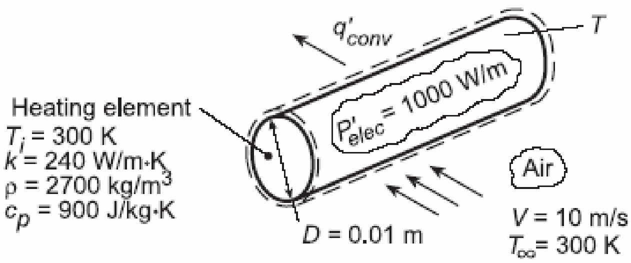
PROBLEM 7.43

KNOWN: Initial temperature, power dissipation, diameter, and properties of heating element. Velocity and temperature of air in cross flow.

FIND: (a) Steady-state temperature, (b) Time to come within 10°C of steady-state temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Uniform heater temperature, (2) Negligible radiation.

PROPERTIES: *Table A.4*, air (assume $T_f \approx 450$ K): $\nu = 32.39 \times 10^{-6}$ m²/s, k = 0.0373 W/m·K, Pr = 0.686.

ANALYSIS: (a) Performing an energy balance for steady-state conditions, we obtain

$$q'_{conv} = \bar{h}(\pi D)(T - T_\infty) = P'_{elec} = 1000\,\text{W/m}$$

With

$$Re_D = \frac{VD}{\nu} = \frac{(10\,\text{m/s})0.01\,\text{m}}{32.39\times10^{-6}\,\text{m}^2/\text{s}} = 3,087$$

the Churchill and Bernstein correlation, Eq. 7.54, yields

$$\overline{Nu}_D = 0.3 + \frac{0.62\,Re_D^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re_D}{282,000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu}_D = 0.3 + \frac{0.62(3087)^{1/2}(0.686)^{1/3}}{\left[1+(0.4/0.686)^{2/3}\right]^{1/4}}\left[1+\left(\frac{3087}{282,000}\right)^{5/8}\right]^{4/5} = 28.2$$

$$\bar{h} = \frac{k}{D}\overline{Nu}_D = \frac{0.0373\,\text{W/m}\cdot\text{K}}{0.010\,\text{m}}28.2 = 105.2\,\text{W/m}^2\cdot\text{K}$$

Hence, the steady-state temperature is

$$T = T_\infty + \frac{P'_{elec}}{\pi D\bar{h}} = 300\,\text{K} + \frac{1000\,\text{W/m}}{\pi(0.01\,\text{m})105.2\,\text{W/m}^2\cdot\text{K}} = 603\,\text{K}$$ <

(b) With Bi = $\bar{h}r_o/k$ = 105.2 W/m²·K(0.005 m)/240 W/m·K = 0.0022, a lumped capacitance analysis may be performed. The time response of the heater is given by Eq. 5.25, which, for $T_i = T_\infty$, reduces to

$$T = T_\infty + (b/a)\left[1 - \exp(-at)\right]$$

Continued...

PROBLEM 7.43 (Cont.)

where $a = 4\bar{h}/D\rho c_p = (4 \times 105.2\,W/m^2 \cdot K)/(0.01\,m \times 2700\,kg/m^3 \times 900\,J/kg \cdot K) = 0.0173\ s^{-1}$ and b/a = $P'_{elec}/\pi D\bar{h} = 1000\,W/m / \pi(0.01\,m \times 105.2\,W/m^2 \cdot K) = 302.6\,K$. Hence,

$$\left[1 - \exp(-0.0173t)\right] = \frac{(593 - 300)K}{302.6K} = 0.968$$

$$t \approx 200s$$ <

COMMENTS: (1) For T = 603 K and a representative emissivity of ε = 0.8, net radiation exchange between the heater and surroundings at $T_{sur} = T_\infty = 300$ K would be $q'_{rad} = \varepsilon\sigma(\pi D)\left(T^4 - T_{sur}^4\right) = 0.8 \times 5.67 \times 10^{-8}\ W/m^2 \cdot K^4\ (\pi \times 0.01\ m)(603^4 - 300^4)K^4 = 177\ W/m$. Hence, although small, radiation exchange is not negligible. The effects of radiation are considered in Problem 7.46.

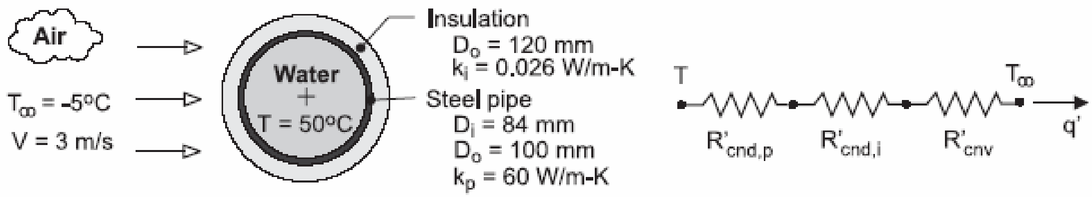
(2) The assumed value of T_f is very close to the actual value, rendering the selected air properties accurate.

PROBLEM 7.56

KNOWN: Diameter, thickness and thermal conductivity of steel pipe. Temperature of water flow in pipe. Temperature and velocity of air in cross flow over pipe. Cost of producing hot water.

FIND: (a) Cost of daily heat loss from an uninsulated pipe, (b) Savings associated with insulating the pipe.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, (2) Negligible convection resistance for water flow, (3) Negligible contact resistance between insulation and pipe, (4) Negligible radiation.

PROPERTIES: *Table A-4*, air $(p = 1\,atm,\ T_f \approx 300K)$: $k_a = 0.0263\,W/m\cdot K$, $\nu = 15.89\times10^{-6}\,m^2/s$, $Pr = 0.707$.

ANALYSIS: (a) With $Re_D = VD_o/\nu = 3\,m/s \times 0.1m/15.89\times10^{-6}\,m^2/s = 18,880$, application of the Churchill-Bernstein correlation yields

$$\overline{Nu}_D = 0.3 + \frac{0.62(18,800)^{1/2}(0.707)^{1/3}}{\left[1+(0.4/0.707)^{2/3}\right]^{1/4}}\left[1+\left(\frac{18,880}{282,000}\right)^{5/8}\right]^{4/5} = 76.6$$

$$\bar{h} = \frac{k_a}{D_o}\overline{NU}_D = \frac{0.0263\,W/m\cdot K}{0.1m}76.6 = 20.1\,W/m^2\cdot K$$

Without the insulation, the total thermal resistance and heat loss per length of pipe are then

$$R'_{tot(wo)} = \frac{\ln(D_o/D_i)}{2\pi k_p} + \frac{1}{\pi D_o \bar{h}} = \frac{\ln(100/84)}{2\pi\times 60\,W/m\cdot K} + \frac{1}{\pi(0.1m)20.1\,W/m^2\cdot K}$$

$$= \left(4.63\times10^{-4} + 0.158\right)m\cdot K/W = 0.159\,m\cdot K/W$$

$$q'_{wo} = \frac{T-T_\infty}{R'_{tot(wo)}} = \frac{55°C}{0.159\,m\cdot K/W} = 346\,W/m = 0.346\,kW/m$$

The corresponding daily energy loss is

$$Q'_{wo} = 0.346\,kW/m \times 24\,h/d = 8.3\,kW\cdot h/m\cdot d$$

and the associated cost is

$$C'_{wo} = (8.3\,kW\cdot h/m\cdot d)(\$0.05/kW\cdot h) = \$0.415/m\cdot d$$ <

(b) The conduction resistance of the insulation is

Continued …..

$$R'_{cnd} = \frac{\ln(D_o / D_i)}{2\pi k_i} = \frac{\ln(120/100)}{2\pi(0.026\,W/m\cdot K)} = 1.116\,m\cdot K/W$$

Using the Churchill-Bernstein correlation with an outside diameter of $D_o = 0.12m$, $Re_D = 22,660$, $\overline{Nu}_D = 83.9$ and $\overline{h} = 18.4\,W/m^2\cdot K$. The convection resistance is then

$$R'_{cnv} = \frac{1}{\pi D_o \overline{h}} = \frac{1}{\pi(0.12m)18.4\,W/m^2\cdot K} = 0.144\,m\cdot K/W$$

and the total resistance is

$$R'_{tot(w)} = \left(4.63\times10^{-4} + 1.116 + 0.144\right)m\cdot K/W = 1.261\,m\cdot K/W$$

The heat loss and cost are then

$$q'_W = \frac{T - T_\infty}{R'_{tot(w)}} = \frac{55°C}{1.261\,m\cdot K/W} = 43.6\,W/m = 0.0436\,kW/m$$

$$C'_W = 0.0436\,kW/m \times 24h/d \times \$0.05/kW\cdot h = \$0.052/m\cdot d$$

The daily savings is then

$$S' = C'_{Wo} - C'_W = \$0.363/m\cdot d$$ <

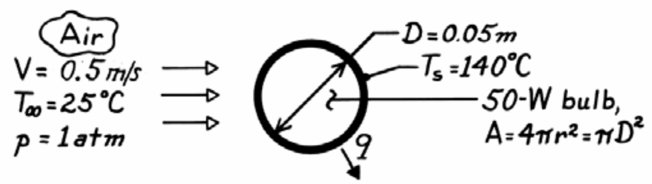
COMMENTS: (1) The savings are significant, and the pipe should be insulated. (2) Assuming a negligible temperature drop across the pipe wall, a pipe emissivity of $\varepsilon_p = 0.6$ and surroundings at $T_{sur} = 268K$, the radiation coefficient associated with the uninsulated pipe is $h_r = \varepsilon\sigma(T + T_{sur})\left(T^2 + T_{sur}^2\right) = 0.6\times5.67\times10^{-8}\,W/m^2\cdot K^4\,(591K)\,\left(323^2 + 268^2\right)K^2 = 3.5\,W/m^2\cdot K$. Accordingly, radiation increases the heat loss estimate of Part (a) by approximately 17%.

PROBLEM 7.68

KNOWN: Conditions associated with airflow over a spherical light bulb of prescribed diameter and surface temperature.

FIND: Heat loss by convection.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Uniform surface temperature.

PROPERTIES: *Table A-4*, Air (T_∞ = 25°C, 1 atm): $\nu = 15.71 \times 10^{-6}$ m^2/s, k = 0.0261 W/m·K, Pr = 0.71, $\mu = 183.6 \times 10^{-7}$ N·s/m^2; *Table A-4*, Air (T_s = 140°C, 1 atm): $\mu = 235.5 \times 10^{-7}$ N·s/m^2.

ANALYSIS: The heat rate by convection is

$$q = \bar{h}\left(\pi D^2\right)\ \left(T_s - T_\infty\right)$$

where $\bar{h}$ may be estimated from the Whitaker relation

$$\bar{h} = \frac{k}{D}\left[2 + \left(0.4\ Re_D^{1/2} + 0.06\ Re_D^{2/3}\right)\ Pr^{0.4}\left(\mu / \mu_s\right)^{1/4}\right]$$

where

$$Re_D = \frac{VD}{\nu} = \frac{0.5\ \text{m/s} \times 0.05\ \text{m}}{15.71 \times 10^{-6}\ \text{m}^2/\text{s}} = 1591.$$

Hence,

$$\bar{h} = \frac{0.0261\ \text{W/m}\cdot\text{K}}{0.05\ \text{m}}\ \left\{2 + \left[0.4(1591)^{1/2} + 0.06(1591)^{2/3}\right]\ (0.71)^{0.4}\left(\frac{183.6}{235.5}\right)^{1/4}\right\}$$

$$\bar{h} = 11.4\ \text{W/m}^2\cdot\text{K}$$

and the heat rate is

$$q = 11.4\frac{\text{W}}{\text{m}^2\cdot\text{K}}\pi(0.05\ \text{m})^2\ \ (140 - 25)^\circ\text{C} = 10.3\ \text{W}.$$ <

COMMENTS: (1) The low value of $\bar{h}$ suggests that heat transfer by free convection may be significant and hence that the total loss by convection exceeds 10.3 W.

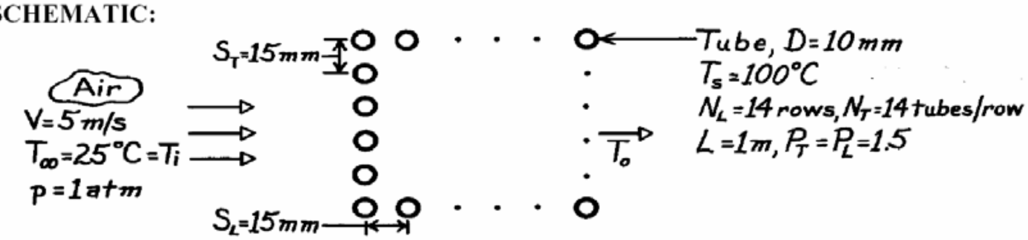
(2) The surface of the bulb also dissipates heat to the surrounding by radiation. Further, in an actual light bulb, there is also heat loss by conduction through the socket.

PROBLEM 7.84

KNOWN: Surface temperature and geometry of a tube bank. Velocity and temperature of air in cross flow.

FIND: (a) Total heat transfer, (b) Air flow pressure drop.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Uniform surface temperature.

PROPERTIES: *Table A-4*, Atmospheric air (T_∞ = 298 K): $\nu = 15.8 \times 10^{-6}$ m^2/s, k = 0.0263 W/m·K, Pr = 0.707, c_p = 1007 J/kg·K, ρ = 1.17 kg/m^3; (T_s = 373 K): Pr = 0.695.

ANALYSIS: (a) The total heat transfer rate is

$$q = \overline{h} N \pi D L \frac{(T_s - T_i) - (T_s - T_o)}{\ell n\left[(T_s - T_i)/(T_s - T_o)\right]} = \overline{h} N \pi D L \, \Delta T_{\ell m}.$$

With $V_{max} = \dfrac{S_T}{S_T - D} V = \dfrac{15 \text{ mm}}{5 \text{ mm}} 5 \text{ m/s} = 15 \text{ m/s}$, $Re_{D,max} = \dfrac{15 \text{ m/s}(0.01 \text{ m})}{15.8 \times 10^{-6} \text{ m}^2/\text{s}} = 9494.$ Tables 7.7 and 7.8 give C = 0.27, m = 0.63 and $C_2 \approx 0.99$. Hence, from the Zukauskas correlation

$$\overline{Nu}_D = 0.99 \times 0.27(9494)^{0.63}(0.707)^{0.36}(0.707/0.695)^{1/4} = 75.9$$

$$\overline{h} = \overline{Nu}_D \; k/D = 75.9 \times 0.0263 \text{ W/m}\cdot\text{K}/0.01 \text{ m} = 200 \text{ W/m}^2\cdot\text{K}$$

$$T_s - T_o = (T_s - T_i)\exp\left(-\frac{\pi D N \overline{h}}{\rho V N_T S_T c_p}\right) = 75^\circ\text{C} \exp\left(-\frac{\pi \times 0.01 \text{ m} \times 196 \times 200 \text{ W/m}^2\cdot\text{K}}{1.17 \text{ kg/m}^3 \times 5 \text{ m/s} \times 14 \times 0.015 \text{ m} \times 1007 \text{ J/kg}\cdot\text{K}}\right)$$

$$T_s - T_o = 27.7^\circ\text{C}.$$

Hence

$$q = 200 \text{ W/m}^2\cdot\text{K} \times 196\pi(0.01 \text{ m}) \; 1 \text{ m} \frac{75^\circ\text{C} - 27.7^\circ\text{C}}{\ell n(75/27.7)} = 58.5 \text{ kW}.$$ <

(b) With $Re_{D,max}$ = 9494, (P_T - 1)/(P_L - 1) = 1, Fig. 7.13 yields f ≈ 0.32 and χ = 1. Hence,

$$\Delta p = N\chi\left(\rho V_{max}^2/2\right) f = 14 \times 1\left(\frac{1.17 \text{ kg/m}^3 (15 \text{ m/s})^2}{2}\right) 0.32$$

$$\Delta p = 590 \text{ N/m}^2 = 5.9 \times 10^{-3} \text{ bar}.$$

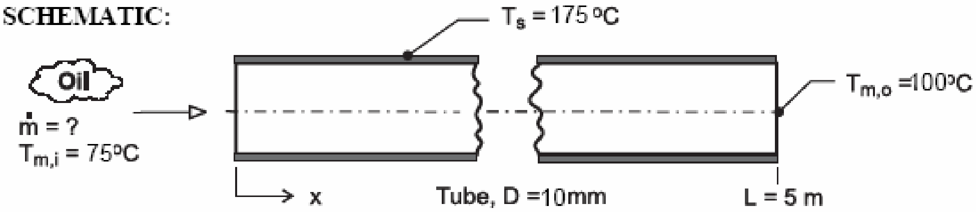
COMMENTS: The heat transfer rate would have been substantially overestimated (93.3 kW) if the inlet temperature difference (T_s - T_i) had been used in lieu of the log-mean temperature difference.

PROBLEM 8.24

KNOWN: Oil at 75°C enters a single-tube preheater of 10-mm diameter and 5-m length; tube surface maintained at 175°C by swirling combustion gases.

FIND: Determine the flow rate and heat transfer rate when the outlet temperature is 95°C.

SCHEMATIC:



ASSUMPTIONS: (1) Combined entry length, laminar flow, (2) Tube wall is isothermal, (3) Incompressible liquid with negligible viscous dissipation, (4) Constant properties.

PROPERTIES: *Table A-5*, Engine oil, new ($T_m = (T_{m,i} + T_{m,o})/2 = 361$ K): $\rho = 847.5$ kg/m^3, c_p = 2163 J/kg·K, $\nu = 2.931 \times 10^{-5}$ m^2/s, k = 0.1379 W/m·K, Pr = 390.2, μ = 0.0245.

ANALYSIS: The overall energy balance, Eq. 8.34, and rate equation, Eq. 8.42b, are

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) \quad (1)$$

$$\frac{T_s - T_{m,o}}{T_s - T_{m,i}} = \exp\left(-\frac{PL\bar{h}}{\dot{m} c_p}\right) \quad (2)$$

Not knowing the flow rate $\dot{m}$, the Reynolds number cannot be calculated. Assume that the flow is laminar, and the combined entry length condition occurs. The average convection coefficient can be estimated using the Hausen correlation, Eq. 8.56,

$$\overline{Nu}_D = 3.66 + \frac{0.0668(D/L)Re_D\,Pr}{1 + 0.04\left[(D/L)Re_D\,Pr\right]^{2/3}} \quad (3)$$

where all properties are evaluated at $T_m = (T_{m,i} + T_{m,o})/2$. The Reynolds number follows from Eq. 8.6,

$$Re_D = 4\dot{m}/\pi D\mu \quad (4)$$

A tedious trial-and-error solution is avoided by using *IHT* to solve the system of equations with the following result:

Re_D	$\overline{Nu}_D$	$\bar{h}_D$ (W/m^2·K)	q(W)	$\dot{m}$ (kg/h)
130	7.25	100	1360	90

<

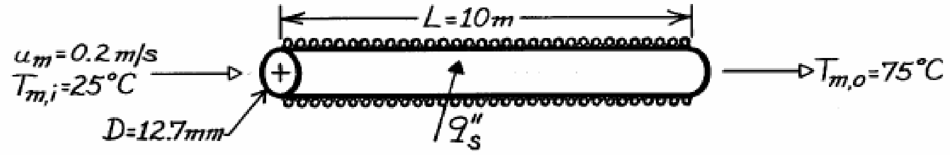
Note that the flow is laminar, and evaluating x_{fd} using Eq. 8.3, find $x_{fd,h}$ = 0.065 m so the thermal entry length condition is appropriate.

PROBLEM 8.27

KNOWN: Inlet and outlet temperatures and velocity of fluid flow in tube. Tube diameter and length.

FIND: Surface heat flux and temperatures at x = 0.5 and 10 m.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Negligible heat loss to surroundings, (4) Incompressible liquid with negligible viscous dissipation, (5) Negligible axial conduction.

PROPERTIES: Pharmaceutical (given): $\rho = 1000$ kg/m^3, $c_p = 4000$ J/kg·K, $\mu = 2 \times 10^{-3}$ kg/s·m, k = 0.80 W/m·K, Pr = 10.

ANALYSIS: With

$$\dot{m} = \rho V A = 1000 \text{ kg/m}^3 (0.2 \text{ m/s}) \pi (0.0127 \text{ m})^2 / 4 = 0.0253 \text{ kg/s}$$

Eq. 8.34 yields

$$q = \dot{m} c_p \left(T_{m,o} - T_{m,i}\right) = 0.0253 \text{ kg/s} (4000 \text{ J/kg}\cdot\text{K}) 50 \text{ K} = 5060 \text{ W}.$$

The required heat flux is then

$$q_s'' = q/A_s = 5060 \text{ W}/\pi(0.0127 \text{ m}) 10 \text{ m} = 12{,}682 \text{ W/m}^2. \quad <$$

With

$$Re_D = \rho V D/\mu = 1000 \text{ kg/m}^3 (0.2 \text{ m/s}) 0.0127 \text{ m}/2\times10^{-3} \text{ kg/s}\cdot\text{m} = 1270$$

the flow is laminar and Eq. 8.23 yields

$$x_{fd,t} = 0.05 Re_D \Pr D = 0.05(1270)10(0.0127 \text{ m}) = 8.06 \text{ m}.$$

Hence, with fully developed hydrodynamic and thermal conditions at x = 10 m, Eq. 8.53 yields

$$h(10 \text{ m}) = Nu_{D,fd}(k/D) = 4.36(0.80 \text{ W/m}\cdot\text{K}/0.0127 \text{ m}) = 274.6 \text{ W/m}^2\cdot\text{K}.$$

Hence, from Newton's law of cooling,

$$T_{s,o} = T_{m,o} + (q_s''/h) = 75^\circ\text{C} + \left(12{,}682 \text{ W/m}^2/274.6 \text{ W/m}^2\cdot\text{K}\right) = 121^\circ\text{C}. \quad <$$

At x = 0.5 m, $(x/D)/(Re_D Pr) = 0.0031$ and Figure 8.10 yields $Nu_D \approx 8$ for a thermal entry region with uniform surface heat flux. Hence, h(0.5 m) = 503.9 W/m^2·K and, since T_m increases linearly with x, $T_m(x = 0.5 \text{ m}) = T_{m,i} + (T_{m,o} - T_{m,i})(x/L) = 27.5$°C. It follows that

$$T_s(x = 0.5 \text{ m}) \approx 27.5^\circ\text{C} + \left(12{,}682 \text{ W/m}^2/503.9 \text{ W/m}^2\cdot\text{K}\right) = 52.7^\circ\text{C}. \quad <$$