CH EN 3453 – Heat Transfer

Heat Transfer from Extended Surfaces Part 2

Reminders…

- Homework #3 due today
- Homework #4 due Friday next week
  - Help session Wednesday
- Bethany's office hours changed to Fridays from 3-4 PM in ICC

Four Scenarios for Treating Pin Tips

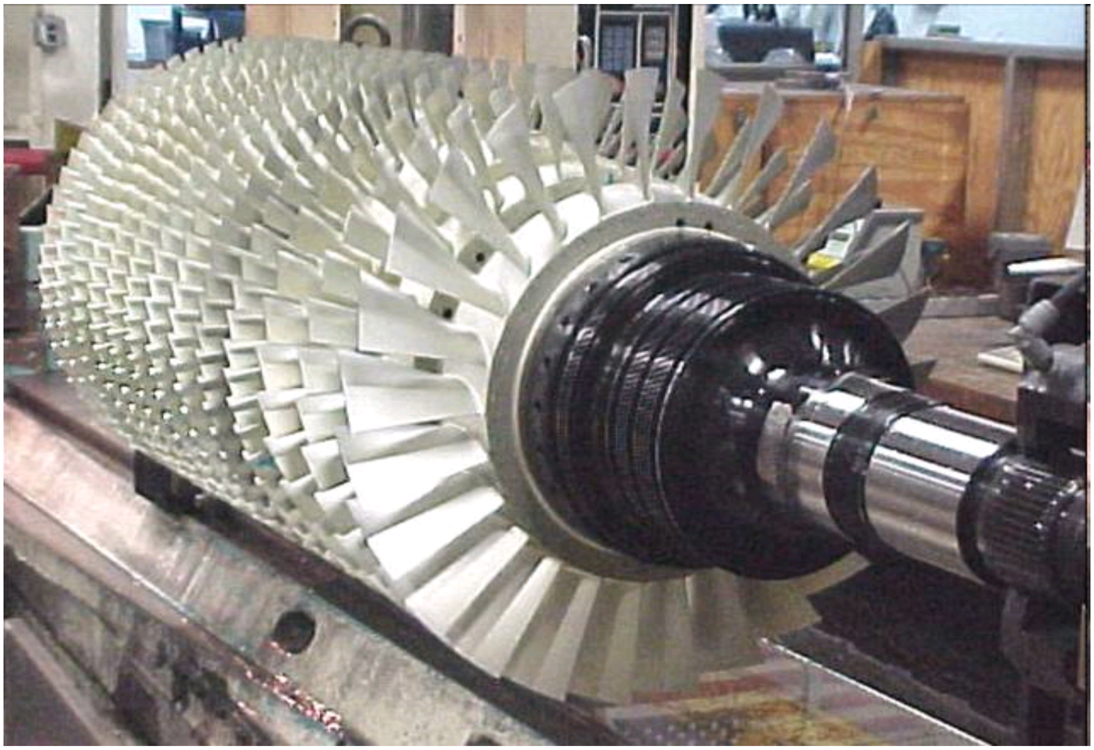
Table 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -k\,d\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M \tanh mL$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx} (3.79)	M (3.80)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

Gas Turbine Blade



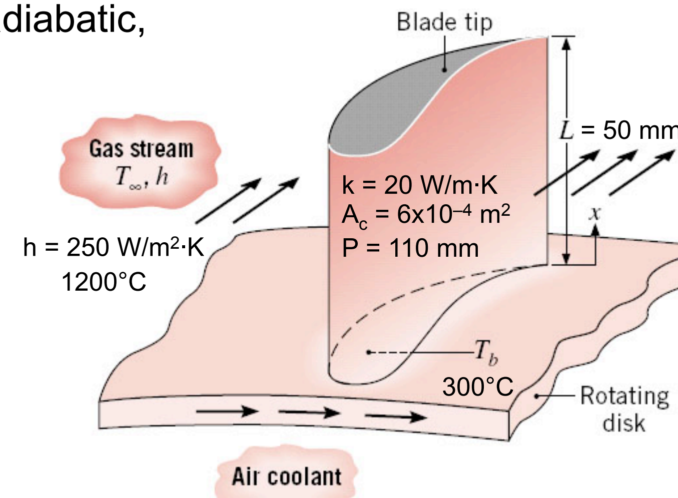
Example – Book Problem 3.116

Turbine blade mounted on proposed air-cooled rotating disc (T_b = 300°C) in a gas turbine with gas stream at T_∞ = 1200°C.

(a) If max allowable blade temperature is 1050°C and blade tip is assumed to be adiabatic, will the air cooling approach work?

(b) What is the rate of heat transfer from blade to coolant?

q_{fin} = 508 Watts



Fin Efficiencies

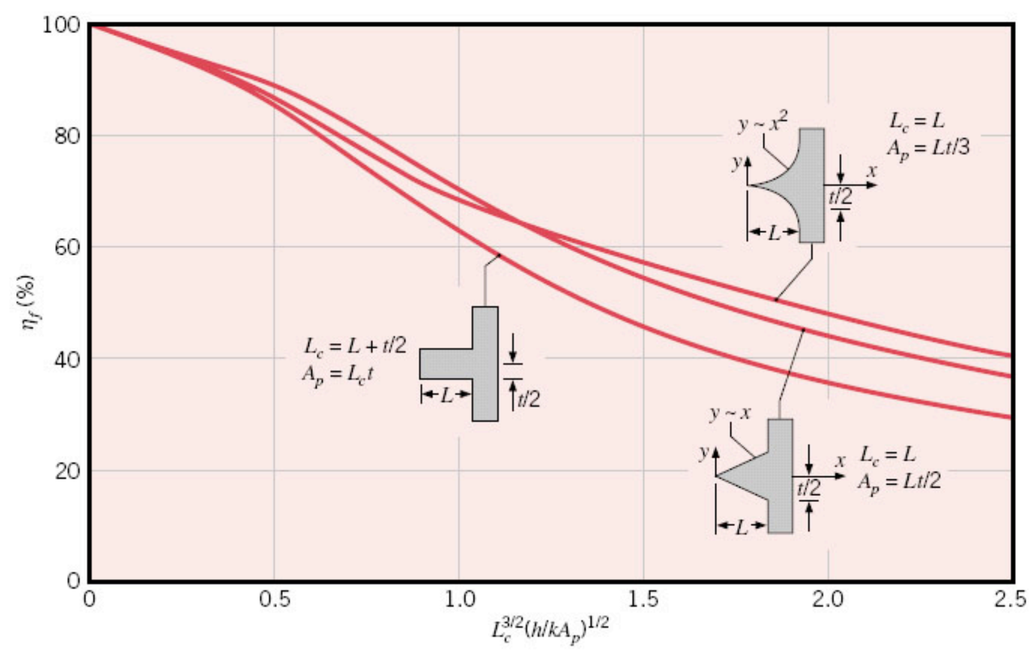


FIGURE 3.18 Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

Fin Efficiencies

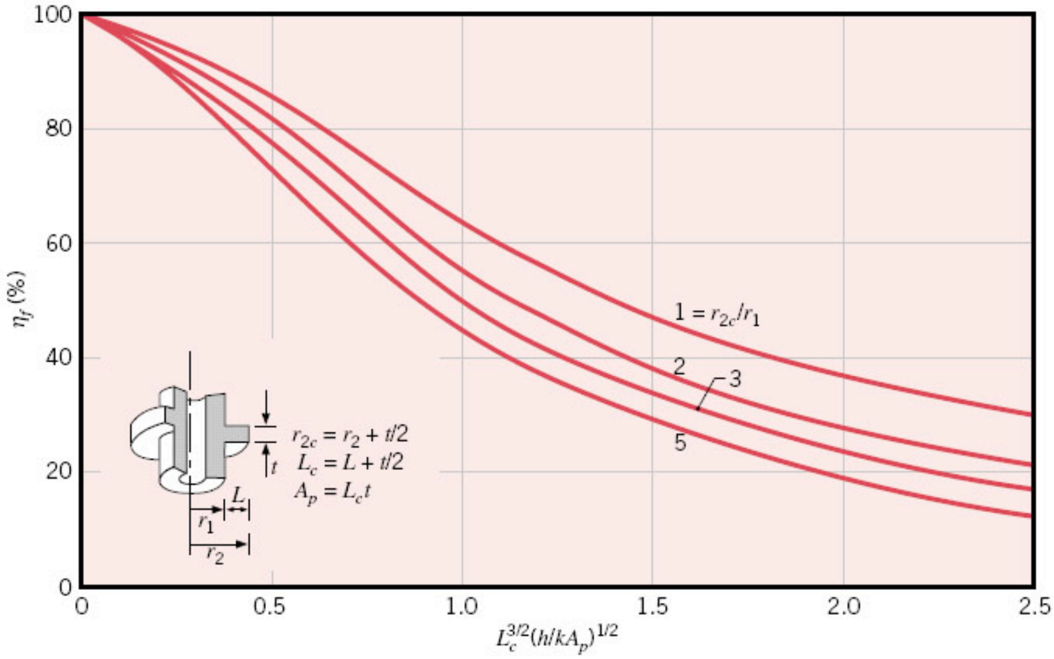
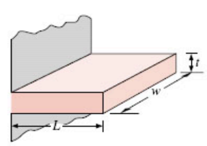
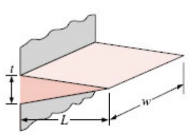
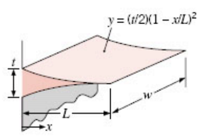
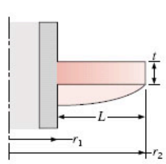


FIGURE 3.19 Efficiency of annular fins of rectangular profile.

Fin Efficiencies

TABLE 3.5 Efficiency of common fin shapes

Straight Fins			
Rectangular[a] $A_f = 2wL_c$ $L_c = L + (t/2)$ $A_p = tL$		$\eta_f = \frac{\tanh mL_c}{mL_c}$	(3.89)
Triangular[a] $A_f = 2w[L^2 + (t/2)^2]^{1/2}$ $A_p = (t/2)L$		$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)}$	(3.93)
Parabolic[a] $A_f = w[C_1 L + (L^2/t)\ln(t/L + C_1)]$ $C_1 = [1 + (t/L)^2]^{1/2}$ $A_p = (t/3)L$		$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1}$	(3.94)
Circular Fin			
Rectangular[a] $A_f = 2\pi(r_{2c}^2 - r_1^2)$ $r_{2c} = r_2 + (t/2)$ $V = \pi(r_2^2 - r_1^2)t$		$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})}$ $C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$	(3.91)

[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

Modified Bessel function of the first kind (Appendix B.5)

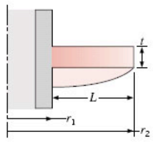
Modified Bessel function of the second kind (Appendix B.5)

Fin Efficiencies, continued

Circular Fin

Rectangular[a]

$A_f = 2\pi(r_{2c}^2 - r_1^2)$
$r_{2c} = r_2 + (t/2)$
$V = \pi(r_2^2 - r_1^2)t$



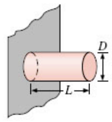
$$\eta_f = C_2 \frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})} \quad (3.91)$$

$$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$$

Pin Fins

Rectangular[b]

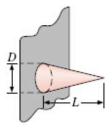
$A_f = \pi D L_c$
$L_c = L + (D/4)$
$V = (\pi D^2/4)L$



$$\eta_f = \frac{\tanh mL_c}{mL_c} \quad (3.95)$$

Triangular[b]

$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$
$V = (\pi/12)D^2L$

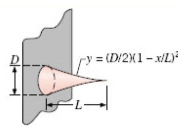


$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} \quad (3.96)$$

Parabolic[b]

$A_f = \frac{\pi L^3}{8D}\{C_3C_4 - \frac{L}{2D}\ln[(2DC_4/L) + C_3]\}$

$C_3 = 1 + 2(D/L)^2$
$C_4 = [1 + (D/L)^2]^{1/2}$
$V = (\pi/20)D^2 L$



$$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1} \quad (3.97)$$

[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

Thermal Circuits

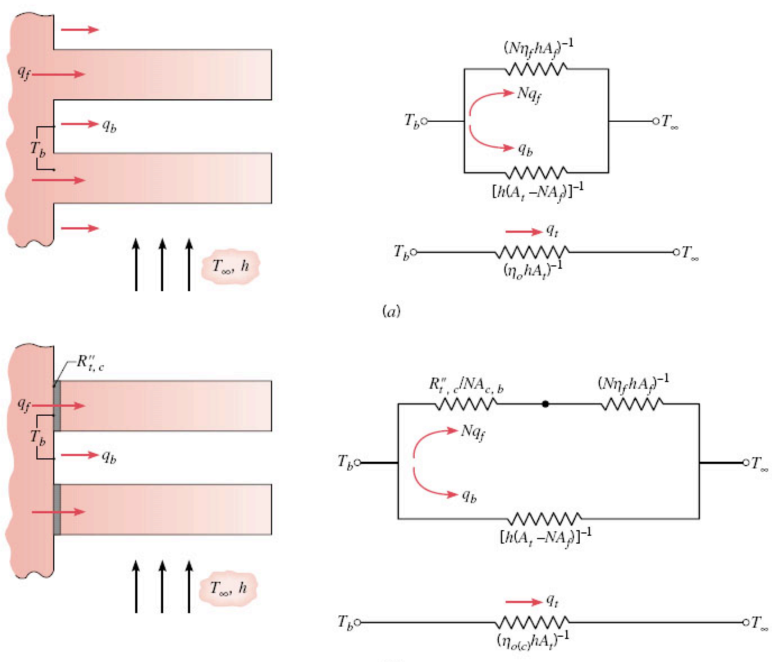


FIGURE 3.21 Fin array and thermal circuit. (*a*) Fins that are integral with the base. (*b*) Fins that are attached to the base.

Example – Finned Pipe

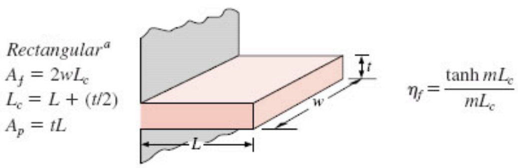
k = 64 W/m·K

h = 12 W/m^2·K

$L_c = L + (t/2) = 0.0175$ m

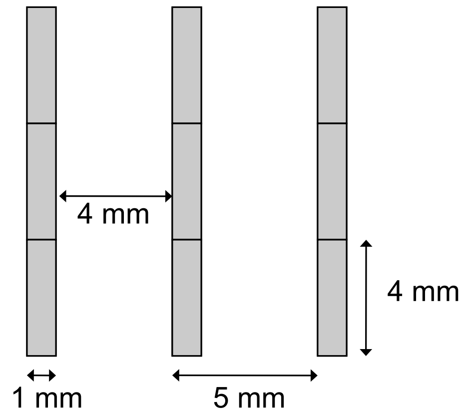
$A_f = 2wL_c = 0.000140$ m^2

$m = (2h/kt)^{0.5} = 19.4$ m^{-1}



$\eta_f = 0.964$

η_o = ?



Pipe OD = 51 mm
ID = 45 mm

Fin L = 17 mm
w = 4 mm
t = 1 mm
pitch = 5 mm