

Homework 8 – Problem 1

- A. Define the following terms: (use symbols and equations if necessary)
- Thermal entrance region
 - Hydrodynamic entrance region
 - Fully developed flow
 - Log mean temperature difference
- B. Draw a simple diagram of each of the following types of concentric heat exchangers (Chapter 11)
- Parallel flow
 - Counterflow
-

- A. Define the following terms: (use symbols and equations if necessary)

- a. Thermal entrance region

For laminar flow:

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} = 0.05 Re_D Pr$$

For turbulent flow on the other hand, the entry region is approximated by Nusselt number correlations in Section 8.4.2.

- b. Hydrodynamic entrance region

For laminar flow:

$$\left(\frac{x_{fd,h}}{D}\right)_{lam} = 0.05 Re_D$$

and for turbulent flow:

$$10 \leq \left(\frac{x_{fd,h}}{D}\right)_{turb} \leq 60$$

- c. Fully developed flow

Once a flow is characterized as ‘fully developed’ (and remains that way), the thermal and physical properties cease changing. This includes density and even the convective heat transfer coefficient.

- d. Log mean temperature difference

For external flow (bank of tubes): (i=in, o=out)

$$\Delta T_{LM} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{(T_s - T_i)}{(T_s - T_o)}\right)}$$

For heat exchangers: (h=hot, c=cold, i=in, o=out)

$$\Delta T_{LM} = \frac{(T_{h,o} - T_{c,o}) - (T_{h,i} - T_{c,i})}{\ln\left(\frac{(T_{h,o} - T_{c,o})}{(T_{h,i} - T_{c,i})}\right)}$$

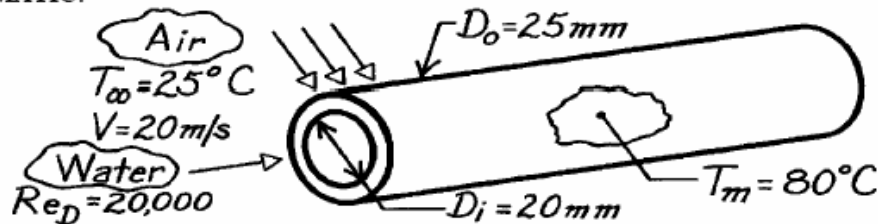
- B. See Figure 11.1

PROBLEM 8.57

KNOWN: Thick-walled pipe of thermal conductivity 60 W/m·K passing hot water with $Re_D = 20,000$, a mean temperature of 80°C, and cooled externally by air in cross-flow at 20 m/s and 25°C.

FIND: Heat transfer rate per unit pipe length, q' .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Internal flow is turbulent and fully developed.

PROPERTIES: Table A-6, Water ($T_m = 80^\circ\text{C} = 353\text{K}$): $k = 0.670\text{ W/m}\cdot\text{K}$, $Pr = 2.20$; Table A-4, Air ($T_\infty = 25^\circ\text{C} \approx 300\text{K}$, 1 atm): $\nu = 15.89 \times 10^{-6}\text{ m}^2/\text{s}$, $k = 0.0263\text{ W/m}\cdot\text{K}$, $Pr = 0.707$.

ANALYSIS: The heat rate per unit length, considering thermal resistances to internal flow, wall conduction (Eq. 3.28) and external flow, with $A = \pi DL$, is

$$q' = \left[1/h_i \pi D_i + (1/2\pi k) \ln(D_o/D_i) + 1/h_o \pi D_o \right]^{-1} (T_m - T_\infty).$$

Internal Flow: Using the Dittus-Boelter correlation with $n = 1/3$ for turbulent, fully developed flow, where $Re_{D_i} = 20,000$

$$h_i = (k/D_i) Nu_D = (k/D_i) 0.023 Re^{4/5} Pr^{1/3}$$

$$h_i = (0.670\text{ W/m}\cdot\text{K}/0.020\text{ m}) 0.023 (20,000)^{4/5} 2.20^{1/3} = 2765\text{ W/m}^2\cdot\text{K}.$$

External Flow: Using the Zukauskas correlation for cross-flow over a circular cylinder with $Pr/Pr_s \approx 1$, find first

$$Re_D = \frac{VD_o}{\nu} = \frac{20\text{ m/s} \times 0.025\text{ m}}{15.89 \times 10^{-6}\text{ m}^2/\text{s}} = 31,466$$

and from Table 7.4, $C = 0.26$ and $m = 0.6$, where $n = 0.37$,

$$Nu_D = \frac{h_o D}{k} = C Re_D^m Pr^n (Pr/Pr_s)^{1/4}$$

$$h_o = (0.0263\text{ W/m}\cdot\text{K}/0.025\text{ m}) 0.26 (31,466)^{0.6} (0.707)^{0.37} = 120\text{ W/m}^2\cdot\text{K}.$$

Hence, the heat rate is

$$q' = \left[\left(1/2765\text{ W/m}^2\cdot\text{K} \times \pi 0.020\text{ m} \right) + (1/2\pi 60\text{ W/m}\cdot\text{K}) \ln(25/20) \right. \\ \left. + \left(1/120\text{ W/m}^2\cdot\text{K} \times \pi 0.025\text{ m} \right) \right]^{-1} (80 - 25)^\circ\text{C}$$

$$q' = \left[5.756 \times 10^{-3} + 5.919 \times 10^{-4} + 1.061 \times 10^{-1} \right]^{-1} \text{ W/m}\cdot\text{K} (80 - 25)^\circ\text{C}$$

$$q' = 489\text{ W/m}.$$

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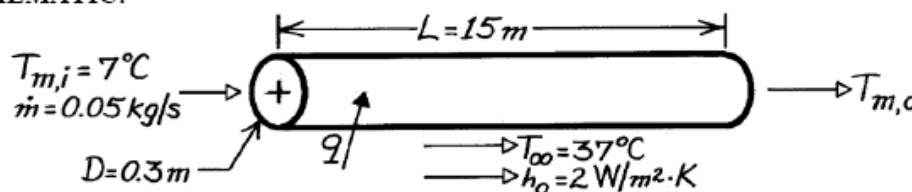
COMMENTS: Note that the external flow represents the major thermal resistance to heat transfer.

PROBLEM 8.62

KNOWN: Length and diameter of air conditioning duct. Inlet temperature of chilled air. Temperature and convection coefficient associated with outer air. Chilled air flowrate.

FIND: Chilled air exit temperature and heat flow rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible tube wall conduction resistance, (3) Ideal gas with negligible viscous dissipation, pressure variation, and axial conduction.

PROPERTIES: Table A-4, Air (300K, 1 atm): $c_p = 1007 \text{ J/kg}\cdot\text{K}$, $\mu = 184.6 \times 10^{-7} \text{ kg/s}\cdot\text{m}$, $k = 0.0263 \text{ W/m}\cdot\text{K}$, $Pr = 0.707$.

ANALYSIS: The exit temperature may be obtained from Eq. 8.45a, where

$$\bar{U} = (h_i^{-1} + h_o^{-1})^{-1}$$

$$\text{With } Re_D = (4\dot{m}/\pi D\mu) = \frac{4(0.05 \text{ kg/s})}{\pi(0.3 \text{ m})184.6 \times 10^{-7} \text{ kg/s}\cdot\text{m}} = 11,495$$

the flow is turbulent and, assuming fully developed conditions over the entire length, the Dittus-Boelter correlation yields

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(11,495)^{4/5} (0.707)^{0.4} = 35.5$$

$$h_i = Nu_D (k/D) = 35.5(0.0263 \text{ W/m}\cdot\text{K}/0.3 \text{ m}) = 3.11 \text{ W/m}^2 \cdot \text{K}$$

$$\text{and } \bar{U} = (3.11^{-1} + 2.0^{-1})^{-1} (\text{W/m}^2 \cdot \text{K}) = 1.22 \text{ W/m}^2 \cdot \text{K}.$$

$$\text{Eq. 8.45a yields } T_{m,o} = T_{\infty} - (T_{\infty} - T_{m,i}) \exp\left[-(\pi DL/\dot{m} c_p) \bar{U}\right]$$

$$T_{m,o} = 37^\circ\text{C} - 30^\circ\text{C} \exp\left[-\frac{\pi(0.3 \text{ m})15 \text{ m}(1.22 \text{ W/m}^2 \cdot \text{K})}{0.05 \text{ kg/s}(1007 \text{ J/kg}\cdot\text{K})}\right] = 15.7^\circ\text{C} \quad <$$

and the heat rate is

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.05 \text{ kg/s}(1007 \text{ J/kg}\cdot\text{K})(8.7^\circ\text{C}) = 438 \text{ W}. \quad <$$

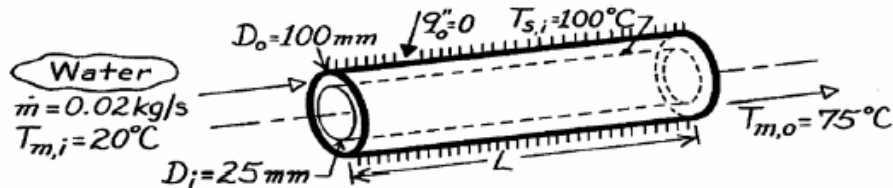
COMMENTS: The temperature rise of the chilled air is excessive, and the outer surface of the duct should be insulated to reduce \bar{U} and thereby $T_{m,o}$ and q .

PROBLEM 8.93

KNOWN: Surface thermal conditions and diameters associated with a concentric tube annulus. Water flow rate and inlet temperature.

FIND: (a) Length required to achieve desired outlet temperature, (b) Heat flux from inner tube at outlet.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed conditions throughout, (3) Adiabatic outer surface, (4) Uniform temperature at inner surface, (5) Constant properties, (6) Water is incompressible liquid with negligible viscous dissipation.

PROPERTIES: Table A-6, Water ($\bar{T}_m = 320\text{K}$): $c_p = 4180\text{ J/kg}\cdot\text{K}$, $\mu = 577 \times 10^{-6}\text{ N}\cdot\text{s/m}^2$, $k = 0.640\text{ W/m}\cdot\text{K}$, $\text{Pr} = 3.77$.

ANALYSIS: (a) From Eq. 8.41a,

$$L = -\frac{\dot{m} c_p}{Ph} \ln \frac{\Delta T_o}{\Delta T_i} = -\frac{\dot{m} c_p}{\pi D_i h} \ln \frac{T_s - T_{m,o}}{T_s - T_{m,i}}$$

$$\text{With } \text{Re}_D = \frac{\rho u_m D_h}{\mu} = \frac{\dot{m} (D_o - D_i)}{(\pi/4) (D_o^2 - D_i^2) \mu} = \frac{4 \dot{m}}{\pi (D_o + D_i) \mu}$$

$$\text{Re}_D = \frac{4 \times 0.02\text{ kg/s}}{\pi (0.125\text{ m}) 577 \times 10^{-6}\text{ N}\cdot\text{s/m}^2} = 353$$

the flow is laminar. Hence, from Eq. 8.69 and Table 8.2,

$$\bar{h} = h_i = \frac{k}{D_h} \text{Nu}_i = \frac{0.64\text{ W/m}\cdot\text{K}}{(0.100 - 0.025)\text{ m}} 7.37 = 63\text{ W/m}^2\cdot\text{K}$$

$$\text{and } L = -\frac{0.02\text{ kg/s} (4180\text{ J/kg}\cdot\text{K})}{\pi (0.025\text{ m}) 63\text{ W/m}^2\cdot\text{K}} \ln \frac{(100 - 75)^\circ\text{C}}{(100 - 20)^\circ\text{C}} = 19.7\text{ m.} \quad <$$

(b) From Eq. 8.68

$$q_i''(L) = h_i (T_{s,i} - T_{m,o}) = 63 \frac{\text{W}}{\text{m}^2\cdot\text{K}} (100 - 75)^\circ\text{C} = 1575\text{ W/m}^2. \quad <$$

COMMENTS: The total heat rate to the water is

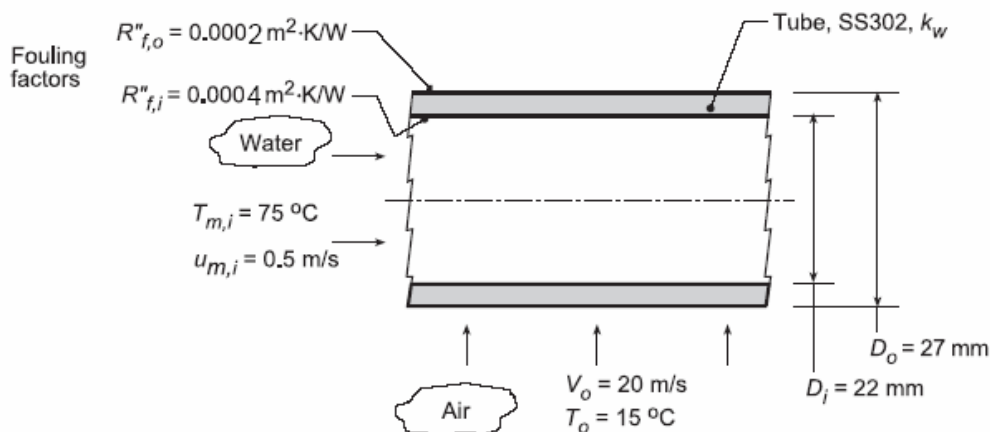
$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.02\text{ kg/s} \times 4180\text{ J/kg}\cdot\text{K} (55^\circ\text{C}) = 4598\text{ W.}$$

PROBLEM 11.2

KNOWN: Type-302 stainless tube with prescribed inner and outer diameters used in a cross-flow heat exchanger. Prescribed fouling factors and internal water flow conditions.

FIND: (a) Overall coefficient based upon the outer surface, U_o , with air at $T_o = 15^\circ\text{C}$ and velocity $V_o = 20$ m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (b) Overall coefficient, U_o , with water (rather than air) at $T_o = 15^\circ\text{C}$ and velocity $V_o = 1$ m/s in cross-flow; compare thermal resistances due to convection, tube wall conduction and fouling; (c) For the water-air conditions of part (a), compute and plot U_o as a function of the air cross-flow velocity for $5 \leq V_o \leq 30$ m/s for water mean velocities of $u_{m,i} = 0.2, 0.5$ and 1.0 m/s; and (d) For the water-water conditions of part (b), compute and plot U_o as a function of the water mean velocity for $0.5 \leq u_{m,i} \leq 2.5$ m/s for air cross-flow velocities of $V_o = 1, 3$ and 8 m/s.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Fully developed internal flow.

PROPERTIES: Table A.1, Stainless steel, AISI 302 (300 K): $k_w = 15.1$ W/m·K; Table A.6, Water ($\bar{T}_{m,i} = 348$ K): $\rho_i = 974.8$ kg/m³, $\mu_i = 3.746 \times 10^{-4}$ N·s/m², $k_i = 0.668$ W/m·K, $Pr_i = 2.354$; Table A.4, Air (assume $\bar{T}_{f,o} = 315$ K, 1 atm): $k_o = 0.02737$ W/m·K, $\nu_o = 17.35 \times 10^{-6}$ m²/s, $Pr_o = 0.705$.

ANALYSIS: (a) For the water-air condition, the overall coefficient, Eq. 11.1, based upon the outer area can be expressed as the sum of the thermal resistances due to convection (cv), tube wall conduction (w) and fouling (f):

$$1/U_o A_o = R_{\text{tot}} = R_{\text{cv},i} + R_{\text{f},i} + R_w + R_{\text{f},o} + R_{\text{cv},o}$$

$$R_{\text{cv},i} = 1/\bar{h}_i A_i \quad R_{\text{cv},o} = 1/\bar{h}_o A_o$$

$$R_{\text{f},i} = R''_{\text{f},i}/A_i \quad R_{\text{f},o} = R''_{\text{f},o}/A_o$$

and from Eq. 3.28,

$$R_w = \ln(D_o/D_i)/(2\pi L k_w)$$

The convection coefficients can be estimated from appropriate correlations.

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PROBLEM 11.2 (Cont.)

Estimating \bar{h}_i : For internal flow, characterize the flow evaluating thermophysical properties at $T_{m,i}$ with

$$Re_{D,i} = \frac{u_{m,i} D_i}{\nu_i} = \frac{0.5 \text{ m/s} \times 0.022 \text{ m}}{3.746 \times 10^{-4} \text{ N} \cdot \text{s/m}^2 / 974.8 \text{ kg/m}^3} = 28,625$$

For the turbulent flow, use the Dittus-Boelter correlation, Eq. 8.60,

$$Nu_{D,i} = 0.023 Re_{D,i}^{0.8} Pr_i^{0.3}$$

$$Nu_{D,i} = 0.023 (28,625)^{0.8} (2.354)^{0.3} = 109.3$$

$$\bar{h}_i = Nu_{D,i} k_i / D_i = 109.3 \times 0.668 \text{ W/m}^2 \cdot \text{K} / 0.022 \text{ m} = 3313 \text{ W/m}^2 \cdot \text{K}$$

Estimating \bar{h}_o : For external flow, characterize the flow with

$$Re_{D,o} = \frac{V_o D_o}{\nu_o} = \frac{20 \text{ m/s} \times 0.027 \text{ m}}{17.35 \times 10^{-6} \text{ m}^2/\text{s}} = 31,124$$

evaluating thermophysical properties at $T_{f,o} = (T_{s,o} + T_o)/2$ when the surface temperature is determined from the thermal circuit analysis result,

$$(T_{m,i} - T_o) / R_{\text{tot}} = (T_{s,o} - T_o) / R_{cv,o}$$

Assume $T_{f,o} = 315 \text{ K}$, and check later. Using the Churchill-Bernstein correlation, Eq. 7.54, find

$$\bar{Nu}_{D,o} = 0.3 + \frac{0.62 Re_{D,o}^{1/2} Pr_o^{1/3}}{\left[1 + (0.4/Pr_o)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{Re_{D,o}}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\bar{Nu}_{D,o} = 0.3 + \frac{0.62 (31,124)^{1/2} (0.705)^{1/3}}{\left[1 + (0.4/0.705)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{31,124}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\bar{Nu}_{D,o} = 102.6$$

$$\bar{h}_o = \bar{Nu}_{D,o} k_o / D_o = 102.6 \times 0.02737 \text{ W/m} \cdot \text{K} / 0.027 \text{ m} = 104.0 \text{ W/m} \cdot \text{K}$$

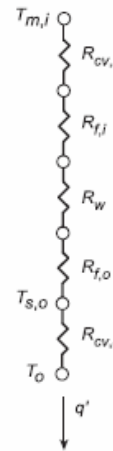
Using the above values for \bar{h}_i , and \bar{h}_o , and other prescribed values, the thermal resistances and overall coefficient can be evaluated and are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	U_o (W/m ² ·K)	R_{tot} (K/W)
0.00436	0.00578	0.00216	0.00236	0.1134	92.1	0.128

The major thermal resistance is due to outside (air) convection, accounting for 89% of the total resistance. The other thermal resistances are of similar magnitude, nearly 50 times smaller than $R_{cv,o}$.

(b) For the water-water condition, the method of analysis follows that of part (a). For the internal flow, the estimated convection coefficient is the same as part (a). For an assumed outer film coefficient, $\bar{T}_{f,o} = 292 \text{ K}$, the convection correlation for the outer water flow condition $V_o = 1 \text{ m/s}$ and $T_o = 15^\circ\text{C}$,

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PROBLEM 11.2 (Cont.)

find

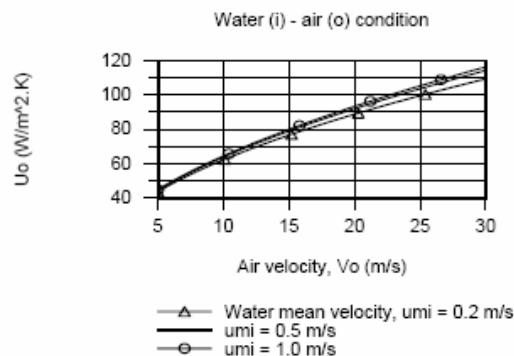
$$Re_{D,o} = 26,260 \quad Nu_{D,o} = 220.6 \quad \bar{h}_o = 4914 \text{ W/m}^2 \cdot \text{K}$$

The thermal resistances and overall coefficient are tabulated below.

$R_{cv,i}$ (K/W)	$R_{f,i}$ (K/W)	R_w (K/W)	$R_{f,o}$ (K/W)	$R_{cv,o}$ (K/W)	R_{tot} (K/W)	U_o (W/m ² ·K)
0.00436	0.00579	0.00216	0.00236	0.00240	0.0171	691

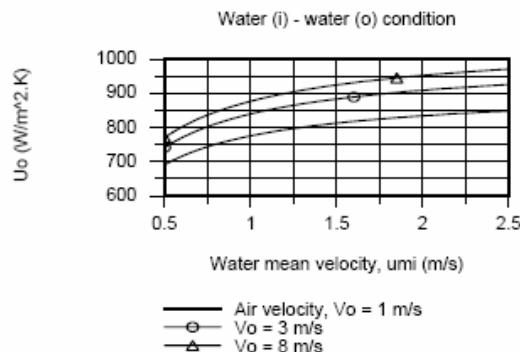
Note that the thermal resistances are of similar magnitude. In contrast with the results for the water-air condition of part (a), the thermal resistance of the outside convection process, $R_{cv,o}$, is nearly 50 times smaller. The overall coefficient for the water-water condition is 7.5 times greater than that for the water-air condition.

(c) For the water-air condition, using the IHT workspace with the analysis of part (a), U_o was calculated as a function of the air cross-flow velocity for selected mean water velocities.



The effect of increasing the cross-flow air velocity is to increase U_o since the $R_{cv,o}$ is the dominant thermal resistance for the system. While increasing the water mean velocity will increase \bar{h}_i , because $R_{cv,i} \ll R_{cv,o}$, this increase has only a small effect on U_o .

(d) For the water-water condition, using the IHT workplace with the analysis of part (b), U_o was calculated as a function of the mean water velocity for selected air cross-flow velocities.



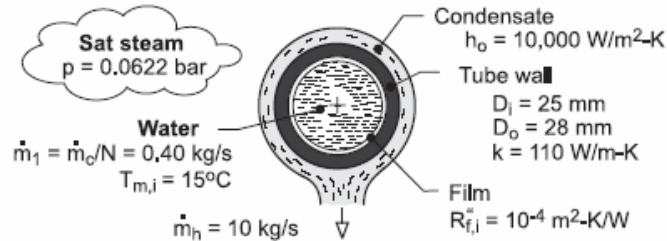
Because the thermal resistances for the convection processes, $R_{cv,i}$ and $R_{cv,o}$, are of similar magnitude according to the results of part (b), we expect to see U_o significantly increase with increasing water mean velocity and air cross-flow velocity.

PROBLEM 11.7

KNOWN: Number, inner and outer diameters, and thermal conductivity of condenser tubes. Convection coefficient at outer surface. Overall flow rate, inlet temperature and properties of water flow through the tubes. Flow rate and pressure of condensing steam. Fouling factor for inner surface.

FIND: (a) Overall coefficient based on outer surface area, U_o , without fouling, (b) Overall coefficient with fouling, (c) Temperature of water leaving the condenser.

SCHEMATIC:



ASSUMPTIONS: (1) Water is incompressible with negligible viscous dissipation, (2) Fully-developed flow in tubes, (3) Negligible effect of fouling on D_i .

PROPERTIES: Water (Given): $c_p = 4180 \text{ J/kg}\cdot\text{K}$, $\mu = 9.6 \times 10^{-4} \text{ N}\cdot\text{s/m}^2$, $k = 0.60 \text{ W/m}\cdot\text{K}$, $\text{Pr} = 6.6$.
Table A-6, Water, saturated vapor ($p = 0.0622 \text{ bars}$): $T_{\text{sat}} = 310 \text{ K}$, $h_{\text{fg}} = 2.414 \times 10^6 \text{ J/kg}$.

ANALYSIS: (a) Without fouling, Eq. 11.5 yields

$$\frac{1}{U_o} = \frac{1}{h_i} \left(\frac{D_o}{D_i} \right) + \frac{D_o \ln(D_o/D_i)}{2k_t} + \frac{1}{h_o}$$

With $\text{Re}_{D_i} = 4\dot{m}_1 / \pi D_i \mu = 1.60 \text{ kg/s} / (\pi \times 0.025 \text{ m} \times 9.6 \times 10^{-4} \text{ N}\cdot\text{s/m}^2) = 21,220$, flow in the tubes is turbulent, and from Eq. 8.60

$$h_i = \left(\frac{k}{D_i} \right) 0.023 \text{Re}_{D_i}^{4/5} \text{Pr}^{0.4} = \left(\frac{0.60 \text{ W/m}\cdot\text{K}}{0.025 \text{ m}} \right) 0.023 (21,200)^{4/5} (6.6)^{0.4} = 3400 \text{ W/m}^2 \cdot \text{K}$$

$$U_o = \left[\frac{1}{3400} \left(\frac{28}{25} \right) + \frac{0.028 \ln(28/25)}{2 \times 110} + \frac{1}{10,000} \right]^{-1} \text{ W/m}^2 \cdot \text{K} = \left(3.29 \times 10^{-4} + 1.44 \times 10^{-5} + 10^{-4} \right)^{-1} \text{ W/m}^2 \cdot \text{K} = 2255 \text{ W/m}^2 \cdot \text{K} <$$

(b) With fouling, Eq. 11.5 yields

$$U_o = \left[4.43 \times 10^{-4} + (D_o/D_i) R_{f,i} \right]^{-1} = \left(5.55 \times 10^{-4} \right)^{-1} = 1800 \text{ W/m}^2 \cdot \text{K} <$$

(c) The rate at which energy is extracted from the steam equals the rate of heat transfer to the water, $\dot{m}_h h_{\text{fg}} = \dot{m}_c c_p (T_{m,o} - T_{m,i})$, in which case

$$T_{m,o} = T_{m,i} + \frac{\dot{m}_h h_{\text{fg}}}{\dot{m}_c c_p} = 15^\circ\text{C} + \frac{10 \text{ kg/s} \times 2.414 \times 10^6 \text{ J/kg}}{400 \text{ kg/s} \times 4180 \text{ J/kg}\cdot\text{K}} = 29.4^\circ\text{C} <$$

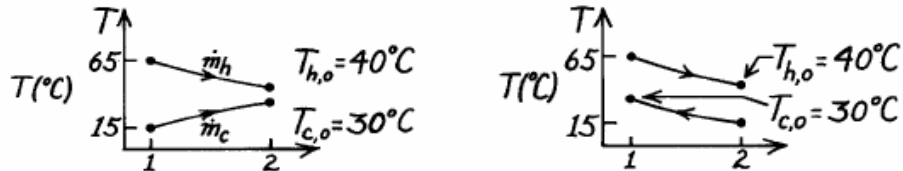
COMMENTS: (1) The largest contribution to the thermal resistance is due to convection at the interior of the tube. To increase U_o , h_i could be increased by increasing \dot{m}_1 , either by increasing \dot{m}_c or decreasing N . (2) Note that $T_{m,o} = 302.4 \text{ K} < T_{\text{sat}} = 310 \text{ K}$, as must be the case.

PROBLEM 11.40

KNOWN: Two-fluid heat exchanger with prescribed inlet and outlet temperatures of the two fluids.

FIND: (a) Whether exchanger is operating in parallel or counter flow, (b) Effectiveness of the exchanger when $C_c = C_{\min}$.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat loss to the surroundings.

ANALYSIS: (a) To determine whether operation is PF or CF, consider the temperature distributions. From the distributions we note that PF or CF operation is possible.

(b) The effectiveness of the exchanger follows from Eq. 11.19,

$$\varepsilon = q / q_{\max} \quad (1)$$

where from Eq. 11.18,

$$q_{\max} = C_{\min} (T_{h,i} - T_{c,i}). \quad (2)$$

Since the hot fluid undergoes a larger temperature change than the cold fluid, $C_{\min} = C_h$ and performing an energy balance on the cold fluid, Eq. (1) with Eq. (2) becomes

$$\varepsilon = C_h (T_{h,i} - T_{h,o}) / C_{\min} (T_{h,i} - T_{c,i}) = (T_{h,i} - T_{h,o}) / (T_{h,i} - T_{c,i})$$

$$\varepsilon = (65 - 40)^\circ\text{C} / (65 - 15)^\circ\text{C} = 0.50. \quad <$$

COMMENTS: If $T_{c,o}$ were greater than $T_{h,o}$, parallel-flow operation would not be possible.