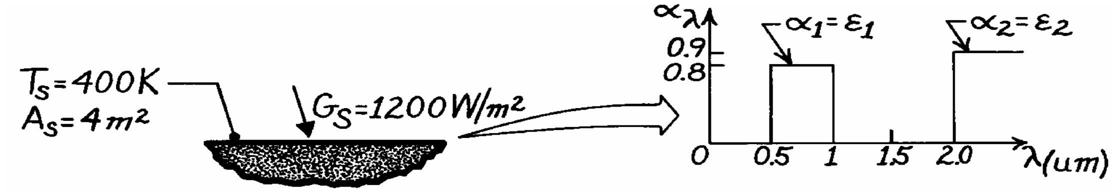
PROBLEM 12.46

KNOWN: Area, temperature, irradiation and spectral absorptivity of a surface.

FIND: Absorbed irradiation, emissive power, radiosity and net radiation transfer from the surface.

SCHEMATIC:



ASSUMPTIONS: (1) Opaque, diffuse surface behavior, (2) Spectral distribution of solar radiation corresponds to emission from a blackbody at 5800 K.

ANALYSIS: The absorptivity to solar irradiation is

$$\alpha_S = \frac{\int_0^\infty \alpha_\lambda G_\lambda d\lambda}{G} = \frac{\int_0^\infty \alpha_\lambda E_{\lambda b}(5800\text{ K})d\lambda}{E_b} = \alpha_1 F_{(0.5\to 1\mu m)} + \alpha_2 F_{(2\to\infty)}.$$

From Table 12.1,	$\lambda T = 2900\ \mu m\cdot K$:	$F_{(0\to 0.5\ \mu m)} = 0.250$
	$\lambda T = 5800\ \mu m\cdot K$:	$F_{(0\to 1\ \mu m)} = 0.720$
	$\lambda T = 11{,}600\ \mu m\cdot K$:	$F_{(0\to 2\ \mu m)} = 0.941$

$$\alpha_S = 0.8(0.720-0.250)+0.9(1-0.941) = 0.429.$$

Hence, $$G_{abs} = \alpha_S G_S = 0.429\left(1200\text{ W/m}^2\right) = 515\text{ W/m}^2.$$ <

The emissivity is

$$\varepsilon = \int_0^\infty \varepsilon_\lambda E_{\lambda b}(400\text{ K})d\lambda / E_b = \varepsilon_1 F_{(0.5\to 1\mu m)} + \varepsilon_2 F_{(2\to\infty)}.$$

From Table 12.1,	$\lambda T = 200\ \mu m\cdot K$:	$F_{(0\to 0.5\ \mu m)} = 0$
	$\lambda T = 400\ \mu m\cdot K$:	$F_{(0\to 1\ \mu m)} = 0$
	$\lambda T = 800\ \mu m\cdot K$	$F_{(0\to 2\ \mu m)} = 0.$

Hence, $\varepsilon = \varepsilon_2 = 0.9$,

$$E = \varepsilon\sigma T_s^4 = 0.9\times 5.67\times 10^{-8}\text{ W/m}^2\cdot K^4(400\text{ K})^4 = 1306\text{ W/m}^2.$$ <

The radiosity is

$$J = E + \rho_S G_S = E + (1-\alpha_S)G_S = [1306 + 0.571\times 1200]\text{W/m}^2 = 1991\text{ W/m}^2.$$ <

The net radiation transfer from the surface is

$$q_{net} = (E - \alpha_S G_S)A_s = (1306-515)\text{W/m}^2\times 4\text{ m}^2 = 3164\text{ W}.$$ <

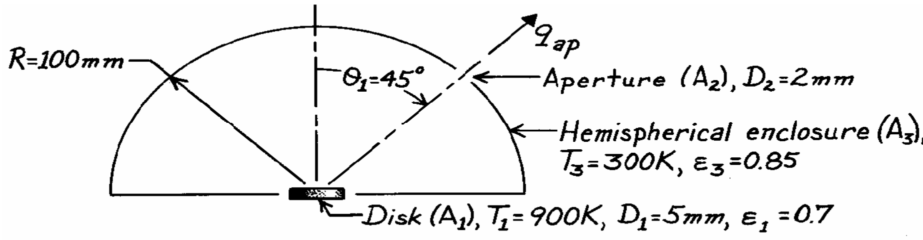
COMMENTS: Unless 3164 W are supplied to the surface by other means (for example, by convection), the surface temperature will decrease with time.

PROBLEM 12.48

KNOWN: Small disk positioned at center of an isothermal, hemispherical enclosure with a small aperture.

FIND: Radiant power [μW] leaving the aperture.

SCHEMATIC:



ASSUMPTIONS: (1) Disk is diffuse-gray, (2) Enclosure is isothermal and has area much larger than disk, (3) Aperture area is very small compared to enclosure area, (4) Areas of disk and aperture are small compared to radius squared of the enclosure.

ANALYSIS: The radiant power leaving the aperture is due to radiation leaving the disk and to irradiation on the aperture from the enclosure. That is,

$$q_{ap} = q_{1\to 2} + G_2 \cdot A_2. \qquad (1)$$

The radiation leaving the disk can be written in terms of the radiosity of the disk. For the diffuse disk,

$$q_{1\to 2} = \frac{1}{\pi} J_1 \cdot A_1 \cos\theta_1 \cdot \omega_{2-1} \qquad (2)$$

and with $\varepsilon = \alpha$ for the gray behavior, the radiosity is

$$J_1 = \varepsilon_1 E_b(T_1) + \rho G_1 = \varepsilon_1 \sigma T_1^4 + (1-\varepsilon_1)\sigma T_3^4 \qquad (3)$$

where the irradiation G_1 is the emissive power of the black enclosure, $E_b(T_3)$; $G_1 = G_2 = E_b(T_3)$. The solid angle ω_{2-1} follows from Eq. 12.2,

$$\omega_{2-1} = A_2 / R^2. \qquad (4)$$

Combining Eqs. (2), (3) and (4) into Eq. (1) with $G_2 = \sigma T_3^4$, the radiant power is

$$q_{ap} = \frac{1}{\pi}\sigma\left[\varepsilon_1 T_1^4 + (1-\varepsilon_1)T_3^4\right]A_1\cos\theta_1 \cdot \frac{A_2}{R^2} + A_2\sigma T_3^4$$

$$q_{ap} = \frac{1}{\pi}5.67\times10^{-8}\frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\left[0.7(900\text{K})^4 + (1-0.7)(300\text{K})^4\right]\frac{\pi}{4}(0.005\text{m})^2\cos45°\times$$

$$\frac{\pi/4(0.002\text{m})^2}{(0.100\text{m})^2} + \frac{\pi}{4}(0.002\text{m})^2\,5.67\times10^{-8}\,\text{W/m}^2\cdot\text{K}^4(300\text{K})^4$$

$$q_{ap} = (36.2 + 0.19 + 1443)\,\mu\text{W} = 1479\,\mu\text{W}.$$ <

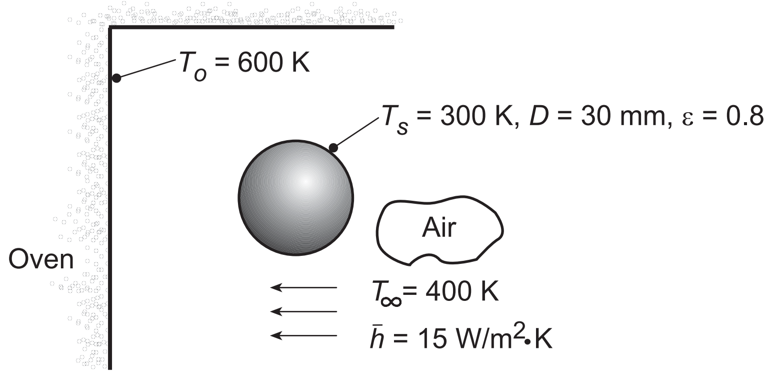
COMMENTS: Note the relative magnitudes of the three radiation components. Also, recognize that the emissivity of the enclosure ε_3 doesn't enter into the analysis. Why?

PROBLEM 12.77

KNOWN: Diffuse-gray sphere is placed in large oven with known wall temperature and experiences convection process.

FIND: (a) Net heat transfer rate to the sphere when its temperature is 300 K, (b) Steady-state temperature of the sphere, (c) Time required for the sphere, initially at 300 K, to come within 20 K of the steady-state temperature, and (d) Elapsed time of part (c) as a function of the convection coefficient for $10 \le h \le 25$ $W/m^2 \cdot K$ for emissivities 0.2, 0.4 and 0.8.

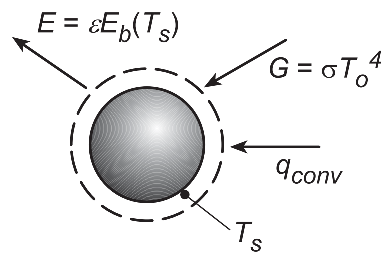
SCHEMATIC:



ASSUMPTIONS: (1) Sphere surface is diffuse-gray, (2) Sphere area is much smaller than the oven wall area, (3) Sphere surface is isothermal.

PROPERTIES: Sphere (Given) : $\alpha = 7.25 \times 10^{-5}$ m^2/s, k = 185 W/m·K.

ANALYSIS: (a) From an energy balance on the sphere find



$$q_{net} = q_{in} - q_{out}$$

$$q_{net} = \alpha G A_s + q_{conv} - E A_s$$

$$q_{net} = \alpha \sigma T_o^4 A_s + h A_s \left(T_\infty - T_s\right) - \varepsilon \sigma T_s^4 A_s . \qquad (1)$$

Note that the irradiation to the sphere is the emissive power of a blackbody at the temperature of the oven walls. This follows since the oven walls are isothermal and have a much larger area than the sphere area. Substituting numerical values, noting that α = ε since the surface is diffuse-gray and that $A_s = \pi D^2$, find

$$q_{net} = \Big[0.8 \times 5.67 \times 10^{-8}\, W/m^2 \cdot K^4 \left(600K\right)^4 + 15\, W/m^2 \cdot K \times \left(400 - 300\right) K$$

$$- 0.8 \times 5.67 \times 10^{-8}\, W/m^2 \cdot K^4 \left(300K\right)^4 \Big] \pi \left(30 \times 10^{-3} m\right)^2$$

$$q_{net} = \left[16.6 + 4.2 - 1.0\right] W = 19.8\, W . \qquad (1)$$ <

(b) For steady-state conditions, q_{net} in the energy balance of Eq. (1) will be zero,

$$0 = \alpha \sigma T_o^4 A_s + h A_s \left(T_\infty - T_{ss}\right) - \varepsilon \sigma T_{ss}^4 A_s \qquad (2)$$

Substitute numerical values and find the steady-state temperature as

$$T_{ss} = 538.2K$$ <

PROBLEM 13.1

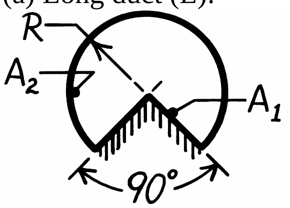
KNOWN: Various geometric shapes involving two areas A_1 and A_2.

FIND: Shape factors, F_{12} and F_{21}, for each configuration.

ASSUMPTIONS: Surfaces are diffuse.

ANALYSIS: The analysis is not to make use of tables or charts. The approach involves use of the reciprocity relation, Eq. 13.3, and summation rule, Eq. 13.4. Recognize that reciprocity applies to two surfaces; summation applies to an enclosure. Certain shape factors will be identified by inspection. Note L is the length normal to page.

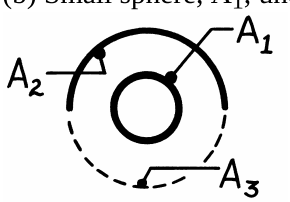
(a) Long duct (L):



By inspection, $F_{12} = 1.0$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2\,RL}{(3/4)\cdot 2\pi RL} \times 1.0 = \frac{4}{3\pi} = 0.424$ <

(b) Small sphere, A_1, under concentric hemisphere, A_2, where $A_2 = 2A$

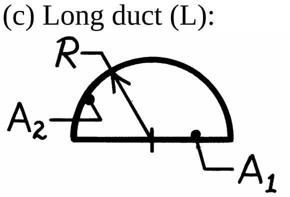


Summation rule $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{A_1}{2A_1} \times 0.5 = 0.25.$ <

(c) Long duct (L):

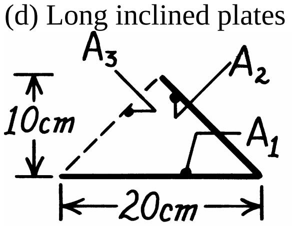


By inspection, $F_{12} = 1.0$

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2RL}{\pi RL} \times 1.0 = \frac{2}{\pi} = 0.637$ <

Summation rule, $F_{22} = 1 - F_{21} = 1 - 0.64 = 0.363.$ <

(d) Long inclined plates (L):

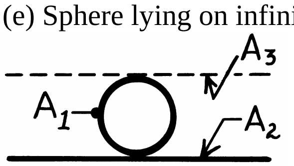


Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} = \frac{20L}{10(2)^{1/2} L} \times 0.5 = 0.707.$ <

(e) Sphere lying on infinite plane



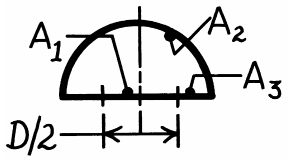
Summation rule, $F_{11} + F_{12} + F_{13} = 1$

But $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.5$ <

By reciprocity, $F_{21} = \frac{A_1}{A_2} F_{12} \to 0$ since $A_2 \to \infty.$ <

Continued …..

(f) Hemisphere over a disc of diameter D/2; find also F_{22} and F_{23}.



By inspection, $F_{12} = 1.0$ <

Summation rule for surface A_3 is written as

$$F_{31} + F_{32} + F_{33} = 1. \quad \text{Hence,} \ F_{32} = 1.0.$$

By reciprocity, $$F_{23} = \frac{A_3}{A_2} F_{32}$$

$$F_{23} = \left\{ \left[\frac{\pi D^2}{4} - \frac{\pi (D/2)^2}{4} \right] / \frac{\pi D^2}{2} \right\} 1.0 = 0.375.$$

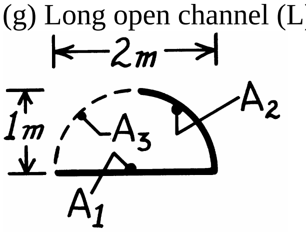
By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \left\{ \frac{\pi}{4} \left[\frac{D}{2} \right]^2 / \frac{\pi D^2}{2} \right\} \times 1.0 = 0.125.$$ <

Summation rule for A_2, $$F_{21} + F_{22} + F_{23} = 1 \text{ or}$$

$$F_{22} = 1 - F_{21} - F_{23} = 1 - 0.125 - 0.375 = 0.5.$$ <

Note that by inspection you can deduce $F_{22} = 0.5$

(g) Long open channel (L):



Summation rule for A_1

$$F_{11} + F_{12} + F_{13} = 0$$

but $F_{12} = F_{13}$ by symmetry, hence $F_{12} = 0.50$. <

By reciprocity, $$F_{21} = \frac{A_1}{A_2} F_{12} = \frac{2 \times L}{(2\pi 1)/4 \times L} = \frac{4}{\pi} \times 0.50 = 0.637.$$

COMMENTS: (1) Note that the summation rule is applied to an enclosure. To complete the enclosure, it was necessary in several cases to define a third surface which was shown by dashed lines.

(2) Recognize that the solutions follow a systematic procedure; in many instances it is possible to deduce a shape factor by inspection.

PROBLEM 13.8

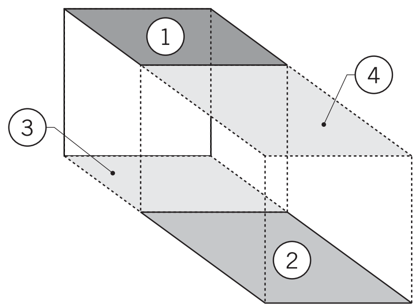
KNOWN: Arrangement of plane parallel rectangles.

FIND: Show that the view factor between A_1 and A_2 can be expressed as

$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\,F_{(1,4)(2,3)} - A_1\,F_{13} - A_4\,F_{42}\right]$$

where all F_{ij} on the right-hand side of the equation can be evaluated from Fig. 13.4 (see Table 13.2) for aligned parallel rectangles.

SCHEMATIC:



ASSUMPTIONS: Diffuse surfaces with uniform radiosity.

ANALYSIS: Using the additive rule where the parenthesis denote a composite surface,

$$A_{(1,4)}\,F^*_{(1,4)(2,3)} = A_1\,F^*_{13} + A_1\,F_{12} + A_4\,F_{43} + A_4\,F^*_{42} \qquad (1)$$

where the asterisk (*) denotes that the F_{ij} can be evaluated using the relation of Figure 13.4. Now, find suitable relation for F_{43}. By symmetry,

$$F_{43} = F_{21} \qquad (2)$$

and from reciprocity between A_1 and A_2,

$$F_{21} = \frac{A_1}{A_2}F_{12} \qquad (3)$$

Multiply Eq. (2) by A_4 and substitute Eq. (3), with $A_4 = A_2$,

$$A_4\,F_{43} = A_4\,F_{21} = A_4\,\frac{A_1}{A_2}F_{12} = A_1\,F_{12} \qquad (4)$$

Substituting for $A_4\,F_{43}$ from Eq. (4) into Eq. (1), and rearranging,

$$F_{12} = \frac{1}{2\,A_1}\left[A_{(1,4)}\,F^*_{(1,4)(2,3)} - A_1\,F^*_{13} - A_4\,F^*_{42}\right] \qquad <$$