SOLUTION – Velocity, thermal and concentration boundary layers

The **velocity boundary layer** develops whenever there is flow over a surface. It is associated with shear stresses parallel to the surface and results in an increase in velocity through the boundary layer from nearly zero right at the surface to the free stream velocity far from the surface. The boundary layer thickness is by convention defined as the distance from the surface at which the velocity is 99% of the free stream velocity.

The **thermal boundary layer** is associated with temperature gradients near the surface, and develops when there is temperature difference between the fluid free stream and the surface. Right at the fluid-surface interface, heat transfer occurs only through conduction. The thickness of the thermal boundary layer is defined as that point at which the temperature difference between the fluid and surface is 99% of the temperature difference between the free stream fluid and the surface.

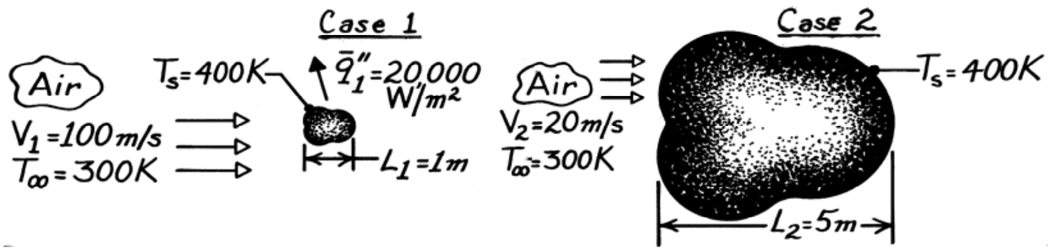
The **concentration boundary layer** develops when there is a difference in concentration of a component between the free stream and the surface. A concentration profile develops, and the thickness of the concentration boundary layer is defined as that point at which the difference in concentration between the fluid and the surface is 99% of the difference in concentration between the free stream fluid and the surface.

PROBLEM 6.18

KNOWN: Characteristic length, surface temperature and average heat flux for an object placed in an airstream of prescribed temperature and velocity.

FIND: Average convection coefficient if characteristic length of object is increased by a factor of five and air velocity is decreased by a factor of five.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

ANALYSIS: For a particular geometry,

$$\overline{Nu}_L = f\left(Re_L, Pr\right).$$

The Reynolds numbers for each case are

Case 1: $$Re_{L,1} = \frac{V_1 L_1}{\nu_1} = \frac{(100\text{m/s})1\text{m}}{\nu_1} = \frac{100\ \text{m}^2/\text{s}}{\nu_1}$$

Case 2: $$Re_{L,2} = \frac{V_2 L_2}{\nu_2} = \frac{(20\text{m/s})5\text{m}}{\nu_2} = \frac{100\ \text{m}^2/\text{s}}{\nu_2}.$$

Hence, with $\nu_1 = \nu_2$, $Re_{L,1} = Re_{L,2}$. Since $Pr_1 = Pr_2$, it follows that

$$\overline{Nu}_{L,2} = \overline{Nu}_{L,1}.$$

Hence,

$$\bar{h}_2 L_2 / k_2 = \bar{h}_1 L_1 / k_1$$

$$\bar{h}_2 = \bar{h}_1 \frac{L_1}{L_2} = 0.2\ \bar{h}_1.$$

For *Case 1*, using the rate equation, the convection coefficient is

$$q_1 = \bar{h}_1 A_1 \left(T_s - T_\infty\right)_1$$

$$\bar{h}_1 = \frac{\left(q_1 / A_1\right)}{\left(T_s - T_\infty\right)_1} = \frac{q_1''}{\left(T_s - T_\infty\right)_1} = \frac{20,000\ \text{W/m}^2}{(400-300)\text{K}} = 200\ \text{W/m}^2 \cdot \text{K}.$$

Hence, it follows that for *Case 2*

$$\bar{h}_2 = 0.2 \times 200\ \text{W/m}^2 \cdot \text{K} = 40\ \text{W/m}^2 \cdot \text{K}.$$ <

COMMENTS: If $Re_{L,2}$ were *not* equal to $Re_{L,1}$, it would be necessary to know the specific form of $f(Re_L, Pr)$ before $\bar{h}_2$ could be determined.

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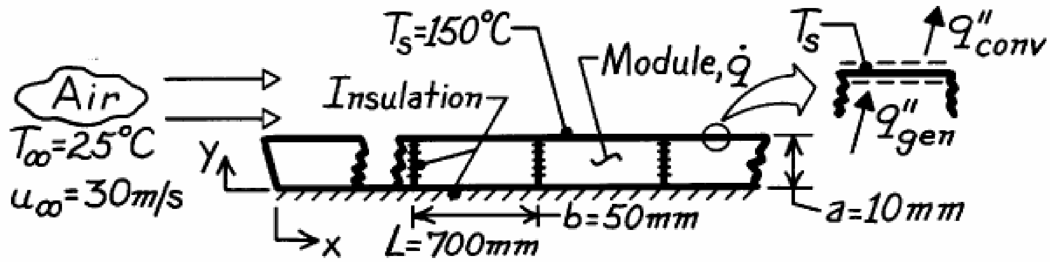
PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s, thickness a and length b cooled by air at 25°C and a velocity of 30 m/s. Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5, (3) Heat transfer is one-dimensional in y-direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): $k = 5.2$ W/m·K, $c_p = 320$ J/kg·K, $\rho = 2300$ kg/m^3; *Table A-4*, Air $\left(\overline{T}_f = (T_s + T_\infty)/2 = 360 \text{ K, 1 atm}\right)$: $k = 0.0308$ W/m·K, $\nu = 22.02 \times 10^{-6}$ m^2/s, Pr = 0.698.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q''_{conv} = q''_{gen}$$

$$\overline{h}(T_s - T_\infty) = \dot{q} \cdot a \qquad \text{or} \qquad \dot{q} = \frac{\overline{h}(T_s - T_\infty)}{a}.$$

To select a convection correlation for estimating $\overline{h}$, first find the Reynolds numbers at x = L.

$$Re_L = \frac{u_\infty L}{\nu} = \frac{30 \text{ m/s} \times 0.70 \text{ m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.537 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\overline{h} \approx h_x \ (L + b/2)$ is appropriate, with

$$Re_{L+b/2} = \frac{30 \text{ m/s} \times (0.700 + 0.050/2)\text{m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.877 \times 10^5.$$

Using the turbulent flow correlation with x = L + b/2 = 0.725 m,

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0296\left(9.877 \times 10^5\right)^{4/5} (0.698)^{1/3} = 1640$$

$$\overline{h} \approx h_x = \frac{Nu_x k}{x} = \frac{1640 \times 0.0308 \text{ W/m} \cdot \text{K}}{0.725} = 69.7 \text{ W/m}^2 \cdot \text{K}.$$

Continued …..

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PROBLEM 7.8 (Cont.)

Hence,

$$\dot{q} = \frac{69.7\ \text{W/m}^2 \cdot \text{K}(150-25)\text{K}}{0.010\ \text{m}} = 8.713 \times 10^5\ \text{W/m}^3. \qquad <$$

(b) The maximum temperature within the module occurs at the surface next to the insulation (y = 0). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T(0) = \frac{\dot{q}a^2}{2k} + T_s = \frac{8.713 \times 10^5\ \text{W/m}^3 \times (0.010\ \text{m})^2}{2 \times 5.2\ \text{W/m} \cdot \text{K}} + 150^\circ\text{C} = 158.4^\circ\text{C}. \qquad <$$

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$q_{module} = q_{0 \to L+b} - q_{0 \to L}$$

$$\bar{h} \cdot b = \bar{h}_{L+b} \cdot (L+b) - \bar{h}_L \cdot L \qquad \text{or} \qquad \bar{h} = \bar{h}_{L+b}\frac{L+b}{b} - \bar{h}_L\frac{L}{b}.$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{Nu}_x = \left(0.037 Re_x^{4/5} - 871\right) Pr^{1/3}$$

With x = L + b and x = L, find

$$\bar{h}_{L+b} = 54.79\ \text{W/m}^2 \cdot \text{K} \qquad \text{and} \qquad \bar{h}_L = 53.73\ \text{W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h} = \left[54.79\frac{0.750}{0.050} - 53.73\frac{0.700}{0.05}\right]\ \text{W/m}^2 \cdot \text{K} = 69.7\ \text{W/m}^2 \cdot \text{K}.$$

which is in excellent agreement with the approximate result employed in part (a).

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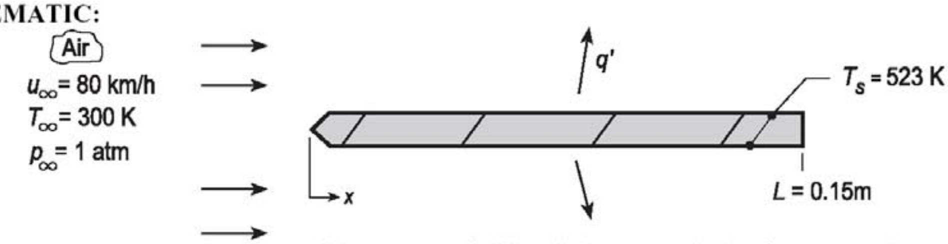
PROBLEM 7.16

KNOWN: Length and surface temperature of a rectangular fin.

FIND: (a) Heat removal per unit width, q', when air at a prescribed temperature and velocity is in parallel, turbulent flow over the fin, and (b) Calculate and plot q' for motorcycle speeds ranging from 10 to 100 km/h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Turbulent flow over entire surface.

PROPERTIES: *Table A.4*, Air (412 K, 1 atm): $\nu = 27.85 \times 10^{-6}$ m²/s, k = 0.0346 W/m·K, Pr = 0.69.

ANALYSIS: (a) The heat loss per unit width is

$$q' = 2\times\left[\overline{h}_L L\left(T_s - T_\infty\right)\right]$$

where $\overline{h}$ is obtained from the correlation, Eq. 7.38 but with turbulent flow over the entire surface,

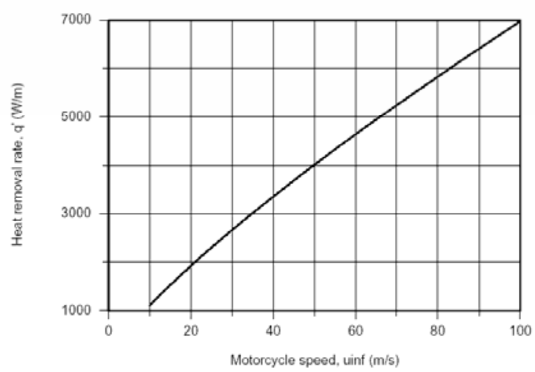
$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left[\frac{80\,km/h \times 1000\,m/km \times 1/3600\,h/s \times 0.15\,m}{27.85\times10^{-6}\,m^2/s}\right]^{4/5}(0.69)^{1/3} = 378$$

Hence,

$$\overline{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0346\,W/m\cdot K}{0.15\,m}378 = 87\,W/m^2\cdot K$$

$$q' = 2\times\left[87\,W/m^2\cdot K\times0.15\,m\left(523-300\right)K\right] = 5826\,W/m.$$ <

(b) Using the foregoing equations in the IHT Workspace, q' as a function of speed was calculated and is plotted as shown.



COMMENTS: (1) Radiation emission from the fin is not negligible. With an assumed emissivity of ε = 1, the rate of emission per unit width at 80 km/h would be $q' = \left(\sigma T_s^4\right)2L$ = 1273 W/m. If the fin received negligible radiation from its surroundings, its loss by radiation would then be approximately 20% of that by convection.

(2) From the correlation and heat rate expression, it follows that $q' \sim u_\infty^{4/5}$. That is, q' vs. u_∞ is nearly linear as evident from the above plot.

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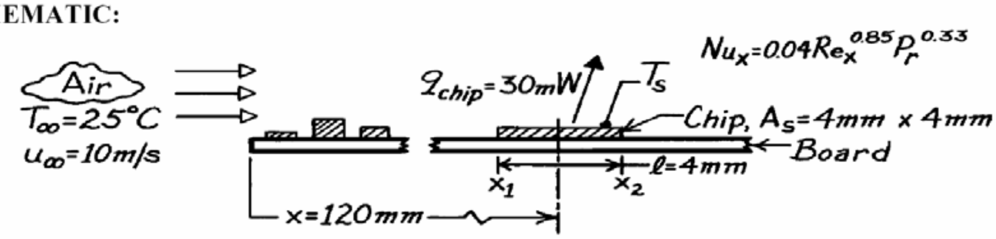
PROBLEM 7.34

KNOWN: Convection correlation for irregular surface due to electronic elements mounted on a circuit board experiencing forced air cooling with prescribed temperature and velocity

FIND: Surface temperature when heat dissipation rate is 30 mW for chip of prescribed area located a specific distance from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Situation approximates parallel flow over a flat plate with prescribed correlation, (2) Heat rate is from top surface of chip.

PROPERTIES: *Table A-4*, Air (assume $T_s \approx 45°C$, then $\overline{T} = (45 + 25)°C/2 \approx 310$ K, 1 atm): k = 0.027 W/m·K, $\nu = 16.90 \times 10^{-6}$ m^2/s, Pr = 0.706.

ANALYSIS: For the chip upper surface, the heat rate is

$$q_{chip} = \overline{h}_{chip} A_s (T_s - T_\infty) \qquad \text{or} \qquad T_s = T_\infty + q_{chip} / \overline{h}_{chip} A_s$$

Assuming the average convection coefficient over the chip length to be equal to the local value at the center of the chip ($x = x_o$), $\overline{h}_{chip} \approx h_x(x_o)$, where

$$Nu_x = 0.04 Re_x^{0.85} Pr^{0.33}$$

$$Nu_x = 0.04\left(10 \text{ m/s} \times 0.120 \text{ m}/16.90 \times 10^{-6} \text{ m}^2/\text{s}\right)^{0.85} (0.706)^{0.33} = 473.4$$

$$h_x = \frac{Nu_x k}{x_o} = \frac{473.4 \times 0.027 \text{ W/m}\cdot\text{K}}{0.120 \text{ m}} = 107 \text{ W/m}^2\cdot\text{K}$$

Hence,

$$T_s = 25°C + 30 \times 10^{-3} \text{ W}/107 \text{ W/m}^2\cdot\text{K} \times \left(4 \times 10^{-3} \text{m}\right)^2 = (25 + 17.5)°C = 42.5°C. \quad <$$

COMMENTS: (1) Note that the assumed value of $\overline{T}$ used to evaluate the thermophysical properties was reasonable. (2) We could have evaluated $\overline{h}_{chip}$ by two other approaches. In one case the average coefficient is approximated as the arithmetic mean of local values at the leading and trailing edges of the chip.

$$\overline{h}_{chip} \approx \left[h_{x2}(x_2) + h_{x1}(x_1)\right]/2 = 107 \text{ W/m}^2\cdot\text{K}.$$

The exact approach is of the form

$$\overline{h}_{chip} \cdot \ell = \overline{h}_{x2} \cdot x_2 - \overline{h}_{x1} \cdot x_1$$

Recognizing that $h_x \sim x^{-0.15}$, it follows that

$$\overline{h}_x = \frac{1}{x}\int_0^x h_x \cdot dx = 1.176 h_x$$

and $\overline{h}_{chip} = 108$ W/m$^2\cdot$K. Why do results for the two approximate methods and the exact method compare so favorably?

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FLAT PLATE PROBLEM

First, let's look up the properties of nitrogen at $T_{film} = \frac{23+131}{2} = 77°C = 350K$

$$\nu = 2.078 \times 10^{-5} \; m^2/s$$
$$k = 2.93 \times 10^{-2} \; W/m \cdot K$$
$$Pr = 0.711$$

Then, let's evaluate the Reynolds number halfway down and at the trailing edge

$$\text{Halfway } Re = \frac{U_\infty L}{\nu} = \frac{(8\ m/s)(0.5\ m)}{2.078 \times 10^{-5}\ m^2/s}$$

$$= 192{,}490 \quad \text{Laminar!}$$

$$\text{Trailing } Re = 384{,}990 \quad \text{Turbulent!}$$

Part a.) $\delta = 5x Re_x^{-1/2}$

$$= 5(0.5\ m)(192{,}490)^{-1/2}$$
$$= \boxed{0.0057\ m}$$

Part b.) $\delta = 5(1.0\ m)(384{,}990)^{-1/2}$

$$= \boxed{0.0081\ m}$$

Part c.) $\frac{\delta}{\delta_t} = Pr^{1/3} \Rightarrow \delta_t = \delta Pr^{-1/3}$

$$= (0.0057\ m)(0.711)^{-1/3} = \boxed{0.0064\ m}$$

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Part d.) $\delta_t = (0.0081\,m)(0.711)^{-1/3} = \boxed{0.0090\ m}$

Part e.) $Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2} Pr^{1/3}$

$= 0.332(192,490)^{1/2}(0.711)^{1/3}$

$= 130.0$

$h_x = \frac{Nu_x k}{x} = \frac{130(0.0293\ W/m\cdot K)}{0.5\ m}$

$= \boxed{7.6\ W/m^2\cdot K}$

Part f.) $Nu_x = 0.332(384,990)^{1/2}(0.711)^{1/3}$

$= 184$

$h_x = \frac{184(0.0293)}{1.0\,m} = \boxed{5.4\ W/m^2\cdot K}$

Part g.) $\overline{Nu_x} = 0.664\, Re_x^{1/2} Pr^{1/3}$

$= 0.664(384,990)^{1/2}(0.711)^{1/3}$

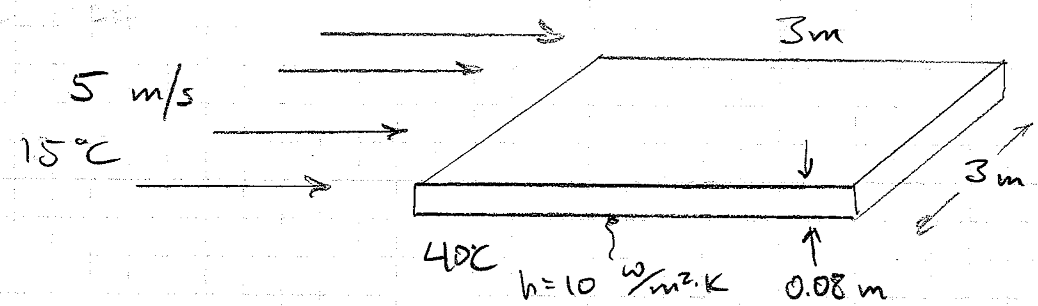
$= 368$

$\overline{h_x} = \frac{368(0.0293)}{1.0\ m} = \boxed{10.8\ W/m^2\cdot K}$

Part h.) $q = \overline{h} A \Delta T = (10.8\ \frac{W}{m^2\cdot K})(0.25\ m^2)(131-23\ ^\circ C)$

$= \boxed{291\ Watts}$

Hot Tub Problem



→ The key here is to find the average convective heat transfer coefficient $\bar{h}$

First, lets get the fluid properties at $\overline{T_{film}}$

$$\overline{T_{film}} = (15 + 40)/2 = 27.5°C = 300 K$$ (about)

$\rho = 1.1614 \ kg/m^3$

$\nu = 15.89 \times 10^{-6} \ m^2/s$

$k = 26.3 \times 10^{-3} \ W/m \cdot K$

$Pr = 0.707$

→ Let's check the Reynolds number at the trailing edge to see what sort of flow we have.

$$Re_L = \frac{u_\infty L}{\nu} = \frac{(5 \ m/s)(3 \ m)}{15.89 \times 10^{-6} \ m^2/s} = 9.44 \times 10^5$$

So, it's turbulent at the trailing edge.
But it transitions from laminar to turbulent about halfway along the way!

Hot tub problem, cont.

For a "mixed" boundary layer that transitions from laminar to turbulent we use Eq. 7.38 to get $\overline{Nu}_L$

$$\overline{Nu}_L = \left(0.037\, Re_L^{4/5} - A\right) Pr^{1/3}$$

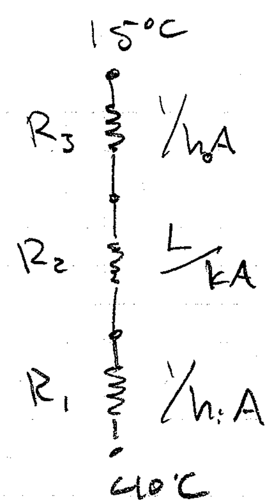
A = 871 for mixed and $Re_{x,c} = 5\times10^5$

$$= \left(0.037\left(9.44\times10^5\right)^{4/5} - 871\right) 0.707^{1/3}$$

$$= 1210$$

$$h = \frac{Nu \cdot k}{L} = \frac{1210\left(26.3\times10^{-3}\ W/m\cdot K\right)}{3\ m}$$

$$= 10.6\ W/m^2\cdot K$$



From here its just a series resistance problem:

$$R_1 = 1/\left(10\ W/m^2 K\right)\left(9\ m^2\right) = 0.0111\ \frac{K}{W}$$

$$R_2 = 0.08\ m / \left(0.3\ W/m\,K\right)\left(9\ m^2\right) = 0.0296\ \frac{K}{W}$$

$$R_3 = 1/\left(10.6\ W/m^2 K\right)\left(9\ m^2\right) = 0.01048\ \frac{K}{W}$$

$$q = \frac{\Delta T}{\Sigma R} = \frac{40-15\ K}{\left(0.0111 + 0.0296 + 0.0105\right)\frac{K}{W}} = \boxed{488\ \text{Watts}}$$

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