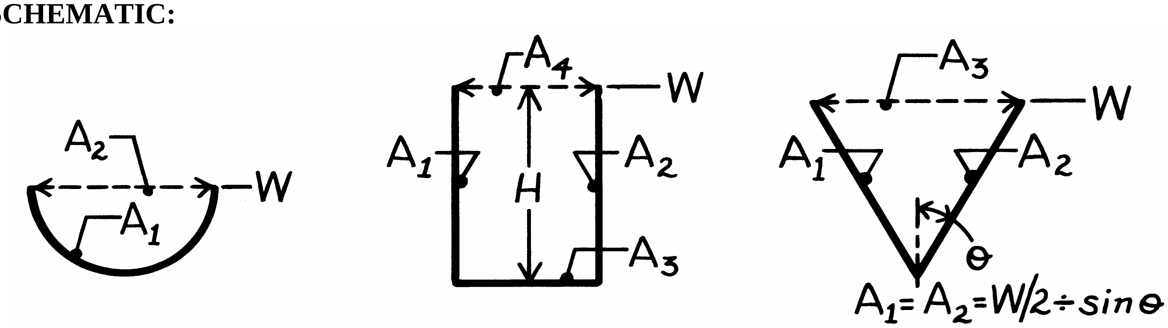
PROBLEM 13.2

KNOWN: Geometry of semi-circular, rectangular and V grooves.

FIND: (a) View factors of grooves with respect to surroundings, (b) View factor for sides of V groove, (c) View factor for sides of rectangular groove.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse surfaces, (2) Negligible end effects, “long grooves”.

ANALYSIS: (a) Consider a unit length of each groove and represent the surroundings by a hypothetical surface (dashed line).

Semi-Circular Groove:

$$F_{21} = 1; \qquad F_{12} = \frac{A_2}{A_1} F_{21} = \frac{W}{(\pi W/2)} \times 1$$

$$F_{12} = 2/\pi.$$ <

Rectangular Groove:

$$F_{4(1,2,3)} = 1; \qquad F_{(1,2,3)4} = \frac{A_4}{A_1 + A_2 + A_3} F_{4(1,2,3)} = \frac{W}{H + W + H} \times 1$$

$$F_{(1,2,3)4} = W/(W + 2H).$$ <

V Groove:

$$F_{3(1,2)} = 1; \qquad F_{(1,2)3} = \frac{A_3}{A_1 + A_2} F_{3(1,2)} = \frac{W}{\frac{W/2}{\sin\theta} + \frac{W/2}{\sin\theta}}$$

$$F_{(1,2)3} = \sin\theta.$$

(b) From Eqs. 13.3 and 13.4, $F_{12} = 1 - F_{13} = 1 - \frac{A_3}{A_1} F_{31}.$

From Symmetry, $F_{31} = 1/2.$

Hence, $F_{12} = 1 - \frac{W}{(W/2)/\sin\theta} \times \frac{1}{2}$ or $F_{12} = 1 - \sin\theta.$ <

(c) From Fig. 13.4, with X/L = H/W =2 and Y/L → ∞,

$$F_{12} \approx 0.62.$$ <

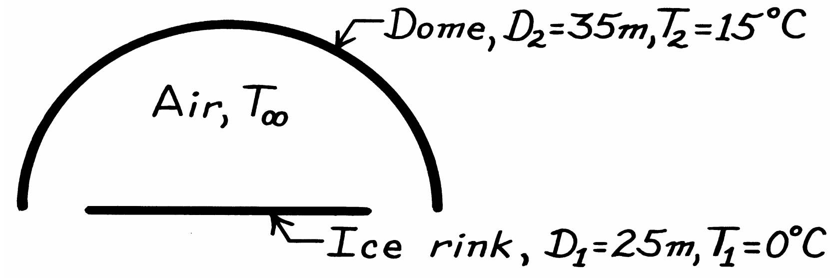
COMMENTS: (1) Note that for the V groove, $F_{13} = F_{23} = F_{(1,2)3} = \sin\theta$, (2) In part (c), Fig. 13.4 could also be used with Y/L = 2 and X/L = ∞. However, obtaining the limit of F_{ij} as X/L → ∞ from the figure is somewhat uncertain.

PROBLEM 13.17

KNOWN: Temperature and diameters of a circular ice rink and a hemispherical dome.

FIND: Net rate of heat transfer to the ice due to radiation exchange with the dome.

SCHEMATIC:



ASSUMPTIONS: (1) Blackbody behavior for dome and ice.

ANALYSIS: From Eq. 13.14, $q_{ij} = A_i F_{ij}(J_i - J_j)$ where $J_i = \sigma T_i^4$ and $J_j = \sigma T_i^4$. Therefore,

$$q_{21} = A_2 F_{21} \sigma \left(T_2^4 - T_1^4\right)$$

From reciprocity, $A_2\ F_{21} = A_1\ F_{12} = \left(\pi D_1^2 / 4\right) 1$

$$A_2 F_{21} = (\pi / 4)(25\text{ m})^2 1 = 491\text{ m}^2.$$

Hence

$$q_{21} = 491\text{ m}^2 \left(5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}^4\right)\left[(288\text{ K})^4 - (273\text{ K})^4\right]$$

$$q_{21} = 3.69 \times 10^4\ \text{W}.$$ <

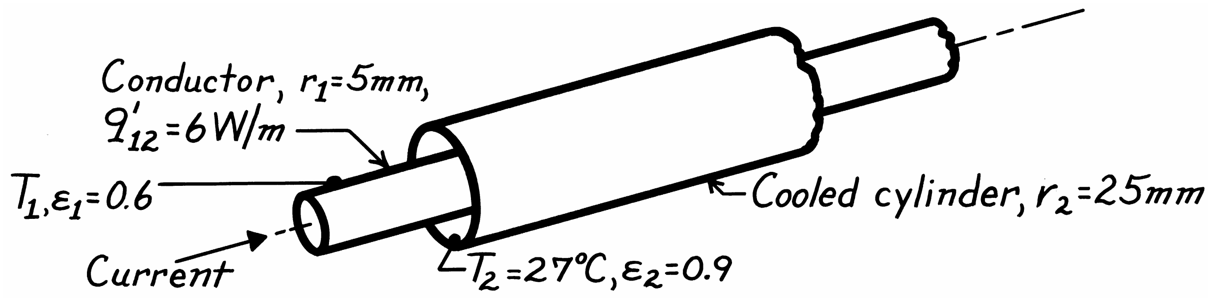
COMMENTS: If the air temperature, T_∞, exceeds T_1, there will also be heat transfer by convection to the ice. The radiation and convection transfer to the ice determine the heat load which must be handled by the cooling system.

PROBLEM 13.50

KNOWN: Long electrical conductor with known heat dissipation is cooled by a concentric tube arrangement.

FIND: Surface temperature of the conductor.

SCHEMATIC:



ASSUMPTIONS: (1) Surfaces are diffuse-gray, (2) Conductor and cooling tube are concentric and very long, (3) Space between surfaces is evacuated.

ANALYSIS: The heat transfer by radiation exchange between the conductor and the concentric, cooled cylinder is given by Eq. 13.20. For a unit length,

$$q'_{12} = \frac{q_{12}}{\ell} = \sigma \cdot 2\pi r_1 \left(T_1^4 - T_2^4\right) / \left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right] \qquad (1)$$

where $A_1 = 2\pi r_1 \cdot \ell$. Solving for T_1 and substituting numerical values, find

$$T_1 = \left\{T_2^4 + \frac{q'_{12}}{\sigma \cdot 2\pi r_1}\left[\frac{1}{\varepsilon_1} + \frac{1-\varepsilon_2}{\varepsilon_2}\left(\frac{r_1}{r_2}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(27+273)^4 \, K^4 + \frac{6 \text{ W/m}}{5.67\times10^{-8} \text{W/m}^2\cdot K^4 \times 2\pi(0.005\text{m})}\left[\frac{1}{0.6} + \frac{1-0.9}{0.9}\left(\frac{5}{25}\right)\right]\right\}^{1/4}$$

$$T_1 = \left\{(300 \text{ K})^4 + 3.368\times10^9 K^4 [1.667 + 0.00222]\right\}^{1/4} \qquad (2)$$

$$T_1 = 342.3 \text{ K} = 69°\text{C}.$$ <

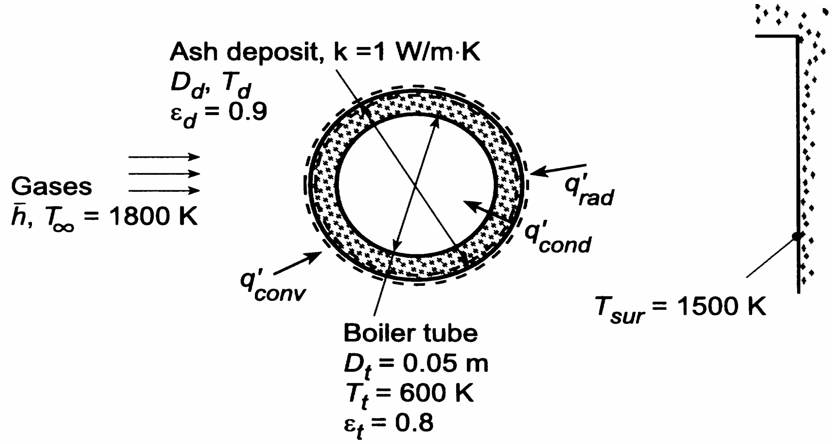
COMMENTS: (1) Note that Eq. (1) implies that $F_{12} = 1$. From Eq. (2) by comparison of the second term in the brackets involving ε_2, note that the influence of ε_2 is small. This follows since $r_1 << r_2$.

PROBLEM 13.94

KNOWN: Diameter, temperature and emissivity of boiler tube. Thermal conductivity and emissivity of ash deposit. Convection coefficient and temperature of gas flow over the tube. Temperature of surroundings.

FIND: (a) Rate of heat transfer to tube without ash deposit, (b) Rate of heat transfer with an ash deposit of diameter $D_d = 0.06$ m, (c) Effect of deposit diameter and convection coefficient on heat rate and contributions due to convection and radiation.

SCHEMATIC:



ASSUMPTIONS: (1) Diffuse/gray surface behavior, (2) Surroundings form a large enclosure about the tube and may be approximated as a blackbody, (3) One-dimensional conduction in ash, (4) Steady-state.

ANALYSIS: (a) Without an ash deposit, the heat rate per unit tube length may be calculated directly.

$$q' = \bar{h}\pi D_t\left(T_\infty - T_t\right) + \varepsilon_t \sigma \pi D_t\left(T_{sur}^4 - T_t^4\right)$$

$$q' = 100\ \text{W/m}^2\cdot\text{K}(\pi)0.05\,\text{m}(1800-600)\text{K} + 0.8\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4(\pi)(0.05\,\text{m})\left(1500^4 - 600^4\right)\text{K}$$

$$q' = (18{,}850 + 35{,}150)\,\text{W/m} = 54{,}000\ \text{W/m}$$ <

(b) Performing an energy balance for a control surface about the outer surface of the ash deposit, $q'_{conv} + q'_{rad} = q'_{cond}$, or

$$\bar{h}\pi D_d\left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d\left(T_{sur}^4 - T_d^4\right) = \frac{2\pi k\left(T_d - T_t\right)}{\ln\left(D_d / D_t\right)}$$

Hence, canceling π and considering an ash deposit for which $D_d = 0.06$ m,

$$100\,\text{W/m}^2\cdot\text{K}(0.06\,\text{m})(1800 - T_d)\text{K} + 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4(0.06\,\text{m})\left(1500^4 - T_d^4\right)\text{K}^4$$

$$= \frac{2(1\ \text{W/m}\cdot\text{K})(T_d - 600)\text{K}}{\ln(0.06/0.05)}$$

A trial-and-error solution yields $T_d \approx 1346$ K, from which it follows that

$$q' = \bar{h}\pi D_d\left(T_\infty - T_d\right) + \varepsilon_d \sigma \pi D_d\left(T_{sur}^4 - T_d^4\right)$$

$$q' = 100\,\text{W/m}^2\cdot\text{K}(\pi)0.06\,\text{m}(1800 - 1346)\text{K} + 0.9\times 5.67\times 10^{-8}\,\text{W/m}^2\cdot\text{K}^4(\pi)0.06\,\text{m}\left(1500^4 - 1346^4\right)\text{K}^4$$

$$q' = (8560 + 17{,}140)\,\text{W/m} = 25{,}700\,\text{W/m}$$