

QUESTION

1. Part a) Define the following terms (3 pts each):

- (i) Isotherm
- (ii) Heat flow line
- (iii) Adiatat
- (iv) Shape factor
- (v) Nodal point

Part b) How are isotherms, adiabats and heat flow lines related? (5 pts)

ANSWER

Part a)

- (i) Isotherm – A vector that is everywhere perpendicular to lines of constant temperature.
- (ii) Heat flow line – The directions of a heat flux vector are represented by heat flow lines, and the vector itself is the resultant of heat flux components in the x- and y-directions.
- (iii) Adiatat – This is another term for specific heat flow lines, where no heat can be conducted across a heat flow line.
- (iv) Shape factor – A factor used to quickly solve two- and three-dimensional conduction problems. The symbol, S , is a steady state dimensionless conduction heat rate term.
- (v) Nodal point – Also called a node, this is a reference point used in numerical solutions. The aggregate of nodal points is called a nodal network.

Part b)

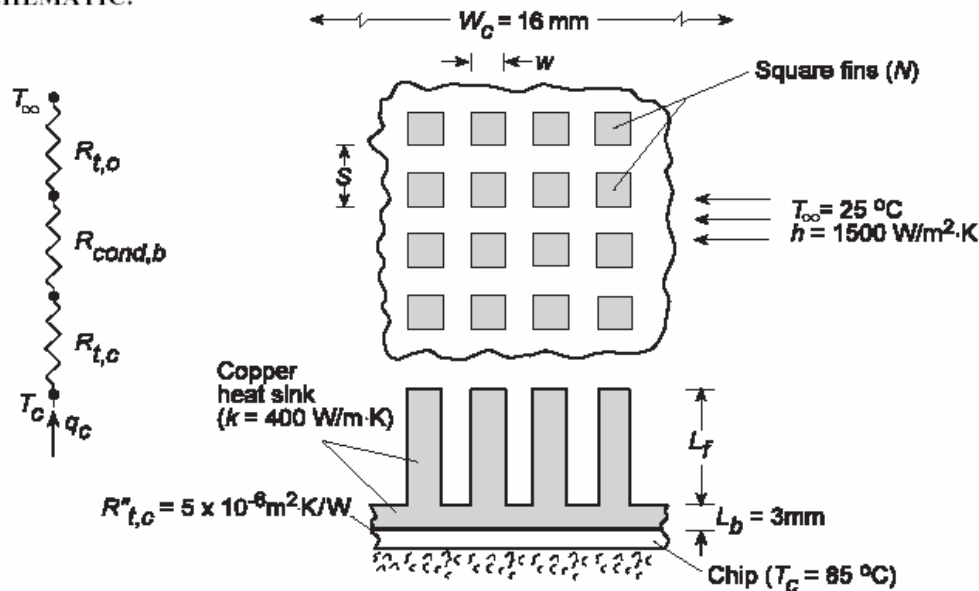
Heat flow lines could be termed as adiabats, given that conduction heat transfer does not occur across the heat line. Plotting temperature versus position allows isotherms to be constructed in the x-direction and heat flow lines to be the heat flux vector in the direction of heat transfer. (See Figure 4.1 on page 202 if Incropera, et. al. 6/e)

PROBLEM 3.136

KNOWN: Copper heat sink dimensions and convection conditions.

FIND: (a) Maximum allowable heat dissipation for a prescribed chip temperature and interfacial chip/heat-sink contact resistance, (b) Effect of fin length and width on heat dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-heat sink assembly, (3) Constant k , (4) Negligible chip thermal resistance, (5) Negligible heat transfer from back of chip, (6) Uniform chip temperature.

ANALYSIS: (a) For the prescribed system, the chip power dissipation may be expressed as

$$q_c = \frac{T_c - T_\infty}{R_{t,c} + R_{cond,b} + R_{t,o}}$$

where $R_{t,c} = \frac{R''_{t,c}}{W_c^2} = \frac{5 \times 10^{-6} \text{ m}^2 \cdot \text{K/W}}{(0.016 \text{ m})^2} = 0.0195 \text{ K/W}$

$$R_{cond,b} = \frac{L_b}{kW_c^2} = \frac{0.003 \text{ m}}{400 \text{ W/m} \cdot \text{K} (0.016 \text{ m})^2} = 0.0293 \text{ K/W}$$

The thermal resistance of the fin array is

$$R_{t,o} = (\eta_o h A_t)^{-1}$$

where $\eta_o = 1 - \frac{N A_f}{A_t} (1 - \eta_f)$

and $A_t = N A_f + A_b = N(4wL_c) + (W_c^2 - Nw^2)$

Continued...

PROBLEM 3.136 (Cont.)

With $w = 0.25$ mm, $S = 0.50$ mm, $L_f = 6$ mm, $N = 1024$, and $L_c \approx L_f + w/4 = 6.063 \times 10^{-3}$ m, it follows that $A_f = 6.06 \times 10^{-6}$ m² and $A_t = 6.40 \times 10^{-3}$ m². The fin efficiency is

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

where $m = (hP/kA_c)^{1/2} = (4h/kw)^{1/2} = 245$ m⁻¹ and $mL_c = 1.49$. It follows that $\eta_f = 0.608$ and $\eta_o = 0.619$, in which case

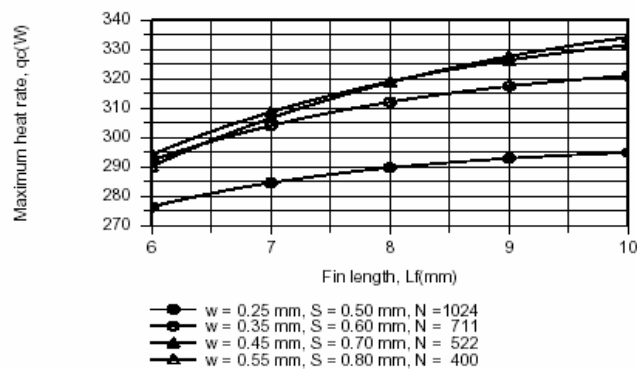
$$R_{t,o} = \left(0.619 \times 1500 \text{ W/m}^2 \cdot \text{K} \times 6.40 \times 10^{-3} \text{ m}^2 \right) = 0.168 \text{ K/W}$$

and the maximum allowable heat dissipation is

$$q_c = \frac{(85 - 25)^\circ \text{C}}{(0.0195 + 0.0293 + 0.168) \text{ K/W}} = 276 \text{ W}$$

(b) The IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array* has been used to determine q_c as a function of L_f for four different cases, each of which is characterized by the closest allowable fin spacing of $(S - w) = 0.25$ mm.

Case	w (mm)	S (mm)	N
A	0.25	0.50	1024
B	0.35	0.60	711
C	0.45	0.70	522
D	0.55	0.80	400



With increasing w and hence decreasing N , there is a reduction in the total area A_t associated with heat transfer from the fin array. However, for Cases A through C, the reduction in A_t is more than balanced by an increase in η_f (and η_o), causing a reduction in $R_{t,o}$ and hence an increase in q_c . As the fin efficiency approaches its limiting value of $\eta_f = 1$, reductions in A_t due to increasing w are no longer balanced by increases in η_f , and q_c begins to decrease. Hence there is an optimum value of w , which depends on L_f . For the conditions of this problem, $L_f = 10$ mm and $w = 0.55$ mm provide the largest heat dissipation.

PROBLEM 4.9

KNOWN: Heat generation in a buried spherical container.

FIND: (a) Outer surface temperature of the container, (b) Representative isotherms and heat flow lines.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Soil is a homogeneous medium with constant properties.

PROPERTIES: Table A-3, Soil (300K): $k = 0.52 \text{ W/m}\cdot\text{K}$.

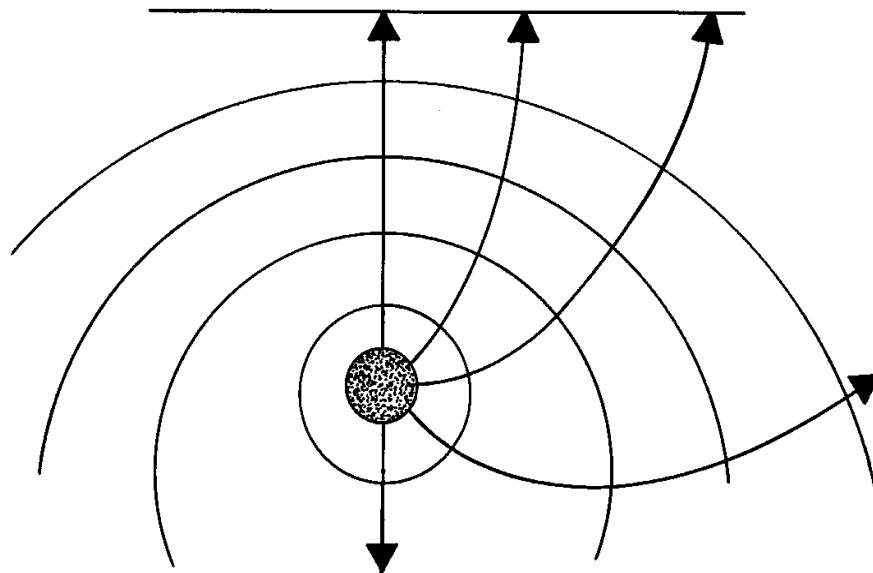
ANALYSIS: (a) From an energy balance on the container, $q = \dot{E}_g$ and from the first entry in Table 4.1,

$$q = \frac{2\pi D}{1 - D/4z} k(T_1 - T_2).$$

Hence,

$$T_1 = T_2 + \frac{q}{k} \frac{1 - D/4z}{2\pi D} = 20^\circ\text{C} + \frac{500\text{W}}{0.52 \frac{\text{W}}{\text{m}\cdot\text{K}}} \frac{1 - 2\text{m}/40\text{m}}{2\pi(2\text{m})} = 92.7^\circ\text{C} \quad <$$

(b) The isotherms may be viewed as spherical surfaces whose center moves downward with increasing radius. The surface of the soil is an isotherm for which the center is at $z = \infty$.

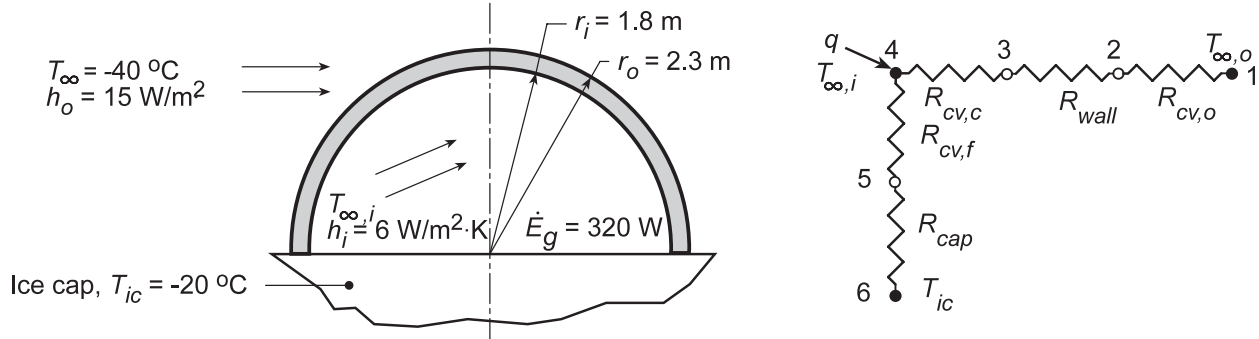


PROBLEM 4.25

KNOWN: Igloo constructed in hemispheric shape sits on ice cap; igloo wall thickness and inside/outside convection coefficients (h_i , h_o) are prescribed.

FIND: (a) Inside air temperature $T_{\infty,i}$ when outside air temperature is $T_{\infty,o} = -40^\circ\text{C}$ assuming occupants provide 320 W within igloo, (b) Perform parameter sensitivity analysis to determine which variables have significant effect on T_i .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficient is the same on floor and ceiling of igloo, (3) Floor and ceiling are at uniform temperatures, (4) Floor-ice cap resembles disk on semi-infinite medium, (5) One-dimensional conduction through igloo walls.

PROPERTIES: Ice and compacted snow (given): $k = 0.15 \text{ W/m}\cdot\text{K}$.

ANALYSIS: (a) The thermal circuit representing the heat loss from the igloo to the outside air and through the floor to the ice cap is shown above. The heat loss is

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{cv,c} + R_{wall} + R_{cv,o}} + \frac{T_{\infty,i} - T_{ic}}{R_{cv,f} + R_{cap}}$$

$$\text{Convection, ceiling: } R_{cv,c} = \frac{2}{h_i (4\pi r_i^2)} = \frac{2}{6 \text{ W/m}^2 \cdot \text{K} \times 4\pi (1.8 \text{ m})^2} = 0.00819 \text{ K/W}$$

$$\text{Convection, outside: } R_{cv,o} = \frac{2}{h_o (4\pi r_o^2)} = \frac{2}{15 \text{ W/m}^2 \cdot \text{K} \times 4\pi (2.3 \text{ m})^2} = 0.00201 \text{ K/W}$$

$$\text{Convection, floor: } R_{cv,f} = \frac{1}{h_i (\pi r_i^2)} = \frac{1}{6 \text{ W/m}^2 \cdot \text{K} \times \pi (1.8 \text{ m})^2} = 0.01637 \text{ K/W}$$

$$\text{Conduction, wall: } R_{wall} = 2 \left[\frac{1}{4\pi k} \left(\frac{1}{r_i} - \frac{1}{r_o} \right) \right] = 2 \left[\frac{1}{4\pi \times 0.15 \text{ W/m}\cdot\text{K}} \left(\frac{1}{1.8} - \frac{1}{2.3} \right) \text{ m} \right] = 0.1281 \text{ K/W}$$

$$\text{Conduction, ice cap: } R_{cap} = \frac{1}{kS} = \frac{1}{4kr_i} = \frac{1}{4 \times 0.15 \text{ W/m}\cdot\text{K} \times 1.8 \text{ m}} = 0.9259 \text{ K/W}$$

where S was determined from the shape factor of Table 4.1. Hence,

$$q = 320 \text{ W} = \frac{T_{\infty,i} - (-40)^\circ\text{C}}{(0.00819 + 0.1281 + 0.00201) \text{ K/W}} + \frac{T_{\infty,i} - (-20)^\circ\text{C}}{(0.01637 + 0.9259) \text{ K/W}}$$

$$320 \text{ W} = 7.231(T_{\infty,i} + 40) + 1.06(T_{\infty,i} + 20) \quad T_{\infty,i} = 1.2^\circ\text{C.}$$

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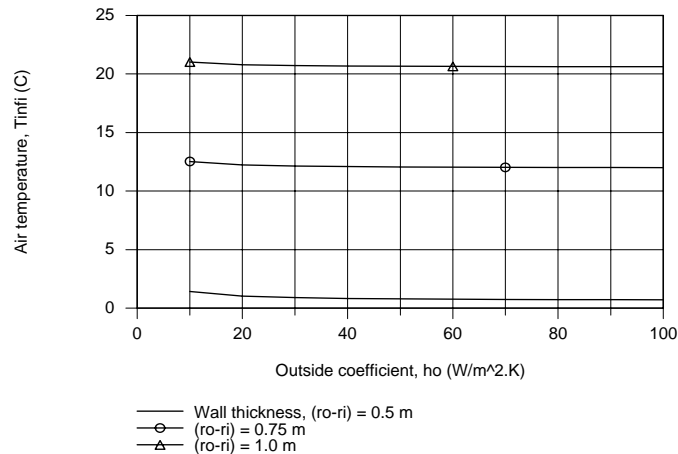
PROBLEM 4.25 (Cont.)

(b) Begin the parameter sensitivity analysis to determine important variables which have a significant influence on the inside air temperature by examining the thermal resistances associated with the processes present in the system and represented by the network.

Process	Symbols		Value (K/W)
Convection, outside	$R_{cv,o}$	R21	0.0020
Conduction, wall	R_{wall}	R32	0.1281
Convection, ceiling	$R_{cv,c}$	R43	0.0082
Convection, floor	$R_{cv,f}$	R54	0.0164
Conduction, ice cap	R_{cap}	R65	0.9259

It follows that the convection resistances are negligible relative to the conduction resistance across the igloo wall. As such, only changes to the wall thickness will have an appreciable effect on the inside air temperature relative to the outside ambient air conditions. We don't want to make the igloo walls thinner and thereby allow the air temperature to dip below freezing for the prescribed environmental conditions.

Using the *IHT Thermal Resistance Network Model*, we used the circuit builder to construct the network and perform the energy balances to obtain the inside air temperature as a function of the outside convection coefficient for selected increased thicknesses of the wall.



COMMENTS: (1) From the plot, we can see that the influence of the outside air velocity which controls the outside convection coefficient h_o is negligible.

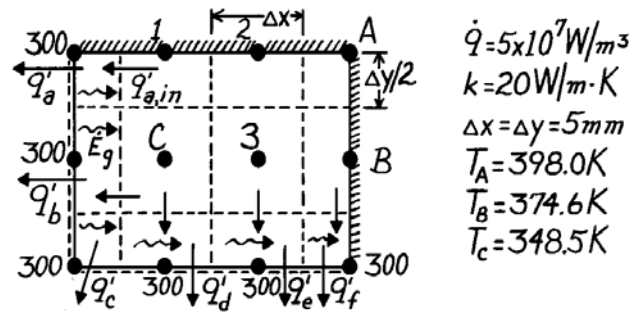
(2) The thickness of the igloo wall is the dominant thermal resistance controlling the inside air temperature.

PROBLEM 4.45

KNOWN: Steady-state temperatures (K) at three nodes of a long rectangular bar.

FIND: (a) Temperatures at remaining nodes and (b) heat transfer per unit length from the bar using nodal temperatures; compare with result calculated using knowledge of \dot{q} .

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equations for the nodes (1,2,3,A,B,C) can be written by inspection using Eq. 4.35 and recognizing that the adiabatic boundary can be represented by a symmetry plane.

$$\sum T_{\text{neighbors}} - 4T_i + \dot{q}\Delta x^2/k = 0 \quad \text{and} \quad \frac{\dot{q}\Delta x^2}{k} = \frac{5 \times 10^7 \text{ W/m}^3 (0.005\text{m})^2}{20 \text{ W/m}\cdot\text{K}} = 62.5\text{K}.$$

Node A (to find T_2):

$$2T_2 + 2T_B - 4T_A + \dot{q}\Delta x^2/k = 0$$

$$T_2 = \frac{1}{2}(-2 \times 374.6 + 4 \times 398.0 - 62.5) \text{ K} = 390.2\text{K} \quad <$$

Node 3 (to find T_3):

$$T_C + T_2 + T_B + 300\text{K} - 4T_3 + \dot{q}\Delta x^2/k = 0$$

$$T_3 = \frac{1}{4}(348.5 + 390.2 + 374.6 + 300 + 62.5) \text{ K} = 369.0\text{K} \quad <$$

Node 1 (to find T_1):

$$300 + 2T_C + T_2 - 4T_1 + \dot{q}\Delta x^2/k = 0$$

$$T_1 = \frac{1}{4}(300 + 2 \times 348.5 + 390.2 + 62.5) = 362.4\text{K} \quad <$$

(b) The heat rate out of the bar is determined by calculating the heat rate out of each control volume around the 300 K nodes. Consider the node in the upper left-hand corner; from an energy balance

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = 0 \quad \text{or} \quad q'_a = q'_{a,\text{in}} + \dot{E}_g \quad \text{where} \quad \dot{E}_g = \dot{q}V.$$

Hence, for the entire bar $q'_{\text{bar}} = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$, or

$$q'_{\text{bar}} = \left[k \frac{\Delta y}{2} \frac{T_1 - 300}{\Delta x} + \dot{q} \left[\frac{\Delta x}{2} \cdot \frac{\Delta y}{2} \right] \right]_a + \left[k \Delta y \frac{T_C - 300}{\Delta x} + \dot{q} \left[\frac{\Delta x}{2} \cdot \Delta y \right] \right]_b + \left[\dot{q} \left[\frac{\Delta x}{2} \cdot \frac{\Delta y}{2} \right] \right]_c + \left[k \Delta x \frac{T_C - 300}{\Delta y} + \dot{q} \left[\Delta x \cdot \frac{\Delta y}{2} \right] \right]_d + \left[k \Delta x \frac{T_3 - 300}{\Delta y} + \dot{q} \left[\Delta x \cdot \frac{\Delta y}{2} \right] \right]_e + \left[k \frac{\Delta x}{2} \frac{T_B - 300}{\Delta y} + \dot{q} \left[\frac{\Delta x}{2} \cdot \frac{\Delta y}{2} \right] \right]_f.$$

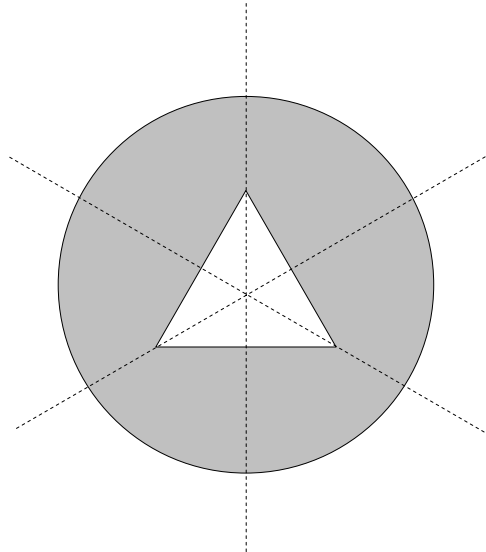
Substituting numerical values, find $q'_{\text{bar}} = 7,502.5 \text{ W/m}$. From an overall energy balance on the bar,

$$q'_{\text{bar}} = \dot{E}'_g = \dot{q}V/\ell = \dot{q}(3\Delta x \cdot 2\Delta y) = 5 \times 10^7 \text{ W/m}^3 \times 6(0.005\text{m})^2 = 7,500 \text{ W/m}. \quad <$$

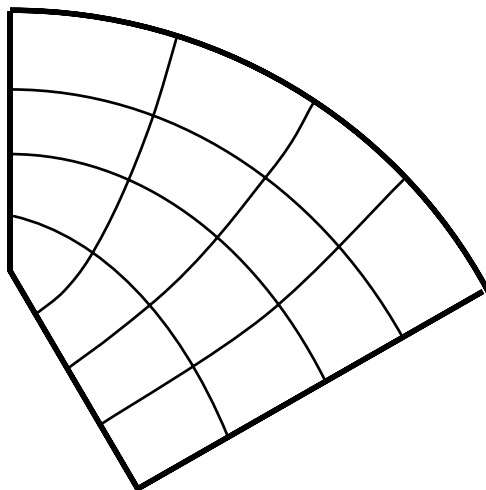
As expected, the results of the two methods agree. Why must that be?

Problem 1.

This problem can be simplified by recognizing that the geometry can be split not just into thirds, but into sixths:



Recognizing that no heat is transferred across the “cut” sides (left and lower right), heat flow lines can be drawn roughly in the direction of the arc, making sure that they contact perpendicular to the adiabatic “cut” lines. The inner and outer edges of the geometry are isothermal, so additional isotherms can be drawn between these, making sure cross the heat flow lines perpendicularly. The result should look something like that below:



GO-CART PROBLEM SOLUTION

→ For a circular fin, use Figure 3.19 on Incropera page 150.

$$r_1 = 5 \text{ cm}$$

$$r_2 = 5 + 5 = 10 \text{ cm}$$

$$r_{2c} \text{ (Fig 3.19)} = 10 + \left(\frac{0.3}{2}\right) = 10.15 \text{ cm}$$

$$L_c = 5 + \frac{0.3}{2} = 5.15 \text{ cm} = 0.0515 \text{ m}$$

$$A_p = L_c t = (0.0515)(0.003) = 0.000155 \text{ m}^2$$

$$r_{2c}/r_1 = \frac{10.15}{5} = 2.03$$

$$\begin{aligned} L_c^{3/2} (h/kA_p)^{1/2} &= 0.0515^{3/2} \left[(80) / (240(0.000155)) \right]^{1/2} \\ &= 0.543 \end{aligned}$$

→ From Figure 3.19, $\eta_f \sim \underline{\underline{80\%}}$

$$q = \eta_f h A_f (T_b - T_\infty)$$

$$\begin{aligned} A_f &= 2\pi(0.1^2 - 0.05^2) + 2\pi(0.1)(0.003) \\ &= 0.049 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} q &= 0.8 \left(80 \frac{\text{W}}{\text{m}^2 \cdot \text{K}} \right) (0.049 \text{ m}^2) (265 - 27 \text{ K}) \\ &= \boxed{747 \text{ Watts}} \end{aligned}$$

GO-CART PROBLEM CONT...

Amount of heat to remove:

$$(8500 \text{ W}) \times (1 - 0.3) \times (0.8) = 4760 \text{ Watts}$$

Fins needed

$$4760 / 747 = 6.38 \text{ fins}$$

7 FINS

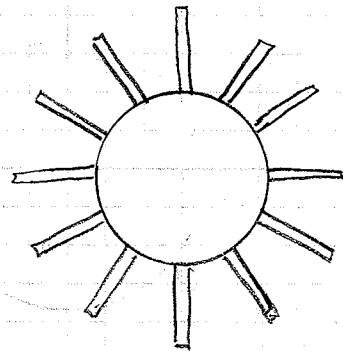
FINNED PIPE PROBLEM

Data:

$$k = 42 \frac{\text{W}}{\text{m}\cdot\text{K}}$$
$$h = 34 \frac{\text{W}}{\text{m}^2\cdot\text{K}}$$
$$D_o = 0.025 \text{ m}$$
$$T_{s,b} = 120^\circ\text{C}$$
$$T_\infty = 25^\circ\text{C}$$

Part a.) Longitudinal fins

→ These fins would simply be "straight fins" in Table 3.5.



→ Let's take a basis length of one meter.

→ For a tube with no fins

$$q = hA\Delta T$$
$$= \left(34 \frac{\text{W}}{\text{m}^2\cdot\text{K}}\right) \left(\pi(0.025 \text{ m})(1 \text{ m})\right) (120 - 25^\circ\text{C})$$
$$= \underline{\underline{254 \text{ Watts for no fins}}}$$

Continued...

FINNED PIPES, CONT.

→ For the 1 meter length, let's do some calculations:

$$L_c = L + t/2 = 0.018 + \frac{0.003}{2} = 0.0195 \text{ m}$$

$$A_f = 2WL_c = 2(1 \text{ m})(0.0195 \text{ m}) = 0.039 \text{ m}^2$$

$$A_p = tL_c = (0.003 \text{ m})(0.0195 \text{ m}) = 0.0000585 \text{ m}^2$$

→ Let's determine the efficiency of one fin from Figure 3.18:

$$L_c^{3/2} \left(\frac{h}{kA_p} \right)^{1/2}$$

$$= (0.0195 \text{ m})^{3/2} \left[\frac{341 \text{ W}}{\text{m} \cdot \text{K}} \left(\frac{\text{m} \cdot \text{K}}{42 \text{ W}} \right) \left(\frac{1}{0.0000585 \text{ m}^2} \right) \right]^{1/2}$$

$$= 0.320$$

→ From Fig 3.18 we get $\eta_f \sim \underline{\underline{0.92}}$

→ For overall efficiency we need the "prime" area A_b

$$A_b = \left(\pi (0.025 \text{ m}) - 12(0.003 \text{ m}) \right) (1 \text{ m})$$

Diameter not
occupied by
fins attached

$$= 0.04254 \text{ m}^2$$

continued...

FINNED PIPE, CONT...

$$\text{Overall efficiency } \eta_o = 1 - \frac{NA_f}{A_t} (1 - \eta_f)$$

$$\begin{aligned} A_t &= NA_f + A_b \\ &= 12(0.039) + 0.04254 \\ &= 0.5105 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \eta_o &= 1 - \frac{12(0.039 \text{ m}^2)}{0.5105 \text{ m}^2} (1 - 0.92) \\ &= 0.927 \end{aligned}$$

So, total heat transfer is

$$\begin{aligned} q_t &= \eta_o h A_t \theta_b \\ &= 0.927 (34 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}) (0.5105 \text{ m}^2) (120 - 25) \\ &= \underline{1529 \text{ Watts}} \text{ for longitudinal fins} \end{aligned}$$

Compared to no fins we get

$$\frac{1529}{254} = 6.027$$

or as an increase

$$(6.03 - 1) \times 100\% = \boxed{503\% \text{ increase for longitudinal fins}}$$

continued...

Finned Pipe, cont.

Part b.) Circular Pins

→ Some calculations:

$$A_f = 2\pi(r_2^2 - r_1^2) \\ = 2\pi(0.061^2 - 0.025^2) = 0.004863 \text{ m}^2$$

$$L_c = L + t/2 = 0.0195 \text{ m}$$

$$A_p = L_c t = 0.0000585 \text{ m}^2$$

→ Per meter, to get the same surface area (A_f) as for the longitudinal case

$$\text{we need } 0.039 / 0.000585 = \underline{96.2 \text{ pins}}$$

(Of course, we would not have 0.2 pins somewhere, but assuming the tube is long enough we can use the fraction for the calculations.)

→ The efficiency of a single pin comes from Figure 3.19

$$L_c^{3/2} \left(\frac{h}{kA_p} \right)^{1/2} \\ = (0.0195 \text{ m})^{3/2} \left[\left(\frac{34 \text{ W}}{\text{m}^2 \cdot \text{K}} \right) \left(\frac{\text{m} \cdot \text{K}}{42 \text{ W}} \right) \left(0.0000585 \text{ m}^2 \right) \right]^{1/2} \\ = \underline{0.320}$$

$$r_{2c} = r_2 + t/2 = \left[(0.025 + 2(0.018)) + \frac{0.003}{2} \right] / 2 \\ = 0.03125 \text{ m}$$

$$r_{2c}/r_1 = 0.0625 / 0.025 = \underline{2.5}$$

continued

FINNED PIPE, CONT...

→ From figure 3.19 we get

$$\eta_f \sim 0.88$$

→ For overall efficiency we need A_b & A_t

$$\begin{aligned} A_b &= \pi D (1 \text{ m} - Nt) \\ &= \pi (0.025 \text{ m}) [1 \text{ m} - 96.2(0.003 \text{ m})] \\ &= 0.05587 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} A_t &= A_b + NA_f \\ &= 0.05587 + 96.2(0.00486) \\ &= 0.524 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \eta_o &= 1 - \frac{NA_f}{A_t} (1 - \eta_f) \\ &= 1 - \frac{96.2(0.00486)}{0.524} (1 - 0.88) \\ &= 0.893 \end{aligned}$$

$$\begin{aligned} \text{So, total } q &= \eta_o h A_t \theta_b \\ &= 0.893 (34 \frac{\text{W}}{\text{m}^2 \cdot \text{K}}) (0.524 \text{ m}^2) (120 - 25 \text{ }^\circ\text{C}) \\ &= \underline{1511 \text{ Watts}} \end{aligned}$$

This is an increase of $\frac{1510 - 254}{254} \times 100\%$

$$= \boxed{496\% \text{ increase for circular fins relative to a bare tube}}$$