QUESTION

1. (20 pts) With respect to free convection:
   a. What is an extensive, quiescent fluid? (4 points)
   b. What are the two major physical considerations or forces for free convection? (4 points)
   c. What is the Grashof number in words (ratio of forces) and mathematically? (6 points)
   d. What is the Rayleigh number in words and mathematically? (6 points)

SOLUTION

a. As defined on page 561 of the text (Incropera, et.al., 4/e), "an extensive medium is, in principle, an infinite medium. Since a quiescent fluid is one that is otherwise at rest". In other words, an extensive, quiescent fluid is one that has infinite bulk properties (extensive) with no forced convection, and by all practical purposes, is stagnant (quiescent).

b. The two major physical considerations or forces for free convection are buoyant forces and viscous forces. Buoyant forces arise from a fluid density gradient and a body force proportional to the density at any given instant; this proportional force is often gravity. Viscous forces are always present and actually provide a counter-force situation to the buoyant force. This is similar to Newton's 3rd Law of Motion. More discussion of these forces is provided in Section 9.1 of the text.

c. The Grashof number is the ratio of buoyant forces to viscous forces acting on a fluid.

$$Gr_L \equiv \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}$$

d. The Rayleigh number is the product of the Grashof number and the Prandtl number, effectively being the ratio of buoyant forces to viscous forces multiplied by the ratio of momentum to thermal diffusivities.

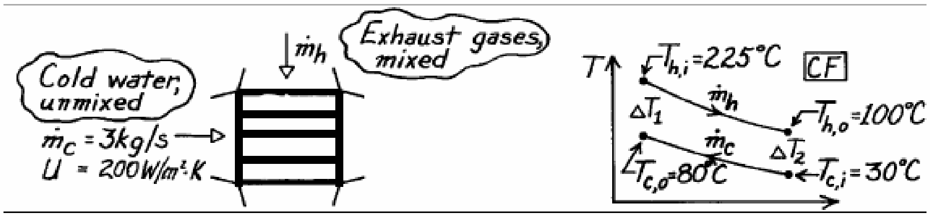
$$Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon\alpha}$$

PROBLEM 11.32

KNOWN: Single pass, cross-flow heat exchanger with hot exhaust gases (mixed) to heat water (unmixed)

FIND: Required surface area.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Exhaust gas properties assumed to be those of air.

PROPERTIES: *Table A-6*, Water ($\overline{T}_c$ = (80 + 30)°C/2 = 328 K): c_p = 4184 J/kg·K; *Table A-4*, Air (1 atm, $\overline{T}_h$ = (100 + 225)°C/2 = 436 K): c_p = 1019 J/kg·K.

ANALYSIS: Using the ε-NTU method,

$$C_c = \dot{m}_c c_c = 3\text{kg/s} \times 4184\ \text{J/kg}\cdot\text{K} = 12{,}552\ \text{W/K}$$

$$q = C_c(T_{c,o} - T_{c,i}) = 12{,}552\text{W/K}(80-30)°\text{C} = 627{,}600\ \text{W}$$

From an energy balance on the hot fluid,

$$C_h = q/(T_{h,i} - T_{h,o}) = 627{,}600\ \text{W}/(225-100)°\text{C} = 5{,}021\ \text{W/K}$$

Thus, $C_r = C_{min}/C_{max} = 0.40$ and $\varepsilon = q/C_{min}(T_{h,i} - T_{c,i}) = 0.641$. With C_{min} mixed and C_{max} unmixed, Eq. 11.34b yields

$$\text{NTU} = -\frac{1}{C_r}\ln\left[C_r \ln(1-\varepsilon)+1\right] = -\frac{1}{0.4}\ln\left[0.4\ln(1-0.641)+1\right] = 1.32$$

Thus,

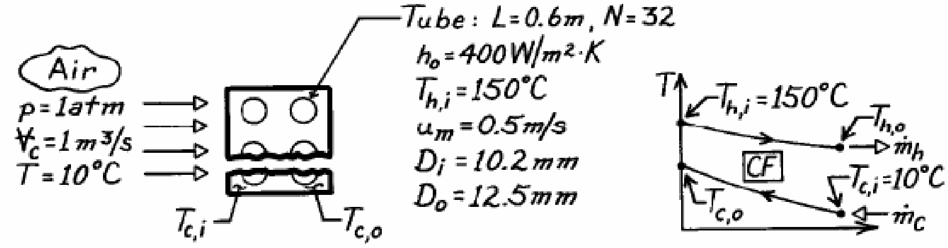
$$A = \text{NTU} \times C_{min}/U = 1.32 \times 5021\ \text{W/K} / 200\text{W/m}^2\cdot\text{K} = 33.1\ \text{m}^2$$ <

PROBLEM 11.58

KNOWN: Hxer consisting of 32 tubes in 0.6m square duct. Hot water enters tubes at 150°C with mean velocity 0.5 m/s. Atmospheric air at 10°C enters exchanger with volumetric flow rate of 1 m^3/s. Heat transfer coefficient on tube outer surfaces is 400 $W/m^2\cdot K$.

FIND: Outlet temperatures of the fluids, $T_{c,o}$ and $T_{h,o}$.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to surroundings, (2) Constant properties, (3) Hxer is a single-pass, cross-flow type with one fluid mixed (air) and the other unmixed (water), (4) Tube water flow is fully developed, (5) Negligible thermal resistance due to tube wall.

PROPERTIES: *Table A-4*, Air ($T_{c,i}$ = 10°C = 283 K, 1 atm): ρ = 1.2407 kg/m^3; *Table A-4*, Air (assume $T_{c,o} \approx 40$°C, $\overline{T}_c$ = (10 + 40)°C/2 = 298 K, 1 atm): c_p = 1007 J/kg·K; *Table A-6*, Water (assume $T_{h,o} \approx 140$°C, $\overline{T}_h$ = (140 + 150)°C/2 = 418 K): $\rho = 1/v_f = 1/1.0850 \times 10^{-3}$ m^3/kg, c_p = 4297 J/kg·K, $\mu_f = 188 \times 10^{-6}$ $N\cdot s/m^2$, k_f = 0.688 W/m·K, Pr_f = 1.18.

ANALYSIS: Using the ε-NTU method, first find the capacity rates.

$$C_h = \dot{m}_h c_{p,h} = (\rho A_c u_m)_h N \cdot c_{p,h}$$

$$C_h = \frac{1}{1.0850\times10^{-3}\,m^3/kg}\times\frac{\pi}{4}\left(10.2\times10^{-3}\,m\right)^2\times0.5\frac{m}{s}\times32\times4297\frac{J}{kg\cdot K}=5178\frac{W}{K}$$

$$C_c = \dot{m}_c c_{p,c} = (\rho V)_c c_{p,c} = 1.2407\frac{kg}{m^3}\times 1m^3/s\times1007\,J/kg\cdot K = 1249\frac{W}{K}. \quad (1,2)$$

Note that the cold fluid is the minimum fluid, $C_c = C_{min}$. The overall heat transfer coefficient follows from Eq. 11.5,

$$U_oA_o = \left[\frac{1}{h_iA_i}+\frac{1}{h_oA_o}\right]^{-1} \quad (3)$$

where h_i must be estimated from an appropriate internal flow correlation. The Reynolds number for water flow is

$$Re_D = \frac{\rho u_m D_i}{\mu} = \frac{\left(1/1.0850\times10^{-3}m^3kg\right)\times0.5m/s\times\left(10.2\times10^{-3}m\right)}{188\times10^{-6}N\cdot s/m^2} = 25,002. \quad (4)$$

Continued .....

The flow is turbulent and since $L/D_i = 0.6\text{m}/10.2 \times 10^{-3}\text{m} = 59$, fully developed conditions may be assumed. The Dittus-Boelter correlation with $n = 0.3$ is appropriate.

$$Nu_D = \frac{h_i D_i}{k} = 0.023\,Re_D^{0.8}\,Pr^{0.3} = 0.023(25{,}002)^{0.8}(1.18)^{0.3} = 79.7$$

$$h_i = \frac{k}{D_i} Nu_D = \frac{0.688\,W/m\cdot K}{10.2\times10^{-3}\,m}\times 79.7 = 5376\,W/m^2\cdot K.$$

Substituting numerical values into Eq. (3), find

$$U_o = \left[\left(\frac{12.5\text{mm}}{10.2\text{mm}}\right)\frac{1}{5376\,W/m^2\cdot K} + \frac{1}{400\,W/m^2\cdot K}\right]^{-1} = 366.6\,W/m^2\cdot K.$$

It follows from Eq. 11.24, with $A_o = N(\pi D_o L)$, that

$$NTU = \frac{U_o A_o}{C_{min}} = 366.6\frac{W}{m^2\cdot K}\times\left(32\times\pi\times12.5\times10^{-3}\,m\times0.6m\right)/1249\frac{W}{K} = 0.22.$$

From Fig. 11.15, noting that $C_{min} = C_c$ is the mixed fluid (solid curves),

$$\frac{C_{mixed}}{C_{unmixed}} = \frac{C_{min}}{C_{max}} = \frac{C_c}{C_h} = \frac{1249\,W/K}{5178\,W/K} = 0.24$$

and with $NTU = 0.22$ find $\varepsilon \approx 0.19$. From the definition of effectiveness, Eq. 11.19,

$$\varepsilon = \frac{q}{q_{max}} = \frac{C_c\left(T_{c,o} - T_{c,i}\right)}{C_{min}\left(T_{h,i} - T_{c,i}\right)}$$

$$T_{c,o} = T_{c,i} + \varepsilon\left(T_{h,i} - T_{c,i}\right) = 10°C + 0.19(150-10)°C = 36.6°C. \qquad <$$

Equating the energy balances on both fluids,

$$C_c\left(T_{c,o} - T_{c,i}\right) = C_h\left(T_{h,i} - T_{h,o}\right)$$

or

$$T_{h,o} = T_{h,i} - \frac{C_c}{C_h}\left(T_{c,o} - T_{c,i}\right)$$

$$T_{h,o} = 150°C - \frac{1249\,W/K}{5178\,W/K}(36.6-10)°C = 143.5°C. \qquad <$$

COMMENTS: (1) Note that the assumptions of $T_{h,o}$ and $T_{c,o}$ used in evaluating properties are reasonable.

(2) Note that to calculate $\dot{m}_c$ from V, the density at 10°C is more appropriate than at $\overline{T}_c$.

PROBLEM 9.10

KNOWN: Interior air and wall temperatures; wall height.

FIND: (a) Average heat transfer coefficient when T_∞ = 20°C and T_s = 10°C, (b) Average heat transfer coefficient when T_∞ = 27°C and T_s = 37°C.

SCHEMATIC:

ASSUMPTIONS: (a) Wall is at a uniform temperature, (b) Room air is quiescent.

PROPERTIES: *Table A-4*, Air (T_f = 288K, 1 atm): $\beta = 1/T_f = 3.472 \times 10^{-3}\ K^{-1}$, $\nu = 14.82 \times 10^{-6}\ m^2/s$, k = 0.0253 W/m·K, $\alpha = 20.9 \times 10^{-6}\ m^2/s$, Pr = 0.710; ($T_f$ = 305K, 1 atm): $\beta = 1/T_f = 3.279 \times 10^{-3}\ K^{-1}$, $\nu = 16.39 \times 10^{-6}\ m^2/s$, k = 0.0267 W/m·K, $\alpha = 23.2 \times 10^{-6}\ m^2/s$, Pr = 0.706.

ANALYSIS: The appropriate correlation for the average heat transfer coefficient for free convection on a vertical wall is Eq. 9.26.

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = \left\{ 0.825 + \frac{0.387\ Ra_L^{0.1667}}{\left[1 + (0.492/Pr)^{0.563}\right]^{0.296}} \right\}^2$$

where $Ra_L = g\,\beta\,\Delta T\,L^3/\nu\alpha$, Eq. 9.25, with $\Delta T = T_s - T_\infty$ or $T_\infty - T_s$.

(a) Substituting numerical values typical of *winter* conditions gives

$$Ra_L = \frac{9.8\ m/s^2 \times 3.472 \times 10^{-3}\ K^{-1}(20-10)\ K\ (2.5m)^3}{14.82 \times 10^{-6} m^2/s \times 20.96 \times 10^{-6} m^2/s} = 1.711 \times 10^{10}$$

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387\left(1.711 \times 10^{10}\right)^{0.1667}}{\left[1 + (0.492/0.710)^{0.563}\right]^{0.296}} \right\}^2 = 299.6.$$

Hence, $\bar{h} = \overline{Nu}_L\ k/L = 299.6(0.0253\ W/m \cdot K)/2.5m = 3.03\ W/m^2 \cdot K.$ <

(b) Substituting numerical values typical of *summer* conditions gives

$$Ra_L = \frac{9.8\ m/s^2 \times 3.279 \times 10^{-3}\ K^{-1}(37-27)\ K\ (2.5\ m)^3}{23.2 \times 10^{-6} m^2/s \times 16.39 \times 10^{-6} m^2/s} = 1.320 \times 10^{10}$$

$$\overline{Nu}_L = \left\{ 0.825 + \frac{0.387\left(1.320 \times 10^{10}\right)^{0.1667}}{\left[1 + (0.492/0.706)^{0.563}\right]^{0.296}} \right\}^2 = 275.8.$$

Hence, $\bar{h} = \overline{Nu}_L\ k/L = 275.8 \times 0.0267\ W/m \cdot K/2.5m = 2.94\ W/m^2 \cdot K.$ <

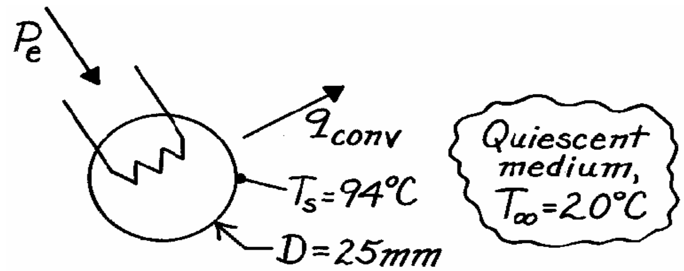
COMMENTS: There is a small influence due to T_f on $\bar{h}$ for these conditions. We should expect radiation effects to be important with such low values of $\bar{h}$.

PROBLEM 9.78

KNOWN: Sphere with embedded electrical heater is maintained at a uniform surface temperature when suspended in various media.

FIND: Required electrical power for these media: (a) atmospheric air, (b) water, (c) ethylene glycol.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible surface radiation effects, (2) Extensive and quiescent media.

PROPERTIES: Evaluated at $T_f = (T_s + T_\infty)/2 = 330K$:

	$\nu\cdot10^6$, m²/s	$k\cdot10^3$, W/m·K	$\alpha\cdot10^6$, m²/s	Pr	$\beta\cdot10^3$, K⁻¹
Table A-4, Air (1 atm)	18.91	28.5	26.9	0.711	3.03
Table A-6, Water	0.497	650	0.158	3.15	0.504
Table A-5, Ethylene glycol	5.15	260	0.0936	55.0	0.65

ANALYSIS: The electrical power (P_e) required to offset convection heat transfer is

$$q_{conv} = \bar{h} A_s (T_s - T_\infty) = \pi \bar{h} D^2 (T_s - T_\infty). \quad (1)$$

The free convection heat transfer coefficient for the sphere can be estimated from Eq. 9.35 using Eq. 9.25 to evaluate Ra_D.

$$\overline{Nu}_D = 2 + \frac{0.589\, Ra_D^{1/4}}{\left[1+(0.469/Pr)^{9/16}\right]^{4/9}} \begin{cases} Pr \ge 0.7 \\ Ra_D \le 10^{11} \end{cases} \qquad Ra_D = \frac{g\beta \Delta T D^3}{\nu\alpha}. \quad (2,3)$$

(a) For air

$$Ra_D = \frac{9.8\,m/s^2 \left(3.03\times10^{-3}K^{-1}\right)(94-20)K(0.025m)^3}{18.91\times10^{-6}m^2/s \times 26.9\times10^{-6}m^2/s} = 6.750\times10^4$$

$$\bar{h}_D = \frac{k}{D}\overline{Nu}_D = \frac{0.0285\,W/m\cdot K}{0.025m}\left\{2 + \frac{0.589\left(6.750\times10^4\right)^{1/4}}{\left[1+(0.469/0.711)^{9/16}\right]^{4/9}}\right\} = 10.6\,W/m^2\cdot K$$

$$q_{conv} = \pi\times10.6\,W/m^2\cdot K(0.025m)^2(94-20)K = 1.55\,W.$$

Continued …..

(b,c) Summary of the calculations above and for water and ethylene glycol:

Fluid	Ra_D	$\bar{h}_D \left(W/m^2 \cdot K\right)$	q(W)	
Air	6.750×10^4	10.6	1.55	<
Water	7.273×10^7	1299	187	<
Ethylene glycol	15.82×10^6	393	57.0	<

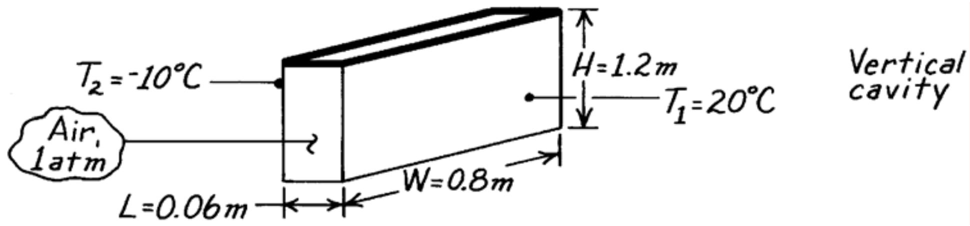
COMMENTS: Note large differences in the coefficients and heat rates for the fluids.

PROBLEM 9.90

KNOWN: Temperatures and dimensions of a window-storm window combination.

FIND: Rate of heat loss by free convection.

SCHEMATIC:



ASSUMPTIONS: (1) Both glass plates are of uniform temperature with insulated interconnecting walls and (2) Negligible radiation exchange.

PROPERTIES: *Table A-4*, Air (278K, 1 atm): $\nu = 13.93 \times 10^{-6}$ m^2/s, k = 0.0245 W/m·K, $\alpha = 19.6 \times 10^{-6}$ m^2/s, Pr = 0.71, $\beta = 0.00360$ K^{-1}.

ANALYSIS: For the vertical cavity,

$$Ra_L = \frac{g\beta(T_1 - T_2)L^3}{\alpha\nu} = \frac{9.8\,m/s^2\left(0.00360\,K^{-1}\right)(30°C)(0.06m)^3}{19.6\times10^{-6}m^2/s\times13.93\times10^{-6}m^2/s}$$

$$Ra_L = 8.37\times10^5.$$

With (H/L) = 20, Eq. 9.52 may be used as a first approximation for Pr = 0.71,

$$\overline{Nu}_L = 0.42\,Ra_L^{1/4}\,Pr^{0.012}(H/L)^{-0.3} = 0.42\left(8.37\times10^5\right)^{1/4}(0.71)^{0.012}(20)^{-0.3}$$

$$\overline{Nu}_L = 5.2$$

$$\bar{h} = \overline{Nu}_L\,\frac{k}{L} = 5.2\,\frac{0.0245\,W/m\cdot K}{0.06m} = 2.1\,W/m^2\cdot K.$$

The heat loss by free convection is then

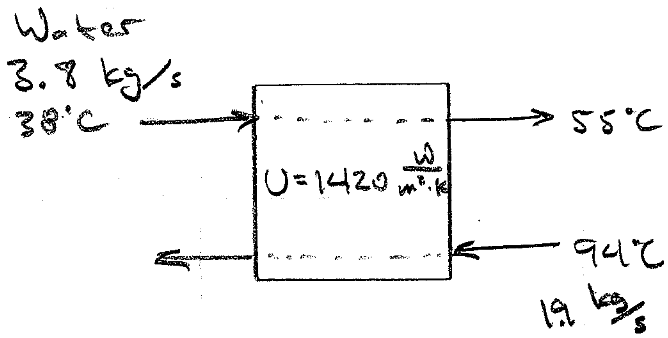
$$q = \bar{h}A(T_1 - T_2)$$

$$q = 2.1\,W/m^2\cdot K(1.2m\times0.8m)(30°C) = 61\,W.$$ <

COMMENTS: In such an application, radiation losses should also be considered, and infiltration effects could render heat loss by free convection significant.

SOLUTION: Water - Water heat exchanger

First, to simplify things let's use a constant c_p of $4184 \ \text{J/kg·K}$.



The hot water flow is exactly half that of the cold water flow, so the temperature decrease will be exactly twice as much as the cold water temperature increase:

$$q = \dot{m} c_p \Delta T \quad \Rightarrow \quad T_{out,h} = 60°C$$

We are given the overall mass flow rate of water in the tubes, plus the velocity in the tubes, so it's straightforward to determine how many tubes there are:

$$\dot{m} = n \rho V A$$

$\dot{m}$: $3.8 \ \frac{kg}{s}$

A: tube cross section area

V: velocity in tubes

ρ: density

n: number of tubes

continued...

$$n = \frac{\dot{m}}{\rho V A}$$

$$= \frac{(3.8 \ \mathrm{kg/s})}{(989 \ \frac{kg}{m^3})(0.366 \ \frac{m}{s})(\frac{\pi}{4} \ 0.01905^2 \ m^2)}$$

$$= \underline{\underline{37 \text{ tubes}}}$$

→ Now these 37 tubes may make multiple passes until the necessary heat transfer area is achieved

$$C_{hot} = \dot{m} C_p = (1.9 \ \tfrac{kg}{s})(4184 \ \tfrac{J}{kg \cdot K})$$

$$= 7950 \ \frac{W}{K} \leftarrow C_{min}$$

$$C_{cold} = 2 \times C_{hot} = 15{,}900 \ W/K \leftarrow C_{max}$$

$$C_r = C_{min}/C_{max} = 0.5$$

$$q = \dot{m} C_p (T_{out} - T_{in})$$

$$= 3.8(4184)(55-38) = 270{,}200 \text{ Watts}$$

continued...

From Eq. 11.22

$$q = \varepsilon C_{min}(T_{h,i} - T_{c,i}) = 270,200 \text{ W}$$

$$\varepsilon = \frac{270,200}{7950(94-38)}$$

$$= 0.607$$

Using this and $C_r = 0.5$ we can look up the NTU value from Fig. 11.12

$$NTU = \frac{UA}{C_{min}} = \sim 1.3$$

$$A = 1.3 \frac{C_{min}}{U}$$

$$= 1.3 \frac{(7950 \text{ W/K})}{1420 \text{ W/m}^2\cdot\text{K}}$$

$$= 7.28 \text{ m}^2$$

Each double pass of the 37 tubes will effectively give 74 tubes. The length for the 74 tubes to achieve $A = 7.28 \text{ m}^2$ can be calculated by...

$$A = n \pi D L$$

$$L = \frac{A}{n \pi D}$$

$$= \frac{7.28 \text{ m}^2}{74(\pi)(0.01905 \text{ m})}$$

$= \underline{1.64 \text{ meters}}$ per double pass

→ This will fit into the 2.24 m we have available, so we only need 2 tube passes.

a) 2 tube passes

b.) 37 tubes per pass

c.) 1.64 m per pass

FLUORESCENT LIGHT BULB PROBLEM

→ Use properties at the film temperature

$= (25 + 140)/2 = 83\ °C = 356\ K$

$\nu = 2.153 \times 10^{-5}\ m^2/s$

$Pr = 0.699$

$\alpha = 3.079 \times 10^{-5}\ m^2/s$

$k = 0.0304\ W/m \cdot K$

→ Treat this as a long horizontal cylinder (section 9.6.3)

$$Ra_D = \frac{g\beta(T_s - T_\infty)D^3}{\nu\alpha}$$

$$= \frac{(9.81\ \frac{m}{s^2})(1/356\ K)(140 - 25\ K)(0.035\ m)^3}{(2.153 \times 10^{-5}\ \frac{m^2}{s})(3.079 \times 10^{-5}\ \frac{m^2}{s})}$$

$$= 2.052 \times 10^5$$

→ Now, we could use Eq. 9.33 or 9.34. I'll use 9.34.

$$\overline{Nu_D} = \left\{ 0.60 + \frac{0.387\, Ra_D^{1/6}}{\left[1 + (0.559/Pr)^{9/16}\right]^{8/27}} \right\}^2$$

→ with $Ra_D = 2.052 \times 10^5$ and
$Pr = 0.699$ we get

$$Nu_D = \frac{hD}{k} = 9.39$$

$$h = \frac{9.39\,(0.0304\ W/m \cdot K)}{0.035\ m} = 8.16\ \frac{W}{m^2 \cdot K}$$

$$q = hA\Delta T$$

$$= \left(8.16\ \frac{W}{m^2 \cdot K}\right)(\pi)(0.035\ m)(0.8\ m)(140 - 25\ K)$$

$$= \boxed{82.5\ \text{Watts}}$$

→ The remaining 17.5 watts go to light, conduction losses, etc.

SOLAR ENERGY COLLECTOR PROBLEM

- This is a "natural convection above a horizontal heated plate" problem, so section 9.6.2 applies and $L = \frac{A_s}{P}$

- Look up air properties at $T_{film} = 37.5°C$

$$\nu = 1.687 \times 10^{-5} \ \frac{m^2}{s}$$

$$\alpha = 2.398 \times 10^{-5} \ \frac{m^2}{s}$$

$$Pr = 0.706$$

$$k = 0.027 \ W/m \cdot K$$

$$\beta = \frac{1}{310 \ K} = 0.00322 \ K^{-1}$$

- Calculate Ra_L, but first

$$L = \frac{A_s}{P} = \frac{(6 \ m)^2}{4(6 \ m)} = 1.5 \ m$$

$$Ra_L = \frac{g\beta(T_s - T_\infty)L^3}{\nu \alpha}$$

$$= \frac{(9.81 \ m/s^2)(0.00322 \ K^{-1})(65 - 10 \ K)(1.5 m)^3}{\left(1.687 \times 10^{-5} \ \frac{m^2}{s}\right)\left(2.398 \times 10^{-5} \ \frac{m^2}{s}\right)}$$

$$= 1.450 \times 10^{10}$$

→ For this Ra, Equation 9.31 is right

$$\overline{Nu}_L = 0.15\, Ra_L^{1/3}$$

$$= 0.15\left(1.450 \times 10^{10}\right)^{1/3}$$

$$= 366$$

$$h = \frac{\overline{Nu}_L k}{L} = \frac{366\left(0.027 \frac{W}{m \cdot K}\right)}{1.5\ m}$$

$$= 6.58\ W/m^2 \cdot K$$

$$q = hA\Delta T = \left(6.58 \frac{W}{m^2 \cdot K}\right)\left(6\ m\right)^2\left(65-10\right)$$

$$= 13{,}000 \text{ Watts} \quad \text{or} \quad 362\ W/m^2$$

$$362/630 = 0.574$$

So, $\boxed{57.4\% \text{ is lost}}$