Answer the following questions concerning radiation exchange between surfaces.
A. What is a view factor?
B. What assumptions are associated with computing view factors between two surfaces?
C. What is the reciprocity relation for view factors?
D. What is the summation rule?

A. A view factor is the fraction of the radiation leaving surface i that is intercepted by surface j. They are also called configuration or shape factors.

B. Assumptions of view factors are that surface i emits and reflects diffusely, and radiosity is uniform of the surface A_i.

C. $A_i F_{ij} = A_j F_{ji}$

D. The summation rule may be applied to each of N surfaces in an enclosure as defined:

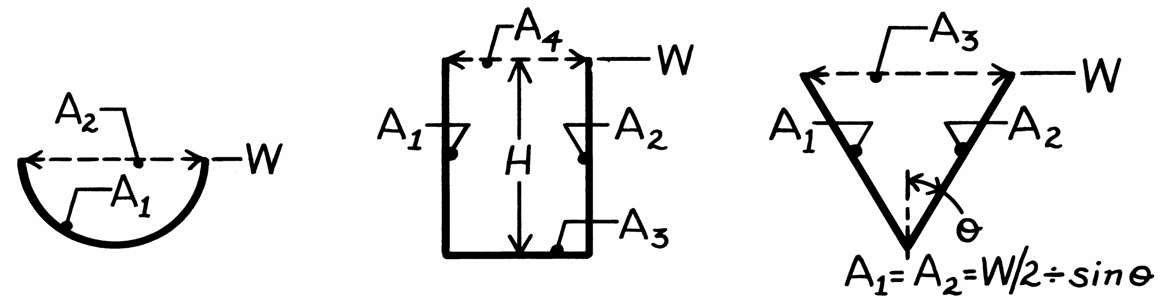
$$\sum_{j=1}^{N} F_{ij} = 1$$

PROBLEM 13.2

KNOWN: Geometry of semi-circular, rectangular and V grooves.

FIND: (a) View factors of grooves with respect to surroundings, (b) View factor for sides of V groove, (c) View factor for sides of rectangular groove.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces, (2) Negligible end effects, “long grooves”.

ANALYSIS: (a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

Semi-Circular Groove:

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi W/2)} \times 1$$

$$F_{12} = 2/\pi.$$ <

Rectangular Groove:

$$F_{4(1,2,3)} = 1; \qquad F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1$$

$$F_{(1,2,3)4} = W/(W + 2H).$$ <

V Groove:

$$F_{3(1,2)} = 1; \qquad F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\frac{W/2}{\sin\theta} + \frac{W/2}{\sin\theta}}$$

$$F_{(1,2)3} = \sin\theta.$$

(b) From Eqs. 13.3 and 13.4, $\quad F_{12} = 1 - F_{13} = 1 - \frac{A_3}{A_1} F_{31}.$

From Symmetry, $\quad F_{31} = 1/2.$

Hence, $F_{12} = 1 - \frac{W}{(W/2)/\sin\theta} \times \frac{1}{2}$ $\quad$ or $\quad$ $F_{12} = 1 - \sin\theta.$ <

(c) From Fig. 13.4, with X/L = H/W =2 and Y/L $\rightarrow \infty$,

$$F_{12} \approx 0.62.$$ <

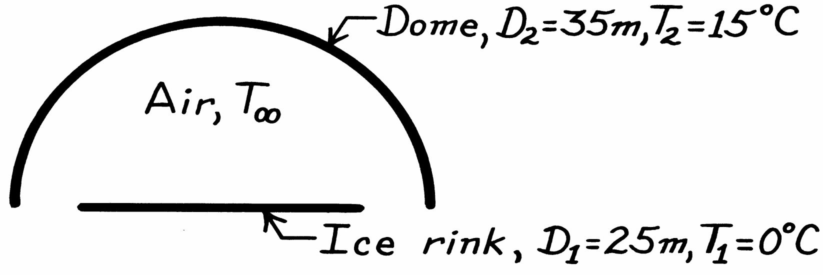
COMMENTS: (1) Note that for the V groove, $F_{13} = F_{23} = F_{(1,2)3} = \sin\theta$, (2) In part (c), Fig. 13.4 could also be used with Y/L = 2 and X/L = ∞. However, obtaining the limit of F_{ij} as X/L $\rightarrow \infty$ from the figure is somewhat uncertain.

PROBLEM 13.17

KNOWN: Temperature and diameters of a circular ice rink and a hemispherical dome.

FIND: Net rate of heat transfer to the ice due to radiation exchange with the dome.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for dome and ice.

ANALYSIS: From Eq. 13.14, $q_{ij} = A_iF_{ij}(J_i - J_j)$ where $J_i = \sigma T_i^4$ and $J_j = \sigma T_i^4$. Therefore,

$$q_{21} = A_2F_{21}\sigma\left(T_2^4 - T_1^4\right)$$

From reciprocity, $A_2 F_{21} = A_1 F_{12} = \left(\pi D_1^2 / 4\right)1$

$$A_2F_{21} = (\pi/4)(25\text{ m})^2 1 = 491\text{ m}^2.$$

Hence

$$q_{21} = 491\text{ m}^2\left(5.67\times10^{-8}\text{W/m}^2\cdot\text{K}^4\right)\left[(288\text{ K})^4 - (273\text{ K})^4\right]$$

$$q_{21} = 3.69\times10^4\text{ W.}$$ <

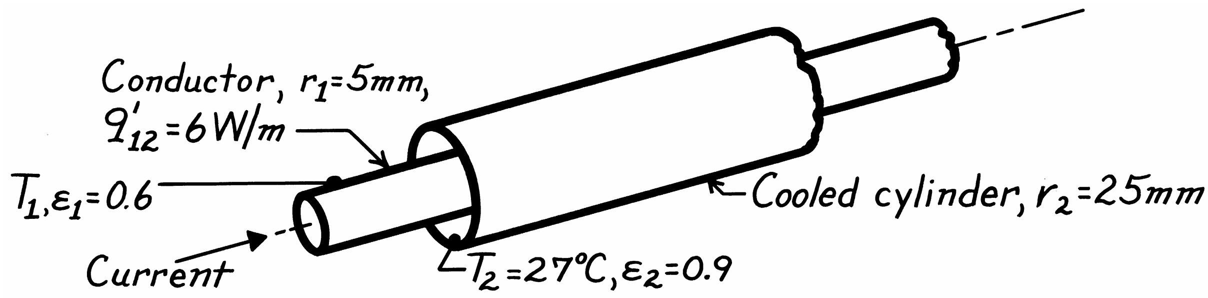
COMMENTS: If the air temperature, T_∞, exceeds T_1, there will also be heat transfer by convection to the ice. The radiation and convection transfer to the ice determine the heat load which must be handled by the cooling system.

PROBLEM 13.50

KNOWN: Long electrical conductor with known heat dissipation is cooled by a concentric tube arrangement.

FIND: Surface temperature of the conductor.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Conductor and cooling tube are concentric and very long, (3) Space between surfaces is evacuated.

ANALYSIS: The heat transfer by radiation exchange between the conductor and the concentric, cooled cylinder is given by Eq. 13.20. For a unit length,

$$q'_{12} = \frac{q_{12}}{\ell} = \sigma \cdot 2\pi r_1 \left(T_1^4 - T_2^4\right) / \left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right] \tag{1}$$

where $A_1 = 2\pi r_1 \cdot \ell$. Solving for T_1 and substituting numerical values, find

$$T_1 = \left\{T_2^4 + \frac{q'_{12}}{\sigma \cdot 2\pi r_1}\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(27+273)^4 \mathrm{K}^4 + \frac{6 \mathrm{~W/m}}{5.67\times10^{-8} \mathrm{W/m^2 \cdot K^4} \times 2\pi(0.005\mathrm{m})}\left[\frac{1}{0.6} + \frac{1-0.9}{0.9}\left(\frac{5}{25}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(300 \mathrm{~K})^4 + 3.368\times10^9 \mathrm{K}^4 [1.667 + 0.00222]\right\}^{1/4} \tag{2}$$

$$T_1 = 342.3 \mathrm{~K} = 69°\mathrm{C}.$$ <

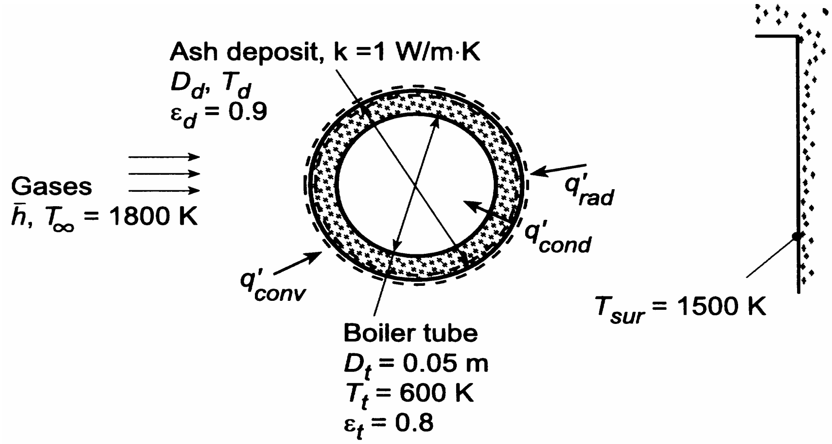
COMMENTS: (1) Note that Eq. (1) implies that $F_{12} = 1$. From Eq. (2) by comparison of the second term in the brackets involving ε_2, note that the influence of ε_2 is small. This follows since $r_1 << r_2$.

PROBLEM 13.94

KNOWN: Diameter, temperature and emissivity of boiler tube. Thermal conductivity and emissivity of ash deposit. Convection coefficient and temperature of gas flow over the tube. Temperature of surroundings.

FIND: (a) Rate of heat transfer to tube without ash deposit, (b) Rate of heat transfer with an ash deposit of diameter D_d = 0.06 m, (c) Effect of deposit diameter and convection coefficient on heat rate and contributions due to convection and radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse/gray surface behavior, (2) Surroundings form a large enclosure about the tube and may be approximated as a blackbody, (3) One-dimensional conduction in ash, (4) Steady-state.

ANALYSIS: (a) Without an ash deposit, the heat rate per unit tube length may be calculated directly.

$$q' = \bar{h}\pi D_t\left(T_\infty - T_t\right) + \varepsilon_t \sigma \pi D_t\left(T_{sur}^4 - T_t^4\right)$$

$$q' = 100\ \mathrm{W/m^2\cdot K}(\pi)0.05\,\mathrm{m}(1800-600)\mathrm{K} + 0.8\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}(\pi)(0.05\,\mathrm{m})\left(1500^4 - 600^4\right)\mathrm{K}$$

$$q' = (18,850 + 35,150)\,\mathrm{W/m} = 54,000\ \mathrm{W/m}$$ <

(b) Performing an energy balance for a control surface about the outer surface of the ash deposit,
$q'_{conv} + q'_{rad} = q'_{cond}$, or

$$\bar{h}\pi D_d\left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d\left(T_{sur}^4 - T_d^4\right) = \frac{2\pi k\left(T_d - T_t\right)}{\ln\left(D_d / D_t\right)}$$

Hence, canceling π and considering an ash deposit for which D_d = 0.06 m,

$$100\,\mathrm{W/m^2\cdot K}(0.06\,\mathrm{m})(1800 - T_d)\mathrm{K} + 0.9\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}(0.06\,\mathrm{m})\left(1500^4 - T_d^4\right)\mathrm{K}^4$$

$$= \frac{2(1\ \mathrm{W/m\cdot K})(T_d - 600)\mathrm{K}}{\ln(0.06/0.05)}$$

A trial-and-error solution yields $T_d \approx 1346$ K, from which it follows that

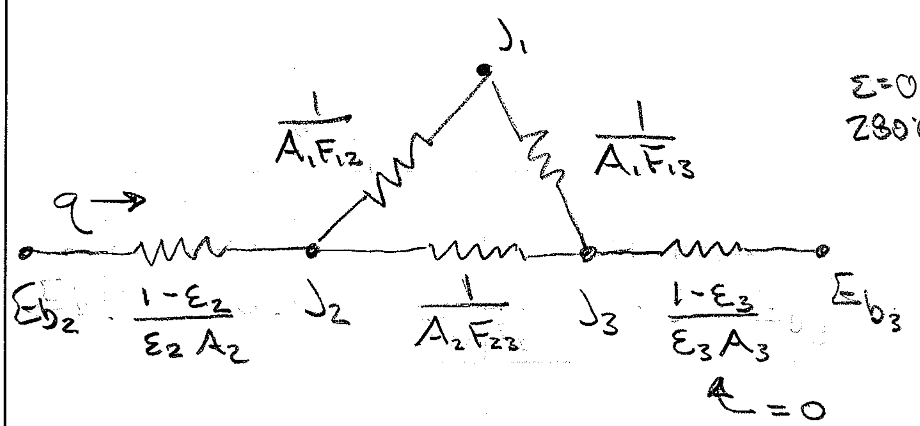
$$q' = \bar{h}\pi D_d\left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d\left(T_{sur}^4 - T_d^4\right)$$

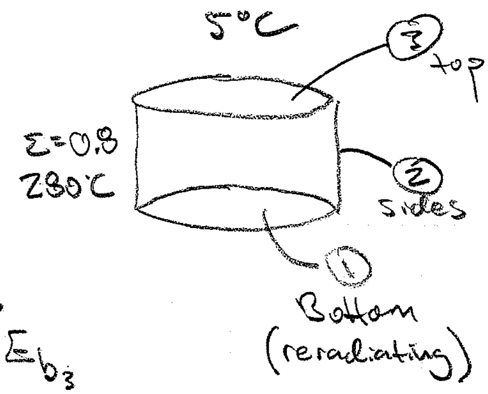
$$q' = 100\,\mathrm{W/m^2\cdot K}(\pi)0.06\,\mathrm{m}(1800-1346)\mathrm{K} + 0.9\times 5.67\times 10^{-8}\,\mathrm{W/m^2\cdot K^4}(\pi)0.06\,\mathrm{m}\left(1500^4 - 1346^4\right)\mathrm{K}^4$$

$$q' = (8560 + 17,140)\,\mathrm{W/m} = 25,700\,\mathrm{W/m}$$

CYLINDRICAL ENCLOSURE PROBLEM

Radiation circuit:





→ First, we note that surface 3 behaves as a blackbody (see page 824) so the resistance $(1-\varepsilon_3)/\varepsilon_3 A_3 = 0$.

View factor analysis

F_{13} from Figure 13.5 with $L/r_i = 1.0$ and $r_j/L = 1.0$

→ $F_{13} \approx 0.38$

Then, from the summation rule we get

$$F_{12} = 0.62$$

This would be the same for F_{32} and from the reciprocity rule we get

$$F_{23} = \frac{A_3 F_{32}}{A_2} = \frac{\pi (2)^2\, 0.62}{2(2\pi 2)}$$

$$= 0.31$$

CYLINDRICAL ENCLOSURE, CONT.

Total Resistance for view factor component:

$$= \left[A_2 F_{23} + \left(\frac{1}{A_1 F_{12}} + \frac{1}{A_1 F_{13}} \right)^{-1} \right]^{-1}$$

$$= \left[2(4\pi)(0.31) + \left(\frac{1}{\pi 2^2 (0.62)} + \frac{1}{\pi 2^2 (0.38)} \right)^{-1} \right]^{-1}$$

$$= 0.0930$$

Adding the radiosity resistance for the walls (surface 2) we get

$$\frac{1-0.8}{0.8(2)(4\pi)} + 0.0930$$

$$= 0.1030 \ m^{-2}$$

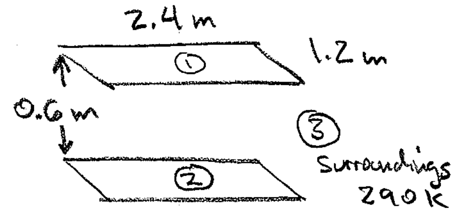
So, between E_{b_2} and E_{b_3} we have

$$q = \frac{E_{b_2} - E_{b_3}}{0.1030 \ m^{-2}} = \frac{\sigma\left[(280+273)^4 + (5+273)^4\right]}{0.1030}$$

$$= \boxed{18{,}200 \text{ Watts}}$$

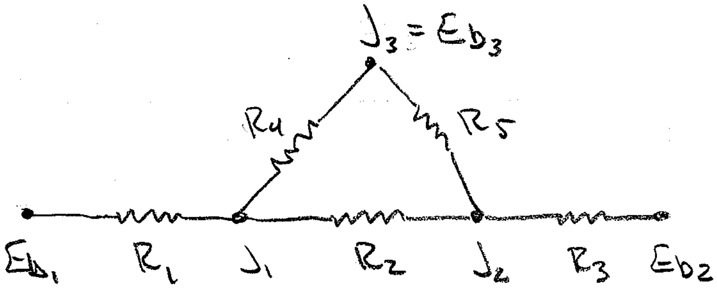
PARALLEL RECTANGLE PROBLEM

First, we will draw the radiation circuit: Note that ③ is effectively a blackbody.



$\varepsilon_1 = 0.6 \quad T_1 = 1000\,K \quad A_1 = 2.88\ m^2$

$\varepsilon_2 = 0.9 \quad T_2 = 420\,K \quad A_2 = 2.88\ m^2$



F_{12} from Fig 13.4 with

$$X/L = \frac{2.4}{0.6} = 4$$

$$Y/L = 1.2/0.6 = 2$$

$$F_{12} \approx 0.52$$

$$F_{13} = 0.48$$

$$F_{23} = 0.48 \text{ also}$$

$$R_1 = \frac{1-0.6}{0.6(2.88)} = 0.231$$

$$R_2 = \frac{1}{A_1F_{12}} = \frac{1}{(2.88)(0.52)} = 0.668$$

$$R_3 = \frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.9}{0.9(2.88)} = 0.0386$$

$$R_4 = \frac{1}{A_1F_{13}} = \frac{1}{2.88(0.48)} = 0.723$$

$$R_5 = \frac{1}{A_2F_{23}} = \frac{1}{2.88(.48)} = 0.723$$

→ For this one, let's try what the book calls the "direct approach" using Equation 13.15

continued...

PARALLEL RECTANGLE PROBLEM, CONT...

$$E_{b_1} = \sigma T_1^4 = \left(5.67\times10^{-8}\ \tfrac{W}{m^2\cdot K^4}\right)\left(1000\ K\right)^4$$
$$= 56{,}700\ W/m^2$$
$$E_{b2} = \sigma(420)^4 = 1764\ W/m^2$$
$$E_{b_3} = \sigma(290)^4 = 401\ W/m^2$$

So, using eq. 13.15 for each "source"

① $$\frac{56{,}700 - J_1}{0.231} = \frac{J_1 - J_2}{0.668} + \frac{J_1 - J_3}{0.723}$$

$$245000 - 4.32 J_1 = 1.50 J_1 - 1.50 J_2 + 1.38 J_1 - 1.38 J_3$$

$$7.32 J_1 - 1.50 J_2 - 1.38 J_3 = 245{,}000 \quad \star$$

② $$\frac{1764 - J_2}{0.0386} = \frac{J_2 - J_1}{0.668} + \frac{J_2 - J_3}{0.723}$$

$$45{,}700 - 25.91 J_2 = 1.50 J_2 - 1.50 J_1 + 1.38 J_2 - 1.38 J_3$$

$$-1.50 J_1 + 28.79 J_2 - 1.38 J_3 = 45{,}700 \quad \star$$

③ $$\frac{401 - J_3}{0} = \frac{J_3 - J_1}{0.723} + \frac{J_3 - J_2}{0.723}$$

→ We must multiply both sides by zero to get

$$J_3 = 401 \quad \star$$

PARALLEL RECTANGLE PROBLEM, CONT.

⇒ Solving the three starred equations simultaneously we get

$$J_1 = 34{,}240 \ \text{W/m}^2$$
$$J_2 = 3390 \ \text{W/m}^2$$
$$J_3 = 400 \ \text{W/m}^2$$

(a) The total energy lost (radiosity) from the hotter plate (Plate 1) is

$$(34{,}240 \ \text{W/m}^2)(2.88 \ \text{m}^2) = \boxed{98{,}600 \ \text{W}}$$

(b) Total energy exchange (q) is

$$q = \frac{J_1 - J_2}{\left(\frac{1}{A_1 F_{12}}\right)} = \frac{34{,}240 - 3390}{0.668} = \boxed{46{,}200 \ \text{W}}$$