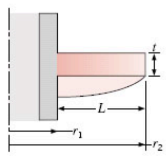
Fin Efficiencies

TABLE 3.5 Efficiency of common fin shapes

Straight Fins

Rectangular[a] $A_f = 2wL_c$ $L_c = L + (t/2)$ $A_p = tL$		$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.89)
Triangular[a] $A_f = 2w[L^2 + (t/2)^2]^{1/2}$ $A_p = (t/2)L$		$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$	(3.93)
Parabolic[a] $A_f = w[C_1 L + (L^2/t)\ln(t/L + C_1)]$ $C_1 = [1 + (t/L)^2]^{1/2}$ $A_p = (t/3)L$		$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$	(3.94)

Circular Fin

Rectangular[a] $A_f = 2\pi(r_{2c}^2 - r_1^2)$ $r_{2c} = r_2 + (t/2)$ $V = \pi(r_2^2 - r_1^2)t$		$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$ $C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$	(3.91)

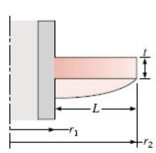
Modified Bessel function of the first kind (Appendix B.5)

Modified Bessel function of the second kind (Appendix B.5)

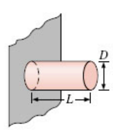
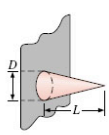
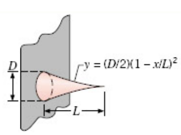
[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

Fin Efficiencies, continued

Circular Fin

Rectangular[a] $A_f = 2\pi(r_{2c}^2 - r_1^2)$ $r_{2c} = r_2 + (t/2)$ $V = \pi(r_2^2 - r_1^2)t$		$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$ $C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$	(3.91)

Pin Fins

Rectangular[b] $A_f = \pi D L_c$ $L_c = L + (D/4)$ $V = (\pi D^2/4)L$		$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.95)
Triangular[b] $A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$ $V = (\pi/12)D^2 L$		$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)}$	(3.96)
Parabolic[b] $A_f = \frac{\pi L^3}{8D}\{C_3 C_4 - \frac{L}{2D}\ln[(2DC_4/L) + C_3]\}$ $C_3 = 1 + 2(D/L)^2$ $C_4 = [1 + (D/L)^2]^{1/2}$ $V = (\pi/20)D^2 L$		$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$	(3.97)

[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

Overall Efficiency

- Definitions
  - η_f efficiency of a *single* fin
  - η_o overall efficiency of a finned surface
  - A_f surface area of a single fin
  - A_b area of base where fins are NOT attached
  - A_t total area $A_f + A_b$
- Overall efficiency of finned surface

$$q_t = N\eta_f h A_f \theta_b + h A_b \theta_b$$

$$= \eta_o h A_t \theta_b$$

where $$\eta_o = 1 - \frac{NA_f}{A_t}(1-\eta_f)$$

Thermal Circuits

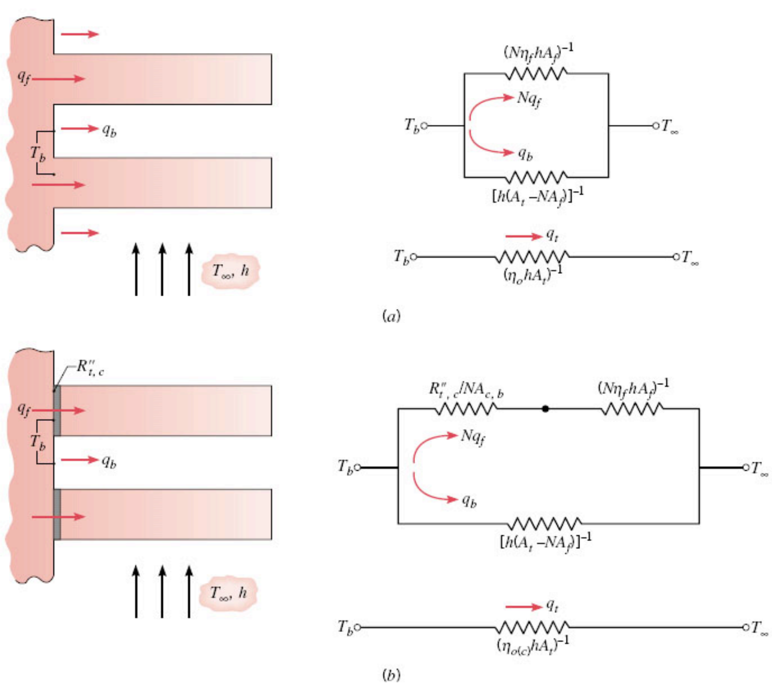


FIGURE 3.21 Fin array and thermal circuit. (*a*) Fins that are integral with the base. (*b*) Fins that are attached to the base.

2-D Heat Flow

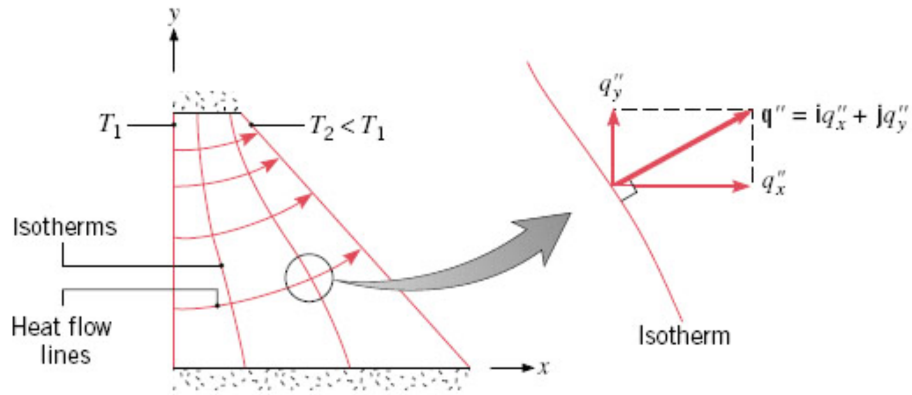
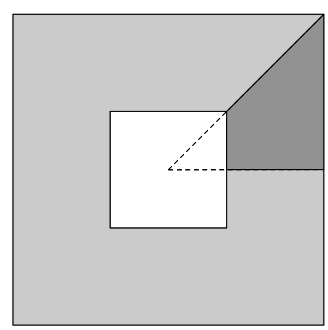


FIGURE 4.1 Two-dimensional conduction.

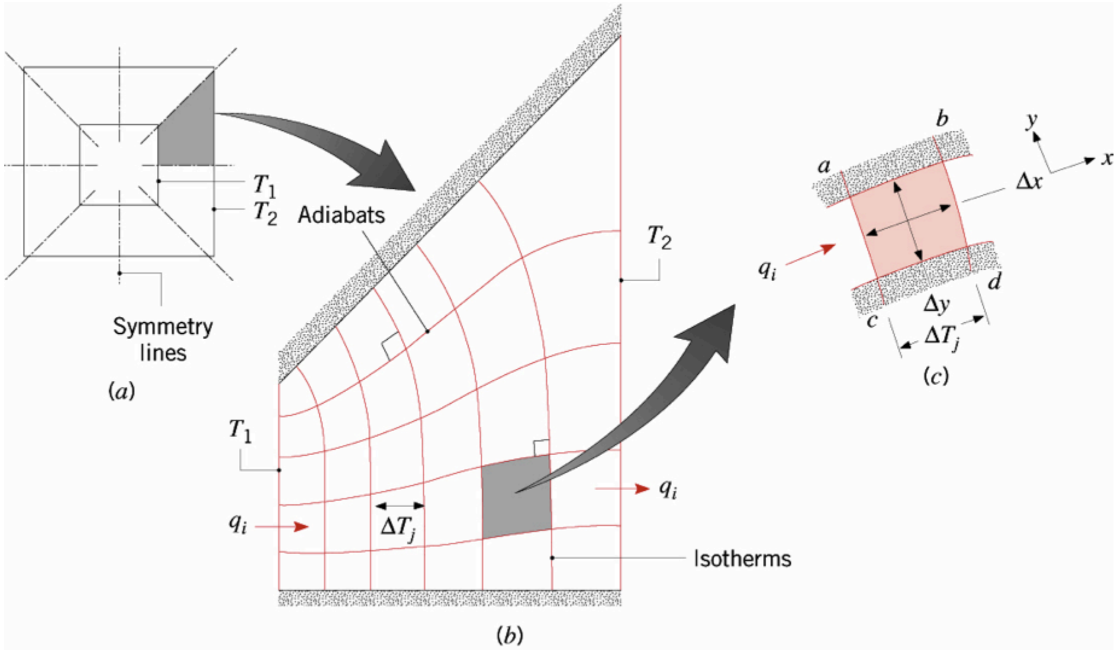
- Heat flow lines ("adiabats") represent how heat "flows."
- There is no heat transfer in a direction perpendicular to heat flow lines
- Isotherms – constant temperature
- Adiabats and isotherms are perpendicular to one another

Graphical Method - Plotting Heat Flux

1. Consider lines of symmetry and choose sub-system if possible.
2. Symmetry lines adiabatic and count as heat flow lines.
3. Identify constant temperature lines at boundaries. Sketch isotherms between the boundaries.
4. Sketch heat flow lines perpendicular to isotherms, attempting to make each cell as square as possible.



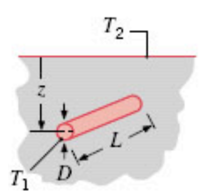
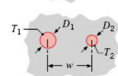
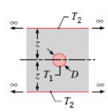
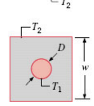
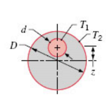
Graphical Solution…



Shape Factors

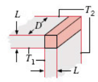
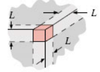
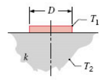
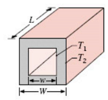
TABLE 4.1 Conduction shape factors and dimensionless conduction heat rates for selected systems.

(*a*) Shape factors [$q = Sk(T_1 - T_2)$]

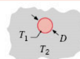
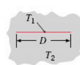
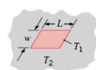
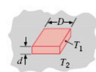
System	Schematic	Restrictions	Shape Factor
Case 1			
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$	$\frac{2\pi L}{\cosh^{-1}(2z/D)}$
		$L \gg D$, $z > 3D/2$	$\frac{2\pi L}{\ln(4z/D)}$
Case 4 Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$, $L \gg w$	$\frac{2\pi L}{\cosh^{-1}\left(\frac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
Case 5 Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$, $L \gg z$	$\frac{2\pi L}{\ln(8z/\pi D)}$
Case 6 Circular cylinder of length L centered in a square solid of equal length		$w > D$, $L \gg w$	$\frac{2\pi L}{\ln(1.08\, w/D)}$
Case 7 Eccentric circular cylinder of length L in a cylinder of equal length		$D > d$, $L \gg D$	$\frac{2\pi L}{\cosh^{-1}\left(\frac{D^2 + d^2 - 4z^2}{2Dd}\right)}$

Shape Factors, Cont.

TABLE 4.1 *Continued*

System	Schematic	Restrictions	Shape Factor
Case 8 Conduction through the edge of adjoining walls		$D > 5L$	$0.54D$
Case 9 Conduction through corner of three walls with a temperature difference ΔT_{1-2} across the walls		$L \ll$ length and width of wall	$0.15L$
Case 10 Disk of diameter D and temperature T_1 on a semi-infinite medium of thermal conductivity k and temperature T_2		None	$2D$
Case 11 Square channel of length L		$\frac{W}{w} < 1.4$ $\frac{W}{w} > 1.4$ $L \gg W$	$\frac{2\pi L}{0.785 \ln (W/w)}$ $\frac{2\pi L}{0.930 \ln (W/w) - 0.050}$

(*b*) Dimensionless conduction heat rates $[q = q_{ss}^* kA_s(T_1 - T_2)/L_c;\ L_c \equiv (A_s/4\pi)^{1/2}]$

System	Schematic	Active Area, A_s	q_{ss}^*
Case 12 Isothermal sphere of diameter D and temperature T_1 in an infinite medium of temperature T_2		πD^2	1
Case 13 Infinitely thin, isothermal disk of diameter D and temperature T_1 in an infinite medium of temperature T_2		$\frac{\pi D^2}{2}$	$\frac{2\sqrt{2}}{\pi} = 0.900$
Case 14 Infinitely thin rectangle of length L, width w, and temperature T_1 in an infinite medium of temperature T_2		$2wL$	0.932
Case 15 Cuboid shape of height d with a square footprint of width D and temperature T_1 in an infinite medium of temperature T_2		$2D^2 + 4Dd$	d/D — q_{ss}^* 0.1 — 0.943 1.0 — 0.956 2.0 — 0.961 10 — 1.111

Example – Book Problem 4.10

A pipeline, used for the transport of crude oil, is buried in the earth such that its centerline is a distance of 1.5 m below the surface. The pipe has an outer diameter of 0.5 m and is insulated with a layer of cellular glass 100 mm thick. What is the heat loss per unit length of pipe under conditions for which heated oil at 120°C flows through the pipe and the surface of the earth is at a temperature of 0°C?