

Homework 7 – Problem 1 Solution

**Part (a)**

| Dimensionless Group | Definition   | Symbols                                    |
|---------------------|--|--|
| Reynolds            | Ratio of the inertia and viscous forces              | $Re_{plate} = \frac{vL\rho}{\mu}$          |
|                     |  | $Re_{pipe} = \frac{vD\rho}{\mu}$           |
| Nusselt             | Ratio of convection to pure conduction heat transfer | $Nu = \frac{hL}{k_f}$ (f=fluid)            |
| Prandtl             | Ratio of the momentum and thermal diffusivities      | $Pr = \frac{c_p\mu}{k} = \frac{v}{\alpha}$ |

**Part (b)**

Laminar flow is typically characterized as a visually even flow profile, similar to a slow-moving stream and has a parabolic velocity profile for the case of a pipe. Turbulent flow is characterized as a more violent flow regime with more dimensions being considered than with laminar flow; think of white-water rapids and the like versus the slow-moving stream. For plate flow, laminar flow exists from a Reynolds number of 0 to ~500,000. The value 500,000 is called the critical Reynolds number and anything above this is termed as turbulent and have already gone through the transition flow regime. For pipe flow, laminar flow exists for Reynolds numbers from 0 to ~2,000, the transition phase is in the range of 2,000 to 4,000, and turbulent flow is deemed anything above 4,000.

**Part (c)**

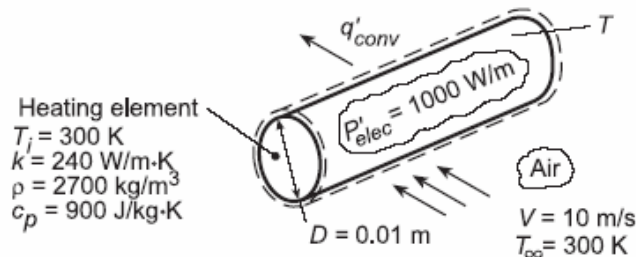
|     | Case  | Temperature  |
|-----|---|--|
| i   | Flow over a flat plate                                    | Film Temp: $T_f \frac{T_s + T_\infty}{2}$              |
| ii  | Flow around a cylinder (Hilpert relation)                 | Film Temp: $T_f \frac{T_s + T_\infty}{2}$              |
| iii | Flow around a cylinder (Zukauskas relation)               | $T_\infty$   |
| iv  | Flow around a cylinder (Churchill and Bernstein relation) | Film Temp: $T_f \frac{T_s + T_\infty}{2}$              |
| v   | Flow around a sphere (Whitaker relation)                  | $T_\infty$   |
| vi  | Flow through a bank of tubes (Grimison relation)          | Film Temp: $T_f \frac{T_s + T_\infty}{2}$              |
| vii | Flow through a bank of tubes (Zukauskas relation)         | Arithmetic mean of fluid inlet and outlet temperatures |

### PROBLEM 7.43

**KNOWN:** Initial temperature, power dissipation, diameter, and properties of heating element. Velocity and temperature of air in cross flow.

**FIND:** (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Uniform heater temperature, (2) Negligible radiation.

**PROPERTIES:** Table A.4, air (assume  $T_f \approx 450 \text{ K}$ ):  $\nu = 32.39 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $k = 0.0373 \text{ W/m}\cdot\text{K}$ ,  $\text{Pr} = 0.686$ .

**ANALYSIS:** (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{\text{conv}} = \bar{h}(\pi D)(T - T_{\infty}) = P'_{\text{elec}} = 1000 \text{ W/m}$$

With

$$\text{Re}_D = \frac{VD}{\nu} = \frac{(10 \text{ m/s})(0.01 \text{ m})}{32.39 \times 10^{-6} \text{ m}^2/\text{s}} = 3,087$$

the Churchill and Bernstein correlation, Eq. 7.54, yields

$$\bar{\text{Nu}}_D = 0.3 + \frac{0.62 \text{Re}_D^{1/2} \text{Pr}^{1/3}}{\left[1 + (0.4/\text{Pr})^{2/3}\right]^{1/4}} \left[1 + \left(\frac{\text{Re}_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\bar{\text{Nu}}_D = 0.3 + \frac{0.62(3087)^{1/2} (0.686)^{1/3}}{\left[1 + (0.4/0.686)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{3087}{282,000}\right)^{5/8}\right]^{4/5} = 28.2$$

$$\bar{h} = \frac{k}{D} \bar{\text{Nu}}_D = \frac{0.0373 \text{ W/m}\cdot\text{K}}{0.010 \text{ m}} 28.2 = 105.2 \text{ W/m}^2 \cdot \text{K}$$

Hence, the steady-state temperature is

$$T = T_{\infty} + \frac{P'_{\text{elec}}}{\pi D \bar{h}} = 300 \text{ K} + \frac{1000 \text{ W/m}}{\pi(0.01 \text{ m})105.2 \text{ W/m}^2 \cdot \text{K}} = 603 \text{ K} \quad <$$

(b) With  $\text{Bi} = \bar{h}r_0/k = 105.2 \text{ W/m}^2 \cdot \text{K}(0.005 \text{ m})/240 \text{ W/m}\cdot\text{K} = 0.0022$ , a lumped capacitance analysis may be performed. The time response of the heater is given by Eq. 5.25, which, for  $T_i = T_{\infty}$ , reduces to

$$T = T_{\infty} + (b/a)[1 - \exp(-at)]$$

Continued...

**PROBLEM 7.43 (Cont.)**

where  $a = 4\bar{h}/D\rho c_p = (4 \times 105.2 \text{ W/m}^2 \cdot \text{K}) / (0.01 \text{ m} \times 2700 \text{ kg/m}^3 \times 900 \text{ J/kg} \cdot \text{K}) = 0.0173 \text{ s}^{-1}$  and  $b/a = P'_{\text{elec}}/\pi D\bar{h} = 1000 \text{ W/m} / \pi (0.01 \text{ m} \times 105.2 \text{ W/m}^2 \cdot \text{K}) = 302.6 \text{ K}$ . Hence,

$$[1 - \exp(-0.0173t)] = \frac{(593 - 300) \text{ K}}{302.6 \text{ K}} = 0.968$$

$$t \approx 200 \text{ s}$$

<

**COMMENTS:** (1) For  $T = 603 \text{ K}$  and a representative emissivity of  $\varepsilon = 0.8$ , net radiation exchange between the heater and surroundings at  $T_{\text{sur}} = T_{\infty} = 300 \text{ K}$  would be  $q'_{\text{rad}} = \varepsilon\sigma(\pi D)(T^4 - T_{\text{sur}}^4) = 0.8 \times 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 (\pi \times 0.01 \text{ m})(603^4 - 300^4) \text{ K}^4 = 177 \text{ W/m}$ . Hence, although small, radiation exchange is not negligible. The effects of radiation are considered in Problem 7.46.

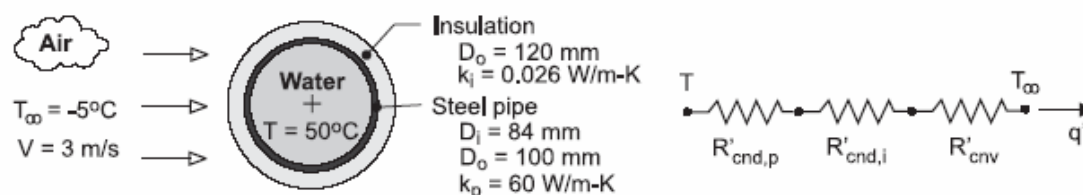
(2) The assumed value of  $T_f$  is very close to the actual value, rendering the selected air properties accurate.

### PROBLEM 7.56

**KNOWN:** Diameter, thickness and thermal conductivity of steel pipe. Temperature of water flow in pipe. Temperature and velocity of air in cross flow over pipe. Cost of producing hot water.

**FIND:** (a) Cost of daily heat loss from an uninsulated pipe, (b) Savings associated with insulating the pipe.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state, (2) Negligible convection resistance for water flow, (3) Negligible contact resistance between insulation and pipe, (4) Negligible radiation.

**PROPERTIES:** Table A-4, air ( $p = 1 \text{ atm}$ ,  $T_f \approx 300 \text{ K}$ ):  $k_a = 0.0263 \text{ W/m}\cdot\text{K}$ .

$$\nu = 15.89 \times 10^{-6} \text{ m}^2/\text{s}, \text{ Pr} = 0.707.$$

**ANALYSIS:** (a) With  $Re_D = VD_o/\nu = 3 \text{ m/s} \times 0.1 \text{ m} / 15.89 \times 10^{-6} \text{ m}^2/\text{s} = 18,880$ , application of the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = 0.3 + \frac{0.62(18,880)^{1/2}(0.707)^{1/3}}{\left[1 + (0.4/0.707)^{2/3}\right]^{1/4}} \left[1 + \left(\frac{18,880}{282,000}\right)^{5/8}\right]^{4/5} = 76.6$$

$$\bar{h} = \frac{k_a \overline{Nu}_D}{D_o} = \frac{0.0263 \text{ W/m}\cdot\text{K}}{0.1 \text{ m}} 76.6 = 20.1 \text{ W/m}^2\cdot\text{K}$$

Without the insulation, the total thermal resistance and heat loss per length of pipe are then

$$R'_{\text{tot(wo)}} = \frac{\ln(D_o/D_i)}{2\pi k_p} + \frac{1}{\pi D_o \bar{h}} = \frac{\ln(100/84)}{2\pi \times 60 \text{ W/m}\cdot\text{K}} + \frac{1}{\pi(0.1 \text{ m})20.1 \text{ W/m}^2\cdot\text{K}}$$

$$= (4.63 \times 10^{-4} + 0.158) \text{ m}\cdot\text{K/W} = 0.159 \text{ m}\cdot\text{K/W}$$

$$q'_{\text{wo}} = \frac{T - T_\infty}{R'_{\text{tot(wo)}}} = \frac{55^\circ\text{C}}{0.159 \text{ m}\cdot\text{K/W}} = 346 \text{ W/m} = 0.346 \text{ kW/m}$$

The corresponding daily energy loss is

$$Q'_{\text{wo}} = 0.346 \text{ kW/m} \times 24 \text{ h/d} = 8.3 \text{ kW}\cdot\text{h/m}\cdot\text{d}$$

and the associated cost is

$$C'_{\text{wo}} = (8.3 \text{ kW}\cdot\text{h/m}\cdot\text{d})(\$0.05/\text{kW}\cdot\text{h}) = \$0.415/\text{m}\cdot\text{d}$$

(b) The conduction resistance of the insulation is

Continued .....

**PROBLEM 7.56 (Cont.)**

$$R'_{\text{cnd}} = \frac{\ln(D_o/D_i)}{2\pi k_i} = \frac{\ln(120/100)}{2\pi(0.026 \text{ W/m}\cdot\text{K})} = 1.116 \text{ m}\cdot\text{K/W}$$

Using the Churchill-Bernstein correlation with an outside diameter of  $D_o = 0.12\text{m}$ ,  $Re_D = 22,660$ ,  $\overline{Nu}_D = 83.9$  and  $\overline{h} = 18.4 \text{ W/m}^2\cdot\text{K}$ . The convection resistance is then

$$R'_{\text{cnv}} = \frac{1}{\pi D_o \overline{h}} = \frac{1}{\pi(0.12\text{m})18.4 \text{ W/m}^2\cdot\text{K}} = 0.144 \text{ m}\cdot\text{K/W}$$

and the total resistance is

$$R'_{\text{tot(w)}} = (4.63 \times 10^{-4} + 1.116 + 0.144) \text{ m}\cdot\text{K/W} = 1.261 \text{ m}\cdot\text{K/W}$$

The heat loss and cost are then

$$q'_w = \frac{T - T_\infty}{R'_{\text{tot(w)}}} = \frac{55^\circ\text{C}}{1.261 \text{ m}\cdot\text{K/W}} = 43.6 \text{ W/m} = 0.0436 \text{ kW/m}$$

$$C'_w = 0.0436 \text{ kW/m} \times 24 \text{ h/d} \times \$0.05/\text{kW}\cdot\text{h} = \$0.052/\text{m}\cdot\text{d}$$

The daily savings is then

$$S' = C'_{w0} - C'_w = \$0.363/\text{m}\cdot\text{d} \quad <$$

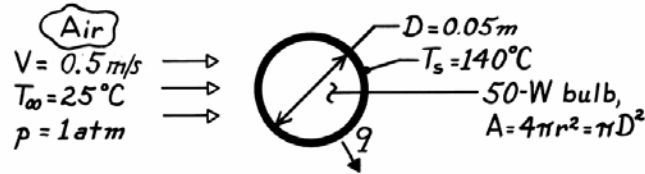
**COMMENTS:** (1) The savings are significant, and the pipe should be insulated. (2) Assuming a negligible temperature drop across the pipe wall, a pipe emissivity of  $\epsilon_p = 0.6$  and surroundings at  $T_{\text{sur}} = 268\text{K}$ , the radiation coefficient associated with the uninsulated pipe is  $h_r = \epsilon\sigma(T + T_{\text{sur}})(T^2 + T_{\text{sur}}^2) = 0.6 \times 5.67 \times 10^{-8} \text{ W/m}^2\cdot\text{K}^4 (591\text{K}) (323^2 + 268^2) \text{ K}^2 = 3.5 \text{ W/m}^2\cdot\text{K}$ . Accordingly, radiation increases the heat loss estimate of Part (a) by approximately 17%.

**PROBLEM 7.68**

**KNOWN:** Conditions associated with airflow over a spherical light bulb of prescribed diameter and surface temperature.

**FIND:** Heat loss by convection.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Uniform surface temperature.

**PROPERTIES:** Table A-4, Air ( $T_\infty = 25^\circ\text{C}$ , 1 atm):  $\nu = 15.71 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $k = 0.0261 \text{ W/m}\cdot\text{K}$ ,  $\text{Pr} = 0.71$ ,  $\mu = 183.6 \times 10^{-7} \text{ N}\cdot\text{s/m}^2$ ; Table A-4, Air ( $T_s = 140^\circ\text{C}$ , 1 atm):  $\mu = 235.5 \times 10^{-7} \text{ N}\cdot\text{s/m}^2$ .

**ANALYSIS:** The heat rate by convection is

$$q = \bar{h}(\pi D^2) (T_s - T_\infty)$$

where  $\bar{h}$  may be estimated from the Whitaker relation

$$\bar{h} = \frac{k}{D} \left[ 2 + \left( 0.4 \text{Re}_D^{1/2} + 0.06 \text{Re}_D^{2/3} \right) \text{Pr}^{0.4} \left( \mu / \mu_s \right)^{1/4} \right]$$

where

$$\text{Re}_D = \frac{VD}{\nu} = \frac{0.5 \text{ m/s} \times 0.05 \text{ m}}{15.71 \times 10^{-6} \text{ m}^2/\text{s}} = 1591.$$

Hence,

$$\bar{h} = \frac{0.0261 \text{ W/m}\cdot\text{K}}{0.05 \text{ m}} \left\{ 2 + \left[ 0.4(1591)^{1/2} + 0.06(1591)^{2/3} \right] (0.71)^{0.4} \left( \frac{183.6}{235.5} \right)^{1/4} \right\}$$

$$\bar{h} = 11.4 \text{ W/m}^2 \cdot \text{K}$$

and the heat rate is

$$q = 11.4 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \pi (0.05 \text{ m})^2 (140 - 25)^\circ\text{C} = 10.3 \text{ W.} \quad <$$

**COMMENTS:** (1) The low value of  $\bar{h}$  suggests that heat transfer by free convection may be significant and hence that the total loss by convection exceeds 10.3 W.

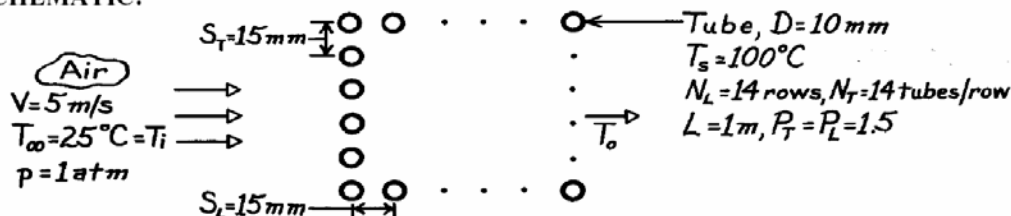
(2) The surface of the bulb also dissipates heat to the surrounding by radiation. Further, in an actual light bulb, there is also heat loss by conduction through the socket.

### PROBLEM 7.84

**KNOWN:** Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross flow.

**FIND:** (a) Total heat transfer, (b) Air flow pressure drop.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible radiation, (3) Uniform surface temperature.

**PROPERTIES:** Table A-4, Atmospheric air ( $T_\infty = 298$  K):  $\nu = 15.8 \times 10^{-6} \text{ m}^2/\text{s}$ ,  $k = 0.0263 \text{ W/m}\cdot\text{K}$ ,  $\text{Pr} = 0.707$ ,  $c_p = 1007 \text{ J/kg}\cdot\text{K}$ ,  $\rho = 1.17 \text{ kg/m}^3$ ; ( $T_s = 373$  K):  $\text{Pr} = 0.695$ .

**ANALYSIS:** (a) The total heat transfer rate is

$$q = \bar{h} N \pi D L \frac{(T_s - T_i) - (T_s - T_o)}{\ln[(T_s - T_i)/(T_s - T_o)]} = \bar{h} N \pi D L \Delta T_{lm}$$

$$\text{With } V_{\max} = \frac{S_T}{S_T - D} V = \frac{15 \text{ mm}}{5 \text{ mm}} 5 \text{ m/s} = 15 \text{ m/s}, \text{Re}_{D,\max} = \frac{15 \text{ m/s}(0.01 \text{ m})}{15.8 \times 10^{-6} \text{ m}^2/\text{s}} = 9494. \quad \text{Tables 7.7}$$

and 7.8 give  $C = 0.27$ ,  $m = 0.63$  and  $C_2 \approx 0.99$ . Hence, from the Zukauskas correlation

$$\overline{\text{Nu}}_D = 0.99 \times 0.27 (9494)^{0.63} (0.707)^{0.36} (0.707/0.695)^{1/4} = 75.9$$

$$\bar{h} = \overline{\text{Nu}}_D k/D = 75.9 \times 0.0263 \text{ W/m}\cdot\text{K}/0.01 \text{ m} = 200 \text{ W/m}^2 \cdot \text{K}$$

$$T_s - T_o = (T_s - T_i) \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right) = 75^\circ\text{C} \exp\left(-\frac{\pi \times 0.01 \text{ m} \times 196 \times 200 \text{ W/m}^2 \cdot \text{K}}{1.17 \text{ kg/m}^3 \times 5 \text{ m/s} \times 14 \times 0.015 \text{ m} \times 1007 \text{ J/kg}\cdot\text{K}}\right)$$

$$T_s - T_o = 27.7^\circ\text{C}.$$

Hence

$$q = 200 \text{ W/m}^2 \cdot \text{K} \times 196 \pi (0.01 \text{ m}) 1 \text{ m} \frac{75^\circ\text{C} - 27.7^\circ\text{C}}{\ln(75/27.7)} = 58.5 \text{ kW}. \quad \leftarrow$$

(b) With  $\text{Re}_{D,\max} = 9494$ ,  $(P_T - 1)/(P_L - 1) = 1$ , Fig. 7.13 yields  $f \approx 0.32$  and  $\chi = 1$ . Hence,

$$\Delta p = N \chi \left( \rho V_{\max}^2 / 2 \right) f = 14 \times 1 \left( \frac{1.17 \text{ kg/m}^3 (15 \text{ m/s})^2}{2} \right) 0.32$$

$$\Delta p = 590 \text{ N/m}^2 = 5.9 \times 10^{-3} \text{ bar}.$$

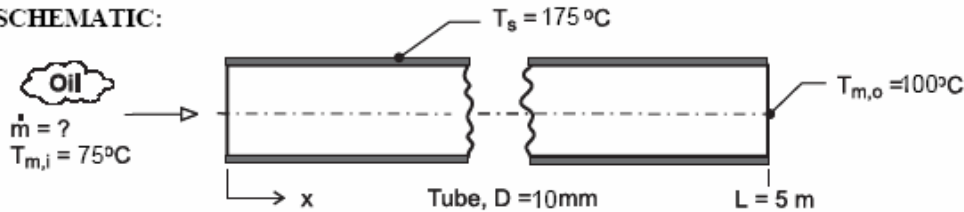
**COMMENTS:** The heat transfer rate would have been substantially overestimated (93.3 kW) if the inlet temperature difference ( $T_s - T_i$ ) had been used in lieu of the log-mean temperature difference.

### PROBLEM 8.24

**KNOWN:** Oil at 75°C enters a single-tube preheater of 10-mm diameter and 5-m length; tube surface maintained at 175°C by swirling combustion gases.

**FIND:** Determine the flow rate and heat transfer rate when the outlet temperature is 95°C.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Combined entry length, laminar flow, (2) Tube wall is isothermal, (3) Incompressible liquid with negligible viscous dissipation, (4) Constant properties.

**PROPERTIES:** Table A-5, Engine oil, new ( $T_m = (T_{m,i} + T_{m,o})/2 = 361$  K):  $\rho = 847.5$  kg/m<sup>3</sup>,  $c_p = 2163$  J/kg·K,  $\nu = 2.931 \times 10^{-5}$  m<sup>2</sup>/s,  $k = 0.1379$  W/m·K,  $Pr = 390.2$ ,  $\mu = 0.0245$ .

**ANALYSIS:** The overall energy balance, Eq. 8.34, and rate equation, Eq. 8.42b, are

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) \quad (1)$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m} c_p}\right) \quad (2)$$

Not knowing the flow rate  $\dot{m}$ , the Reynolds number cannot be calculated. Assume that the flow is laminar, and the combined entry length condition occurs. The average convection coefficient can be estimated using the Hausen correlation, Eq. 8.56,

$$\bar{Nu}_D = 3.66 + \frac{0.0668(D/L) Re_D Pr}{1 + 0.04[(D/L) Re_D Pr]^{2/3}} \quad (3)$$

where all properties are evaluated at  $T_m = (T_{m,i} + T_{m,o})/2$ . The Reynolds number follows from Eq. 8.6,

$$Re_D = 4\dot{m} / \pi D \mu \quad (4)$$

A tedious trial-and-error solution is avoided by using *IHT* to solve the system of equations with the following result:

| $Re_D$ | $\bar{Nu}_D$ | $\bar{h}_D$ (W/m <sup>2</sup> ·K) | $q$ (W) | $\dot{m}$ (kg/h) | < |
|--------|--------------|-----------------------------------|---------|------------------|---|
| 130    | 7.25         | 100                               | 1360    | 90               |   |

Note that the flow is laminar, and evaluating  $x_{fd}$  using Eq. 8.3, find  $x_{fd,h} = 0.065$  m so the thermal entry length condition is appropriate.

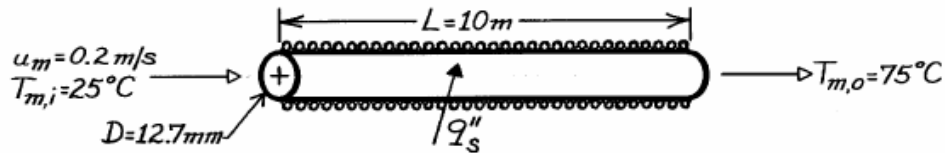


### PROBLEM 8.27

**KNOWN:** Inlet and outlet temperatures and velocity of fluid flow in tube. Tube diameter and length.

**FIND:** Surface heat flux and temperatures at  $x = 0.5$  and  $10$  m.

**SCHEMATIC:**



**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to surroundings, (4) Incompressible liquid with negligible viscous dissipation, (5) Negligible axial conduction.

**PROPERTIES:** Pharmaceutical (given):  $\rho = 1000\text{ kg/m}^3$ ,  $c_p = 4000\text{ J/kg}\cdot\text{K}$ ,  $\mu = 2 \times 10^{-3}\text{ kg/s}\cdot\text{m}$ ,  $k = 0.80\text{ W/m}\cdot\text{K}$ ,  $\text{Pr} = 10$ .

**ANALYSIS:** With

$$\dot{m} = \rho VA = 1000\text{ kg/m}^3 (0.2\text{ m/s}) \pi (0.0127\text{ m})^2 / 4 = 0.0253\text{ kg/s}$$

Eq. 8.34 yields

$$q = \dot{m} c_p (T_{m,o} - T_{m,i}) = 0.0253\text{ kg/s} (4000\text{ J/kg}\cdot\text{K}) 50\text{ K} = 5060\text{ W}.$$

The required heat flux is then

$$q_s'' = q/A_s = 5060\text{ W} / \pi (0.0127\text{ m}) 10\text{ m} = 12,682\text{ W/m}^2. \quad <$$

With

$$\text{Re}_D = \rho VD/\mu = 1000\text{ kg/m}^3 (0.2\text{ m/s}) 0.0127\text{ m} / 2 \times 10^{-3}\text{ kg/s}\cdot\text{m} = 1270$$

the flow is laminar and Eq. 8.23 yields

$$x_{fd,t} = 0.05 \text{Re}_D \text{Pr} D = 0.05 (1270) 10 (0.0127\text{ m}) = 8.06\text{ m}.$$

Hence, with fully developed hydrodynamic and thermal conditions at  $x = 10$  m, Eq. 8.53 yields

$$h(10\text{ m}) = \text{Nu}_{D,fd} (k/D) = 4.36 (0.80\text{ W/m}\cdot\text{K} / 0.0127\text{ m}) = 274.6\text{ W/m}^2\cdot\text{K}.$$

Hence, from Newton's law of cooling,

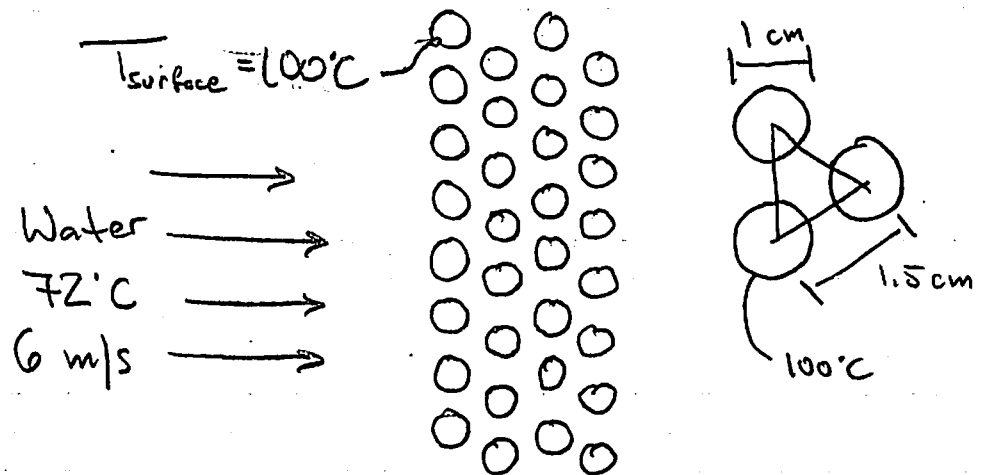
$$T_{s,o} = T_{m,o} + (q_s''/h) = 75^\circ\text{C} + (12,682\text{ W/m}^2 / 274.6\text{ W/m}^2\cdot\text{K}) = 121^\circ\text{C}. \quad <$$

At  $x = 0.5$  m,  $(x/D)/(\text{Re}_D \text{Pr}) = 0.0031$  and Figure 8.10 yields  $\text{Nu}_D \approx 8$  for a thermal entry region with uniform surface heat flux. Hence,  $h(0.5\text{ m}) = 503.9\text{ W/m}^2\cdot\text{K}$  and, since  $T_m$  increases linearly with  $x$ ,  $T_m(x = 0.5\text{ m}) = T_{m,i} + (T_{m,o} - T_{m,i})(x/L) = 27.5^\circ\text{C}$ . It follows that

$$T_s(x = 0.5\text{ m}) \approx 27.5^\circ\text{C} + (12,682\text{ W/m}^2 / 503.9\text{ W/m}^2\cdot\text{K}) = 52.7^\circ\text{C}. \quad <$$

# TUBE BUNDLE PROBLEM

Part a.)



Part b.) First, we need to know  $V_{\text{max}}$ .

For a triangular arrangement,  $S_T = S_D$

From the middle of Page 439

$$2(S_D - D) = 2(1.5 - 1.0) = 1.0 \text{ cm}$$

$$S_T - D = 1.5 - 1.0 = 0.5 \text{ cm}$$

$$\text{Thus, we use } V_{\text{max}} = \frac{S_T}{S_T - D} V$$

$$V_{\text{max}} = \frac{1.5 \text{ cm}}{1.5 - 1.0 \text{ cm}} V$$

$$= 3 V = \underline{\underline{18 \text{ m/s}}}$$

## Tube bundle prob, cont.

For a bank of tubes we should evaluate properties at the average of the inlet and outlet temps, but since we do not yet know the outlet temp we'll use the midpoint of  $T_{inlet}$  and  $T_{surface}$ :  $86^\circ\text{C}$  ( $359\text{K}$ )

$$k = 674 \times 10^{-3} \frac{\text{W}}{\text{m}\cdot\text{K}}$$

$$\mu = 324 \times 10^{-6} \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$\rho = \frac{1}{0.001038} = 964 \frac{\text{kg}}{\text{m}^3}$$

$$Pr = 2.02 \text{ at } T_{avg}, \quad 1.76 \text{ at } T_s = 273\text{K}$$

$$Re_{D,max} = \frac{\rho V_{max} D}{\mu}$$

$$= \frac{(964 \frac{\text{kg}}{\text{m}^3})(18 \text{m/s})(0.01 \text{m})}{324 \times 10^{-6} \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

$$= 535,600 = 5.36 \times 10^5$$

→ For this high of a Reynolds number, Equations 7.58 and 7.60 are not appropriate, so we will use Eq. 7.64\*:

$$\overline{Nu_D} = C Re_{D,max}^m Pr^{0.36} \left( \frac{Pr}{Pr_s} \right)^{1/4}$$

C and m come from Table 7.7 (7.5 in 7<sup>th</sup>)

\* In the 7<sup>th</sup> edition, this is Eq. 7.58 and there are no other equations

### Tube Problem, cont.

For our conditions from Table 7.7  
we get  $C = 0.022$   $m = 0.84$

$$\begin{aligned} \overline{Nu}_D &= 0.022 (535,600)^{0.84} (2.02)^{0.36} \left( \frac{2.02}{1.76} \right)^{1/4} \\ &= 1903 \end{aligned}$$

HOWEVER! Since our tube bank has only four rows we must correct this using Equation 7.65 (7.59 in 7th ed.)

$$\overline{Nu}_D = 0.89 (1903) = 1694$$

$$\overline{h} = \overline{Nu}_D \frac{k}{D} = 1694 \frac{(674 \times 10^{-3} \text{ W/m}\cdot\text{K})}{0.01 \text{ m}}$$

$$= \boxed{114,200 \text{ W/m}^2\cdot\text{K}} \quad \text{Quite high!}$$

## Tube problem, cont.

Part c.) Total heat transfer

→ We should use the relations on Page 442 to solve this

First, we'll estimate the outlet temp

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T C_p}\right)$$

$$\begin{aligned} T_o &= T_s - (T_s - T_i) \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T C_p}\right) \\ &= 100^\circ\text{C} - (100 - 72) \exp\left(\frac{-\pi(0.01)(32)(114,200)}{(964)(6)(4)(0.015)(4200)}\right) \\ &= 74.1^\circ\text{C} \end{aligned}$$

$C_p \left(\frac{\text{J}}{\text{kg}\cdot\text{K}}\right) \uparrow$

$$\begin{aligned} \text{Now, } \Delta T_{lm} &= \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = \frac{28 - 25.9}{\ln\left(\frac{28}{25.9}\right)} \\ &= 26.9^\circ\text{C} \end{aligned}$$

$$\begin{aligned} q &= N(\bar{h} \pi D L \Delta T_{lm}) \\ &= 32(114,200)(\pi)(0.01)(2\text{ m})(26.9^\circ\text{C}) \end{aligned}$$

$$= \boxed{6,177,000 \text{ Watts}}$$

Note: This is an incredibly high number