CH EN 3453 – Heat Transfer

# Radial Conduction in Cylinders and Spheres

## Reminders

- Homework #1 graded
  - Pick up in ChE office
- Homework #2 due today at 4:00 PM
  - Turn in to basket in Chem Eng front office
  - Turn in <u>entire</u> assignment at once
- Homework #3 due Friday next week
  - Note the extra credit problem
- Cody teaches Monday and Wednesday
- Career fair Tuesday, Sept. 23, 9AM-3PM
  - http://careers.utah.edu/students/
  - Many companies, both Utah and national
  - Good leads for internships
- Homework questions
  - If your question is more than ~10 words long, it is best to come meet me or one of the TA's face-to-face instead of emailing it to Kevin
  - Look through the homework <u>before</u> the help session and come with questions

# Complex Heat Transfer

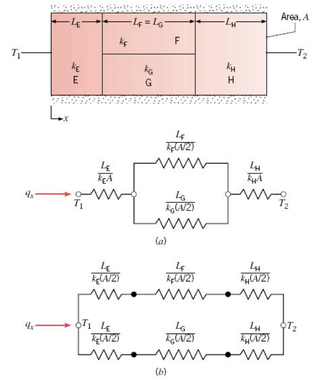



FIGURE 3.3 Equivalent thermal circuits for a series–parallel composite wall.

# Summary of Resistances

- Conduction (flat plane only):

$$R_{cond} = \frac{L}{kA}$$

- Convection:

$$R_{conv} = \frac{1}{hA}$$

- Radiation:

$$R_{rad} = \frac{1}{h_{rad}A} = \frac{1}{\varepsilon\sigma A\left(T_s^2 + T_{sur}^2\right)\left(T_s + T_{sur}\right)}$$

# Contact Resistance

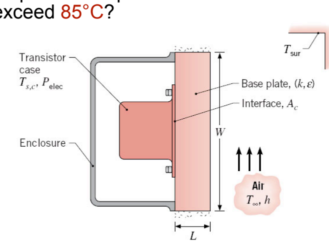
**TABLE 3.1** Thermal contact resistance for (*a*) metallic interfaces under vacuum conditions and (*b*) aluminum interface (10-$\mu$m surface roughness, $10^5$ N/m$^2$) with different interfacial fluids [1]

**Thermal Resistance, $R''_{t,c} \times 10^4$ (m$^2 \cdot$ K/W)**

| (*a*) Vacuum Interface | | | (*b*) Interfacial Fluid | |
|---|---|---|---|---|
| Contact pressure | 100 kN/m$^2$ | 10,000 kN/m$^2$ | Air | 2.75 |
| Stainless steel | 6–25 | 0.7–4.0 | Helium | 1.05 |
| Copper | 1–10 | 0.1–0.5 | Hydrogen | 0.720 |
| Magnesium | 1.5–3.5 | 0.2–0.4 | Silicone oil | 0.525 |
| Aluminum | 1.5–5.0 | 0.2–0.4 | Glycerine | 0.265 |

# Example – Book Problem 3.28

A power transistor is encapsulated in an aluminum case attached to a square aluminum plate ($k$ = 240 W/m·K) which is 6 mm thick and 20 mm square. The transistor and plate are screwed in so the contact pressure is 1 bar and the air-filled interface has a roughness of 10 microns and a contact area of 0.0002 m$^2$. The back side of the plate is exposed to air ($T_\infty = T_{sur}$ = 25°C) and all heat is removed here by convection ($h$ = 4 W/m$^2$·K) and radiation ($\varepsilon$ = 0.9). What is the maximum power dissipation if the case surface temperature should not exceed 85°C?



# Radial Systems: Seventh Grade Geometry

| | Surface Area | Volume |
|---|---|---|
| Cylinder | $A = 2\pi rL$ | $V = \pi r^2 L$ |
| Sphere | $A = 4\pi r^2$ | $V = \frac{4}{3}\pi r^3$ |

# A Cylinder

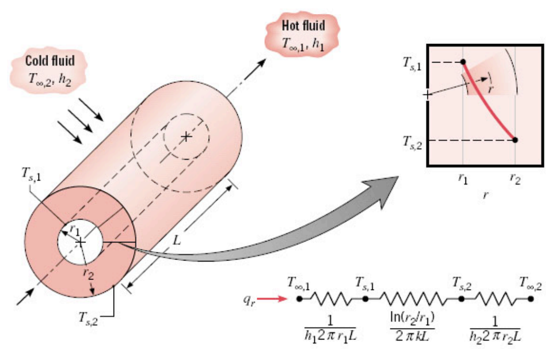



FIGURE 3.6 Hollow cylinder with convective surface conditions.

## Temp. Distribution - Composite Cylinder

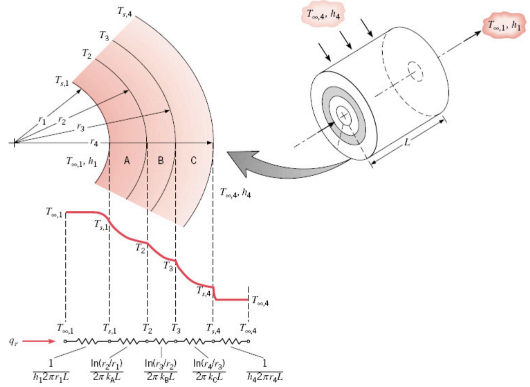


FIGURE 3.7 Temperature distribution for a composite cylindrical wall.

## Conduction through a Sphere

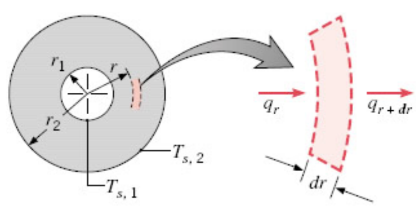


FIGURE 3.8
Conduction in a spherical shell.

# Review of Conduction…

**TABLE 3.3** One-dimensional, steady-state solutions to the heat equation with no generation

| | Plane Wall | Cylindrical Wall[a] | Spherical Wall[a] |
|---|---|---|---|
| Heat equation | $\frac{d^2T}{dx^2} = 0$ | $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$ | $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$ |
| Temperature distribution | $T_{s,1} - \Delta T \frac{x}{L}$ | $T_{s,2} + \Delta T \frac{\ln(r/r_2)}{\ln(r_1/r_2)}$ | $T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$ |
| Heat flux ($q''$) | $k\frac{\Delta T}{L}$ | $\frac{k\,\Delta T}{r\ln(r_2/r_1)}$ | $\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$ |
| Heat rate ($q$) | $kA\frac{\Delta T}{L}$ | $\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$ | $\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$ |
| Thermal resistance ($R_{t,\mathrm{cond}}$) | $\frac{L}{kA}$ | $\frac{\ln(r_2/r_1)}{2\pi Lk}$ | $\frac{(1/r_1)-(1/r_2)}{4\pi k}$ |

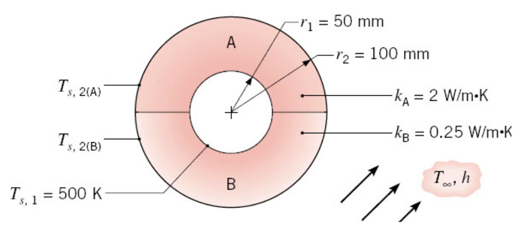
[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

**Page 126 (6th ed) or 143 (7th ed)**

# Example – Book Problem 3.52

Steam flowing through a long pipe maintains the inner pipe wall temperature at 500 K. The pipe is covered with two types of insulation, A and B. The interface between the two insulating layers has infinite contact resistance. The outer surface is exposed to air ($T_\infty$ = 300 K) and h = 25 W/m$^2$·K

(a) Sketch and label the thermal circuit
(b) What are the outer surface temps for materials A and B?



# Example – Book Problem 3.59

A spherical, 3 mm cryogenic probe at temperature –30°C is embedded into skin at 37°C. Frozen tissue develops and the interface between the frozen and normal tissue is 0°C. If the thermal conductivity of frozen tissue is 1.5 W/m·K and heat transfer at the phase front is characterized by a convection coefficient of 50 W/m$^2$·K, what is the thickness of the frozen layer?