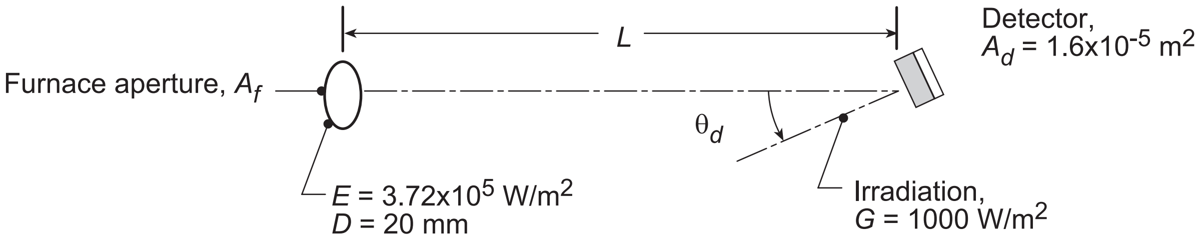
PROBLEM 12.4

KNOWN: Furnace with prescribed aperture and emissive power.

FIND: (a) Position of gauge such that irradiation is $G = 1000\ W/m^2$, (b) Irradiation when gauge is tilted $\theta_d = 20^o$, and (c) Compute and plot the gage irradiation, G, as a function of the separation distance, L, for the range $100 \le L \le 300$ mm and tilt angles of $\theta_d = 0$, 20, and 60^o.

SCHEMATIC:



ASSUMPTIONS: (1) Furnace aperture emits diffusely, (2) $A_d << L^2$.

ANALYSIS: (a) The irradiation on the detector area is defined as the power incident on the surface per unit area of the surface. That is

$$G = q_{f\to d}/A_d \qquad q_{f\to d} = I_e A_f \cos\theta_f\ \omega_{d-f} \qquad (1,2)$$

where $q_{f\to d}$ is the radiant power which leaves A_f and is intercepted by A_d. From Eqs. 12.2 and 12.7, ω_{d-f} is the solid angle subtended by surface A_d with respect to A_f,

$$\omega_{d-f} = A_d \cos\theta_d / L^2 . \qquad (3)$$

Noting that since the aperture emits diffusely, $I_e = E/\pi$ (see Eq. 12.12), and hence

$$G = (E/\pi) A_f \cos\theta_f \left(A_d \cos\theta_d / L^2\right) / A_d \qquad (4)$$

Solving for L^2 and substituting for the condition $\theta_f = 0^o$ and $\theta_d = 0^o$,

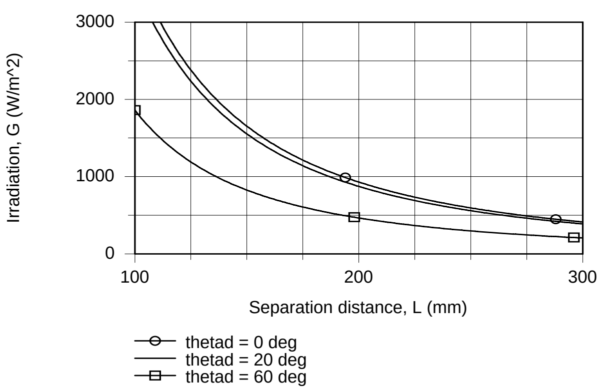
$$L^2 = E\cos\theta_f \cos\theta_d\ A_f / \pi G . \qquad (5)$$

$$L = \left[3.72\times10^5\ W/m^2 \times \frac{\pi}{4}(20\times10^{-3})^2\ m^2 / \pi \times 1000\ W/m^2\right]^{1/2} = 193\ mm . \qquad <$$

(b) When $\theta_d = 20^o$, $q_{f\to d}$ will be reduced by a factor of $\cos\theta_d$ since ω_{d-f} is reduced by a factor $\cos\theta_d$. Hence,

$$G = 1000\ W/m^2 \times \cos\theta_d = 1000 W/m^2 \times \cos 20^o = 940\ W/m^2 . \qquad <$$

(c) Using the IHT workspace with Eq. (4), G is computed and plotted as a function of L for selected θ_d. Note that G decreases inversely as L^2. As expected, G decreases with increasing θ_d and in the limit, approaches zero as θ_d approaches 90^o.

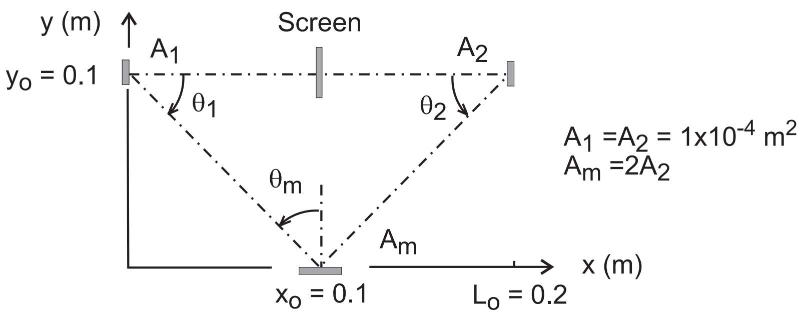


PROBLEM 12.5

KNOWN: Radiation from a diffuse radiant source A_1 with intensity $I_1 = 1.2 \times 10^5$ W/m^2·sr is incident on a mirror A_m, which reflects radiation onto the radiation detector A_2.

FIND: (a) Radiant power incident on A_m due to emission from the source, A_1, $q_{1\rightarrow m}$ (mW), (b) Intensity of radiant power leaving the perfectly reflecting, diffuse mirror A_m, I_m (W/m^2·sr), and (c) Radiant power incident on the detector A_2 due to the reflected radiation leaving A_m, $q_{m\rightarrow 2}$ (μW), (d) Plot the radiant power $q_{m\rightarrow 2}$ as a function of the lateral separation distance y_o for the range $0 \le y_o \le 0.2$ m; explain features of the resulting curve.

SCHEMATIC:



ASSUMPTIONS: (1) Surface A_1 emits diffusely, (2) Surface A_m does not emit, but reflects perfectly and diffusely, and (3) Surface areas are much smaller than the square of their separation distances.

ANALYSIS: (a) The radiant power leaving A_1 that is incident on A_m is

$$q_{1\rightarrow m} = I_1 \cdot A_1 \cdot \cos\theta_1 \cdot \Delta\omega_{m\text{-}1}$$

where $\omega_{m\text{-}1}$ is the solid angle A_m subtends with respect to A_1, Eq. 12.2,

$$\Delta\omega_{m\text{-}1} \equiv \frac{dA_n}{r^2} = \frac{A_m \cos\,\theta_m}{x_o^2 + y_o^2} = \frac{2\times 10^{-4}\ m^2 \cdot \cos 45°}{\left[0.1^2 + 0.1^2\right] m^2} = 7.07\times 10^{-3}\ sr$$

with $\theta_m = 90° - \theta_1$ and $\theta_1 = 45°$,

$$q_{1\rightarrow m} = 1.2\times 10^5\ W/m^2 \cdot sr \times 1\times 10^{-4}\ m^2 \times \cos 45° \times 7.07\times 10^{-3}\ sr = 60\ mW$$ <

(b) The intensity of radiation leaving A_m, after perfect and diffuse reflection, is

$$I_m = \left(q_{1\rightarrow m} / A_m\right)/\pi = \frac{60\times 10^{-3}\ W}{\pi \times 2\times 10^{-4}\ m^2} = 95.5\ W/m^2 \cdot sr$$

(c) The radiant power leaving A_m due to reflected radiation leaving A_m is

$$q_{m\rightarrow 2} = q_2 = I_m \cdot A_m \cdot \cos\,\theta_m \cdot \Delta\omega_{2-m}$$

where $\Delta\omega_{2\text{-}m}$ is the solid angle that A_2 subtends with respect to A_m, Eq. 12.2,

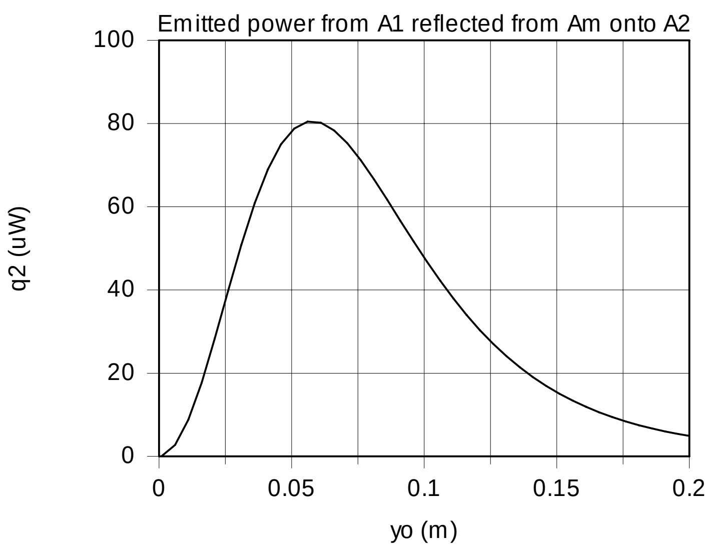
Continued …..

$$\Delta\omega_{2-m} \equiv \frac{dA_n}{r^2} = \frac{A_2 \cos\theta_2}{(L_o - x_o)^2 + y_o^2} = \frac{1\times10^{-4}\ \text{m}^2 \times \cos 45°}{\left[0.1^2 + 0.1^2\right]\text{m}^2} = 3.54\times10^{-3}\ \text{sr}$$

with $\theta_2 = 90° - \theta_m$

$$q_{m\to2} = q_2 = 95.5\ \text{W/m}^2\cdot\text{sr} \times 2\times10^{-4}\ \text{m}^2 \times \cos 45° \times 3.54\times10^{-3}\ \text{sr} = 47.8\ \mu\text{W}$$ <

(d) Using the foregoing equations in the *IHT* workspace, q_2 is calculated and plotted as a function of y_o for the range $0 \le y_o \le 0.2$ m.



From the relations, note that q_2 is dependent upon the geometric arrangement of the surfaces in the following manner. For small values of y_o, that is, when $\theta_1 \approx 0°$, the cos θ_1 term is at a maximum, near unity. But, the solid angles $\Delta\omega_{m-1}$ and $\Delta\omega_{2-m}$ are very small. As y_o increases, the cos θ_1 term doesn't diminish as much as the solid angles increase, causing q_2 to increase. A maximum in the power is reached as the cos θ_1 term decreases and the solid angles increase. The maximum radiant power occurs when y_o = 0.058 m which corresponds to θ_1 = 30°.

PROBLEM 12.22

KNOWN: Various surface temperatures.

FIND: (a) Wavelength corresponding to maximum emission for each surface, (b) Fraction of solar emission in UV, VIS and IR portions of the spectrum.

ASSUMPTIONS: (1) Spectral distribution of emission from each surface is approximately that of a blackbody, (2) The sun emits as a blackbody at 5800 K.

ANALYSIS: (a) From Wien's law, Eq. 12.25, the wavelength of maximum emission for blackbody radiation is

$$\lambda_{max} = \frac{C_3}{T} = \frac{2898\ \mu m \cdot K}{T}.$$

For the prescribed surfaces

Surface	Sun (5800K)	Tungsten (2500K)	Hot metal (1500K)	Skin (305K)	Cool metal (60K)	
$\lambda_{max}(\mu m)$	0.50	1.16	1.93	9.50	48.3	<

(b) From Fig. 12.3, the spectral regions associated with each portion of the spectrum are

Spectrum	**Wavelength limits, μm**
UV	**0.01 – 0.4**
VIS	**0.4 – 0.7**
IR	**0.7 - 100**

For T = 5800K and each of the wavelength limits, from Table 12.1 find:

$\lambda(\mu m)$	10^{-2}	0.4	0.7	10^{2}
$\lambda T(\mu m \cdot K)$	58	2320	4060	5.8×10^{5}
$F_{(0\rightarrow\lambda)}$	0	0.125	0.491	1

Hence, the fraction of the solar emission in each portion of the spectrum is:

$F_{UV} = 0.125 - 0 = 0.125$ <

$F_{VIS} = 0.491 - 0.125 = 0.366$ <

$F_{IR} = 1 - 0.491 = 0.509.$ <

COMMENTS: (1) Spectral concentration of surface radiation depends strongly on surface temperature.

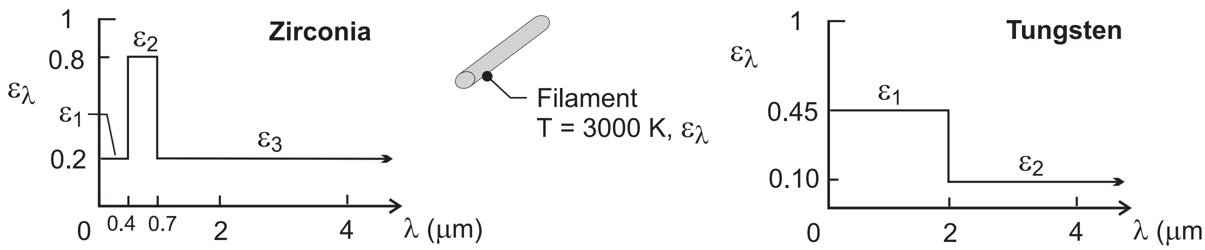
(2) Much of the UV solar radiation is absorbed in the earth's atmosphere.

PROBLEM 12.30

KNOWN: Spectral distribution of emissivity for zirconia and tungsten filaments. Filament temperature.

FIND: (a) Total emissivity of zirconia, (b) Total emissivity of tungsten and comparative power requirement, (c) Efficiency of the two filaments.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible reflection of radiation from bulb back to filament, (2) Equivalent surface areas for the two filaments, (3) Negligible radiation emission from bulb to filament.

ANALYSIS: (a) From Eq. (12.36), the emissivity of the zirconia is

$$\varepsilon = \int_o^\infty \varepsilon_\lambda \left(E_\lambda / E_b\right) d\lambda = \varepsilon_1 F_{(0\to 0.4\mu m)} + \varepsilon_2 F_{(0.4\to 0.7\mu m)} + \varepsilon_3 F_{(0.7\mu m\to\infty)}$$

$$\varepsilon = \varepsilon_1 F_{(0\to 0.4\mu m)} + \varepsilon_2 \left(F_{(0\to 0.7\mu m)} - F_{(0\to 0.4\mu m)}\right) + \varepsilon_3 \left(1 - F_{(0\to 0.7\mu m)}\right)$$

From Table 12.1, with T = 3000 K

$$\lambda T = 0.4\mu m \times 3000 \equiv 1200\mu m \cdot K: \quad F_{(0\to 0.4\mu m)} = 0.0021$$

$$\lambda T = 0.7\mu m \times 3000\ K = 2100\mu m \cdot K: F_{(0\to 0.7\mu m)} = 0.0838$$

$$\varepsilon = 0.2 \times 0.0021 + 0.8(0.0838 - 0.0021) + 0.2 \times (1 - 0.0838) = 0.249$$ <

(b) For the tungsten filament,

$$\varepsilon = \varepsilon_1 F_{(0\to 2\mu m)} + \varepsilon_2 \left(1 - F_{(0\to 2\mu m)}\right)$$

With $\lambda T = 6000\mu m \cdot K$, $F(0 \to 2\mu m) = 0.738$

$$\varepsilon = 0.45 \times 0.738 + 0.1(1 - 0.738) = 0.358$$ <

Assuming, no reflection of radiation from the bulb back to the filament and with no losses due to natural convection, the power consumption per unit surface area of filament is $P''_{elec} = \varepsilon\sigma T^4$.

Continued …..

Zirconia: $P''_{elec} = 0.249 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 (3000\ \text{K})^4 = 1.14 \times 10^6\ \text{W/m}^2$

Tungsten: $P''_{elec} = 0.358 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 (3000\ \text{K})^4 = 1.64 \times 10^6\ \text{W/m}^2$

Hence, for an equivalent surface area and temperature, the tungsten filament has the largest power consumption. <

(c) Efficiency with respect to the production of visible radiation may be defined as

$$\eta_{vis} = \frac{\int_{0.4}^{0.7} \varepsilon_\lambda E_{\lambda,b}\, d\lambda}{E} = \frac{\int_{0.4}^{0.7} \varepsilon_\lambda \left(E_{\lambda,b} / E_b\right)}{\varepsilon} = \frac{\varepsilon_{vis}}{\varepsilon} F_{(0.4 \rightarrow 0.7\,\mu m)}$$

With $F_{(0.4 \rightarrow 0.7\ \mu m)} = 0.0817$ for T = 3000 K,

Zirconia: $\eta_{vis} = (0.8 / 0.249) 0.0817 = 0.263$

Tungsten: $\eta_{vis} = (0.45 / 0.358) 0.0817 = 0.103$

Hence, the zirconia filament is the more efficient. <

COMMENTS: The production of visible radiation per unit filament surface area is $E_{vis} = \eta_{vis}\ P''_{elec}$. Hence,

Zirconia: $E_{vis} = 0.263 \times 1.14 \times 10^6\ \text{W/m}^2 = 3.00 \times 10^5\ \text{W/m}^2$

Tungsten: $E_{vis} = 0.103 \times 1.64 \times 10^6\ \text{W/m}^2 = 1.69 \times 10^5\ \text{W/m}^2$

Hence, not only is the zirconia filament more efficient, but it also produces more visible radiation with less power consumption. This problem illustrates the benefits associated with carefully considering spectral surface characteristics in radiative applications.