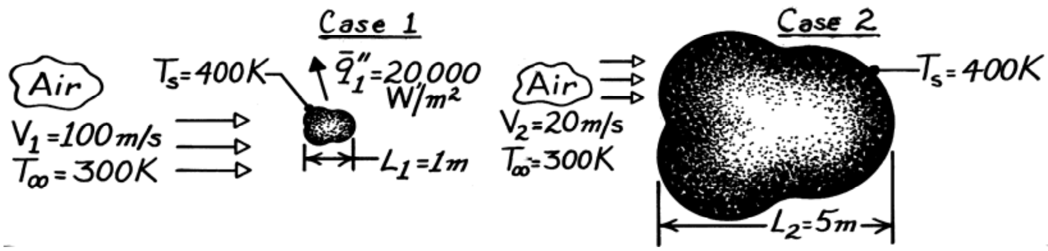
PROBLEM 6.18

KNOWN: Characteristic length, surface temperature and average heat flux for an object placed in an airstream of prescribed temperature and velocity.

FIND: Average convection coefficient if characteristic length of object is increased by a factor of five and air velocity is decreased by a factor of five.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties.

ANALYSIS: For a particular geometry,

$$\overline{Nu}_L = f\left(Re_L, Pr\right).$$

The Reynolds numbers for each case are

Case 1: $$Re_{L,1} = \frac{V_1 L_1}{\nu_1} = \frac{(100\text{m/s})1\text{m}}{\nu_1} = \frac{100\ \text{m}^2/\text{s}}{\nu_1}$$

Case 2: $$Re_{L,2} = \frac{V_2 L_2}{\nu_2} = \frac{(20\text{m/s})5\text{m}}{\nu_2} = \frac{100\ \text{m}^2/\text{s}}{\nu_2}.$$

Hence, with $\nu_1 = \nu_2$, $Re_{L,1} = Re_{L,2}$. Since $Pr_1 = Pr_2$, it follows that

$$\overline{Nu}_{L,2} = \overline{Nu}_{L,1}.$$

Hence,

$$\bar{h}_2 L_2 / k_2 = \bar{h}_1 L_1 / k_1$$

$$\bar{h}_2 = \bar{h}_1 \frac{L_1}{L_2} = 0.2\ \bar{h}_1.$$

For *Case 1*, using the rate equation, the convection coefficient is

$$q_1 = \bar{h}_1 A_1 \left(T_s - T_\infty\right)_1$$

$$\bar{h}_1 = \frac{\left(q_1 / A_1\right)}{\left(T_s - T_\infty\right)_1} = \frac{q_1''}{\left(T_s - T_\infty\right)_1} = \frac{20,000\ \text{W/m}^2}{(400-300)\text{K}} = 200\ \text{W/m}^2\cdot\text{K}.$$

Hence, it follows that for *Case 2*

$$\bar{h}_2 = 0.2 \times 200\ \text{W/m}^2\cdot\text{K} = 40\ \text{W/m}^2\cdot\text{K}.$$ <

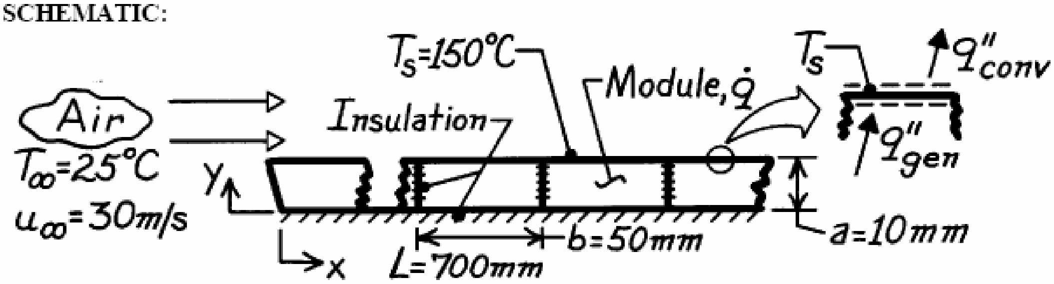
COMMENTS: If $Re_{L,2}$ were *not* equal to $Re_{L,1}$, it would be necessary to know the specific form of $f(Re_L, Pr)$ before $\bar{h}_2$ could be determined.

PROBLEM 7.8

KNOWN: Flat plate comprised of rectangular modules of surface temperature T_s, thickness a and length b cooled by air at 25°C and a velocity of 30 m/s. Prescribed thermophysical properties of the module material.

FIND: (a) Required power generation for the module positioned 700 mm from the leading edge of the plate and (b) Maximum temperature in this module.

SCHEMATIC:



ASSUMPTIONS: (1) Laminar flow at leading edge of plate, (2) Transition Reynolds number of 5×10^5, (3) Heat transfer is one-dimensional in y-direction within each module, (4) $\dot{q}$ is uniform within module, (5) Negligible radiation heat transfer.

PROPERTIES: Module material (given): k = 5.2 W/m·K, c_p = 320 J/kg·K, ρ = 2300 kg/m^3; *Table A-4*, Air $\left(\overline{T}_f = (T_s + T_\infty)/2 = 360 \text{ K, 1 atm}\right)$: k = 0.0308 W/m·K, ν = 22.02 × 10^{-6} m^2/s, Pr = 0.698.

ANALYSIS: (a) The module power generation follows from an energy balance on the module surface,

$$q''_{conv} = q''_{gen}$$

$$\overline{h}(T_s - T_\infty) = \dot{q} \cdot a \qquad \text{or} \qquad \dot{q} = \frac{\overline{h}(T_s - T_\infty)}{a}.$$

To select a convection correlation for estimating $\overline{h}$, first find the Reynolds numbers at x = L.

$$Re_L = \frac{u_\infty L}{\nu} = \frac{30 \text{ m/s} \times 0.70 \text{ m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.537 \times 10^5.$$

Since the flow is turbulent over the module, the approximation $\overline{h} \approx h_x \ (L + b/2)$ is appropriate, with

$$Re_{L+b/2} = \frac{30 \text{ m/s} \times (0.700 + 0.050/2)\text{m}}{22.02 \times 10^{-6} \text{m}^2/\text{s}} = 9.877 \times 10^5.$$

Using the turbulent flow correlation with x = L + b/2 = 0.725 m,

$$Nu_x = \frac{h_x x}{k} = 0.0296 Re_x^{4/5} Pr^{1/3}$$

$$Nu_x = 0.0296\left(9.877 \times 10^5\right)^{4/5}(0.698)^{1/3} = 1640$$

$$\overline{h} \approx h_x = \frac{Nu_x k}{x} = \frac{1640 \times 0.0308 \text{ W/m}\cdot\text{K}}{0.725} = 69.7 \text{ W/m}^2\cdot\text{K}.$$

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Hence,

$$\dot{q} = \frac{69.7\ \text{W/m}^2 \cdot \text{K}(150-25)\text{K}}{0.010\ \text{m}} = 8.713 \times 10^5\ \text{W/m}^3. \quad <$$

(b) The maximum temperature within the module occurs at the surface next to the insulation (y = 0). For one-dimensional conduction with thermal energy generation, use Eq. 3.42 to obtain

$$T(0) = \frac{\dot{q}a^2}{2k} + T_s = \frac{8.713 \times 10^5\ \text{W/m}^3 \times (0.010\ \text{m})^2}{2 \times 5.2\ \text{W/m} \cdot \text{K}} + 150^\circ\text{C} = 158.4^\circ\text{C}. \quad <$$

COMMENTS: An alternative approach for estimating the average heat transfer coefficient for the module follows from the relation

$$q_{\text{module}} = q_{0\to L+b} - q_{0\to L}$$

$$\bar{h} \cdot b = \bar{h}_{L+b} \cdot (L+b) - \bar{h}_L \cdot L \qquad \text{or} \qquad \bar{h} = \bar{h}_{L+b}\frac{L+b}{b} - \bar{h}_L \frac{L}{b}.$$

Recognizing that laminar and turbulent flow conditions exist, the appropriate correlation is

$$\overline{Nu}_x = \left(0.037 Re_x^{4/5} - 871\right) Pr^{1/3}$$

With x = L + b and x = L, find

$$\bar{h}_{L+b} = 54.79\ \text{W/m}^2 \cdot \text{K} \qquad \text{and} \qquad \bar{h}_L = 53.73\ \text{W/m}^2 \cdot \text{K}.$$

Hence,

$$\bar{h} = \left[54.79\frac{0.750}{0.050} - 53.73\frac{0.700}{0.05}\right]\ \text{W/m}^2 \cdot \text{K} = 69.7\ \text{W/m}^2 \cdot \text{K}.$$

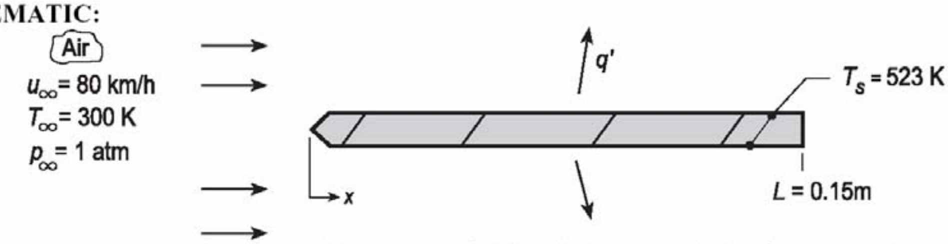
which is in excellent agreement with the approximate result employed in part (a).

PROBLEM 7.16

KNOWN: Length and surface temperature of a rectangular fin.

FIND: (a) Heat removal per unit width, q', when air at a prescribed temperature and velocity is in parallel, turbulent flow over the fin, and (b) Calculate and plot q' for motorcycle speeds ranging from 10 to 100 km/h.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible radiation, (3) Turbulent flow over entire surface.

PROPERTIES: *Table A.4*, Air (412 K, 1 atm): $\nu = 27.85 \times 10^{-6}$ m²/s, k = 0.0346 W/m·K, Pr = 0.69.

ANALYSIS: (a) The heat loss per unit width is

$$q' = 2\times\left[\overline{h}_L L\left(T_s - T_\infty\right)\right]$$

where $\overline{h}$ is obtained from the correlation, Eq. 7.38 but with turbulent flow over the entire surface,

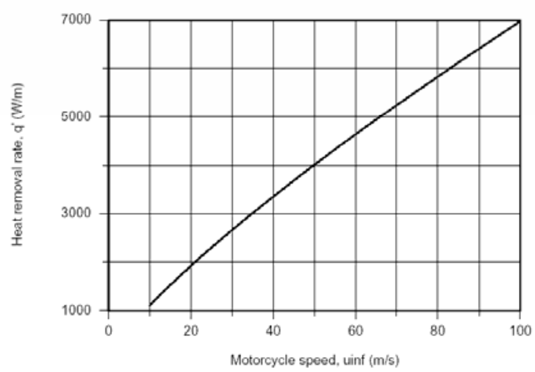
$$\overline{Nu}_L = 0.037\,Re_L^{4/5}\,Pr^{1/3} = 0.037\left[\frac{80\,km/h\times1000\,m/km\times1/3600\,h/s\times0.15\,m}{27.85\times10^{-6}\,m^2/s}\right]^{4/5}(0.69)^{1/3} = 378$$

Hence,

$$\overline{h}_L = \frac{k}{L}\overline{Nu}_L = \frac{0.0346\,W/m\cdot K}{0.15\,m}378 = 87\,W/m^2\cdot K$$

$$q' = 2\times\left[87\,W/m^2\cdot K\times0.15\,m\left(523-300\right)K\right] = 5826\,W/m.$$ <

(b) Using the foregoing equations in the IHT Workspace, q' as a function of speed was calculated and is plotted as shown.



COMMENTS: (1) Radiation emission from the fin is not negligible. With an assumed emissivity of ε = 1, the rate of emission per unit width at 80 km/h would be $q' = \left(\sigma T_s^4\right)2L = 1273$ W/m. If the fin received negligible radiation from its surroundings, its loss by radiation would then be approximately 20% of that by convection.

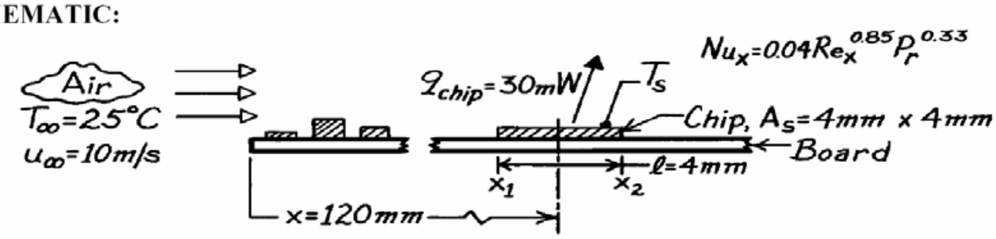
(2) From the correlation and heat rate expression, it follows that $q' \sim u_\infty^{4/5}$. That is, q' vs. u_∞ is nearly linear as evident from the above plot.

PROBLEM 7.34

KNOWN: Convection correlation for irregular surface due to electronic elements mounted on a circuit board experiencing forced air cooling with prescribed temperature and velocity

FIND: Surface temperature when heat dissipation rate is 30 mW for chip of prescribed area located a specific distance from the leading edge.

SCHEMATIC:



ASSUMPTIONS: (1) Situation approximates parallel flow over a flat plate with prescribed correlation, (2) Heat rate is from top surface of chip.

PROPERTIES: *Table A-4*, Air (assume $T_s \approx 45°C$, then $\overline{T} = (45 + 25)°C/2 \approx 310$ K, 1 atm): k = 0.027 W/m·K, $\nu = 16.90 \times 10^{-6}$ m^2/s, Pr = 0.706.

ANALYSIS: For the chip upper surface, the heat rate is

$$q_{chip} = \overline{h}_{chip} A_s (T_s - T_\infty) \qquad \text{or} \qquad T_s = T_\infty + q_{chip} / \overline{h}_{chip} A_s$$

Assuming the average convection coefficient over the chip length to be equal to the local value at the center of the chip ($x = x_o$), $\overline{h}_{chip} \approx h_x(x_o)$, where

$$Nu_x = 0.04 Re_x^{0.85} Pr^{0.33}$$

$$Nu_x = 0.04\left(10 \text{ m/s} \times 0.120 \text{ m}/16.90 \times 10^{-6} \text{ m}^2/\text{s}\right)^{0.85} (0.706)^{0.33} = 473.4$$

$$h_x = \frac{Nu_x k}{x_o} = \frac{473.4 \times 0.027 \text{ W/m} \cdot \text{K}}{0.120 \text{ m}} = 107 \text{ W/m}^2 \cdot \text{K}$$

Hence,

$$T_s = 25°C + 30 \times 10^{-3} \text{ W}/107 \text{ W/m}^2 \cdot \text{K} \times \left(4 \times 10^{-3} \text{m}\right)^2 = (25 + 17.5)° C = 42.5°C. \quad <$$

COMMENTS: (1) Note that the assumed value of $\overline{T}$ used to evaluate the thermophysical properties was reasonable. (2) We could have evaluated $\overline{h}_{chip}$ by two other approaches. In one case the average coefficient is approximated as the arithmetic mean of local values at the leading and trailing edges of the chip.

$$\overline{h}_{chip} \approx \left[h_{x2}(x_2) + h_{x1}(x_1)\right]/2 = 107 \text{ W/m}^2 \cdot \text{K}.$$

The exact approach is of the form

$$\overline{h}_{chip} \cdot \ell = \overline{h}_{x2} \cdot x_2 - \overline{h}_{x1} \cdot x_1$$

Recognizing that $h_x \sim x^{-0.15}$, it follows that

$$\overline{h}_x = \frac{1}{x}\int_0^x h_x \cdot dx = 1.176 h_x$$

and $\overline{h}_{chip} = 108 \text{ W/m}^2 \cdot \text{K}$. Why do results for the two approximate methods and the exact method compare so favorably?