1. (20 pts) The Lumped Capacitance Method
   (a) List and describe the implications of the two major assumptions of the lumped capacitance method. (6 pts)
   (b) Define the Biot number by equations and words. (4 pts)
   (c) Describe what happens when Bi << 1. (3 pts)
   (d) Describe what happens when Bi >> 1. (3 pts)
   (e) Define the Fourier number by equation. (4 pts)

2. (8 pts) Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which \( T_\infty = 325 \) K and \( h = 20 \) W/m\(^2\)·K. Assume the properties of the steel to be \( k = 40 \) W/m·K, \( \rho = 7800 \) kg/m\(^3\) and \( c = 600 \) J/kg·K.
   (a) Estimate the time required for the cooling process.
   (b) At what ball diameter can the lumped capacitance approach no longer be applied if we use the conservative definition for characteristic length that \( L = r \)? (Part b answer not posted online.)

3. (8 pts) Thermal energy storage systems commonly involve a packed bed of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas.

Consider a packed bed of 75-mm-diameter aluminum spheres (\( \rho = 2700 \) kg/m\(^3\), \( c = 950 \) J/kg·K, \( k = 240 \) W/m·K) and a charging process for which gas enters the storage unit at a temperature of \( T_{g,i} = 300^\circ\)C. If the initial temperature of the spheres is \( T_i = 25^\circ\)C and the convection coefficient is \( h = 75 \) W/m\(^2\)·K, how long does it take a sphere near the inlet of the system to accumulate 90% of maximum possible thermal energy? What is the corresponding temperature at the center of the sphere? Is there any advantage to using copper instead of aluminum?

More problems on the other side...

* Solutions for these problems are available on the course website: www.chen3453.com
4*. (8 pts) Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 100-mm-thick steel plate ($\rho = 7830 \text{ kg/m}^3$, $c = 550 \text{ J/kg} \cdot \text{K}$, $k = 48 \text{ W/m} \cdot \text{K}$) which is initially at a uniform temperature of $T_i = 200^\circ \text{C}$ and is to be heated to a minimum temperature of $550^\circ \text{C}$. Heating is effected in a gas-fired furnace, where products of combustion at $T_\infty = 800^\circ \text{C}$ maintain a convection coefficient of $h = 250 \text{ W/m}^2 \cdot \text{K}$ on both surfaces of the plate. How long should the plate be left in the furnace?

5.* (8 pts) In a manufacturing process, long rods of different diameters are at a uniform temperature of $400^\circ \text{C}$ in a curing oven, from which they are removed and cooled by forced convection in air at $25^\circ \text{C}$. One of the line operators has observed that it takes 280 seconds for a 40-mm-diameter rod to cool to a safe-to-handle temperature of $60^\circ \text{C}$. For an equivalent convection coefficient, how long will it take for an 80-mm-diameter rod to cool to the same temperature? The thermophysical properties of the rod are $\rho = 2500 \text{ kg/m}^3$, $c = 900 \text{ J/kg} \cdot \text{K}$ and $k = 15 \text{ W/m} \cdot \text{K}$. Comment on your result did you anticipate this outcome?

6.* (8 pts) Standards for firewalls may be based on their thermal response to a prescribed radiant heat flux. Consider a 0.25-m-thick concrete wall ($\rho = 2300 \text{ kg/m}^3$, $c = 880 \text{ J/kg} \cdot \text{K}$, $k = 1.4 \text{ W/m} \cdot \text{K}$), which is at an initial temperature of $T_i = 25^\circ \text{C}$ and irradiated at one surface by lamps that provide a uniform heat flux of $q'' = 10^5 \text{ W/m}^2$. The absorptivity of the surface to the irradiation is $\alpha_s = 1.0$. If building code requirements dictate that the temperatures of the irradiated and back surfaces must not exceed $325^\circ \text{C}$ and $25^\circ \text{C}$, respectively, after 30 minutes of heating, will the requirements be met?

7. (20 pts) A probe for a high temperature furnace is made of a long solid rod of sapphire (aluminum oxide) and is 40 mm diameter. Suppose that the probe has been in a furnace for days so that its temperature is 800 K throughout. It is then quickly removed and immediately placed into water bath to provide rapid quenching. The water is at 300 K and effects a heat transfer coefficient of 1600 W/m$^2$·K. After just 35 seconds the rod is removed from the water bath and placed in a protective case that is lined with insulation so that the rod experiences no further heat losses. What will be the final temperature of the rod after it has equilibrated to a uniform temperature throughout?

8. (20 pts) A rocky ledge of sandstone near the ocean on the southern Oregon coast has reached a uniform temperature of $40^\circ \text{C}$ on a nice, sunny day. At 3:00 pm, the tide rises enough to suddenly cover the ledge, decreasing the surface temperature to $15^\circ \text{C}$. How much energy (J), per unit area (m$^2$) is transferred from the sandstone ledge in 5 minutes? The ledge can be considered a semi-infinite solid.

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