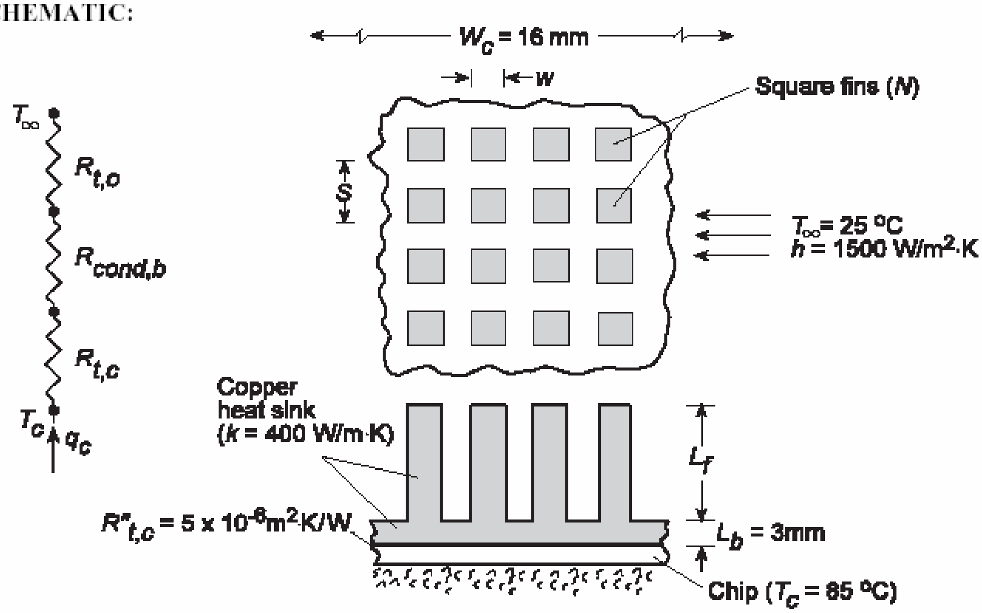
PROBLEM 3.136

KNOWN: Copper heat sink dimensions and convection conditions.

FIND: (a) Maximum allowable heat dissipation for a prescribed chip temperature and interfacial chip/heat-sink contact resistance, (b) Effect of fin length and width on heat dissipation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer in chip-heat sink assembly, (3) Constant k, (4) Negligible chip thermal resistance, (5) Negligible heat transfer from back of chip, (6) Uniform chip temperature.

ANALYSIS: (a) For the prescribed system, the chip power dissipation may be expressed as

$$q_c = \frac{T_c - T_\infty}{R_{t,c} + R_{cond,b} + R_{t,o}}$$

where $R_{t,c} = \frac{R''_{t,c}}{W_c^2} = \frac{5\times10^{-6}\,m^2\cdot K/W}{(0.016m)^2} = 0.0195\,K/W$

$$R_{cond,b} = \frac{L_b}{kW_c^2} = \frac{0.003m}{400\,W/m\cdot K\,(0.016m)^2} = 0.0293\,K/W$$

The thermal resistance of the fin array is

$$R_{t,o} = \left(\eta_o h A_t\right)^{-1}$$

where $\eta_o = 1 - \frac{NA_f}{A_t}\left(1-\eta_f\right)$

and $A_t = NA_f + A_b = N\left(4wL_c\right) + \left(W_c^2 - Nw^2\right)$

Continued...

PROBLEM 3.136 (Cont.)

With w = 0.25 mm, S = 0.50 mm, L_f = 6 mm, N = 1024, and $L_c \approx L_f + w/4 = 6.063\times10^{-3}$ m, it follows that $A_f = 6.06\times10^{-6}\ m^2$ and $A_t = 6.40\times10^{-3}\ m^2$. The fin efficiency is

$$\eta_f = \frac{\tanh mL_c}{mL_c}$$

where $m = (hP/kA_c)^{1/2} = (4h/kw)^{1/2}$ = 245 m^{-1} and mL_c = 1.49. It follows that η_f = 0.608 and η_o = 0.619, in which case

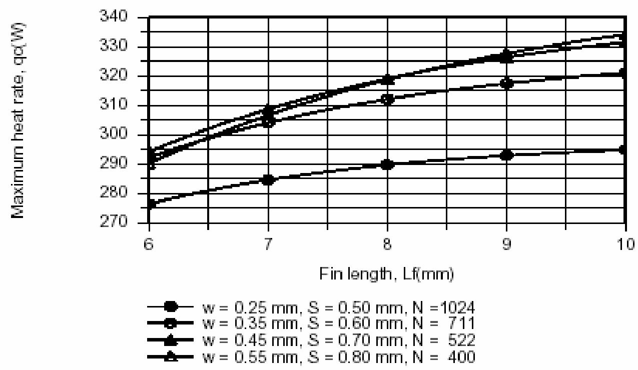
$$R_{t,o} = \left(0.619\times1500\ W/m^2\cdot K\times6.40\times10^{-3}\ m^2\right) = 0.168\ K/W$$

and the maximum allowable heat dissipation is

$$q_c = \frac{(85-25)^\circ C}{(0.0195+0.0293+0.168)K/W} = 276W$$ <

(b) The IHT *Performance Calculation, Extended Surface Model* for the *Pin Fin Array* has been used to determine q_c as a function of L_f for four different cases, each of which is characterized by the closest allowable fin spacing of (S - w) = 0.25 mm.

Case	w (mm)	S (mm)	N
A	0.25	0.50	1024
B	0.35	0.60	711
C	0.45	0.70	522
D	0.55	0.80	400



With increasing w and hence decreasing N, there is a reduction in the total area A_t associated with heat transfer from the fin array. However, for Cases A through C, the reduction in A_t is more than balanced by an increase in η_f (and η_o), causing a reduction in $R_{t,o}$ and hence an increase in q_c. As the fin efficiency approaches its limiting value of η_f = 1, reductions in A_t due to increasing w are no longer balanced by increases in η_f, and q_c begins to decrease. Hence there is an optimum value of w, which depends on L_f. For the conditions of this problem, L_f = 10 mm and w = 0.55 mm provide the largest heat dissipation.

PROBLEM 4.9

KNOWN: Heat generation in a buried spherical container.

FIND: (a) Outer surface temperature of the container, (b) Representative isotherms and heat flow lines.

SCHEMATIC:

ASSUMPTIONS: (1) Steady-state conditions, (2) Soil is a homogeneous medium with constant properties.

PROPERTIES: *Table A-3*, Soil (300K): k = 0.52 W/m·K.

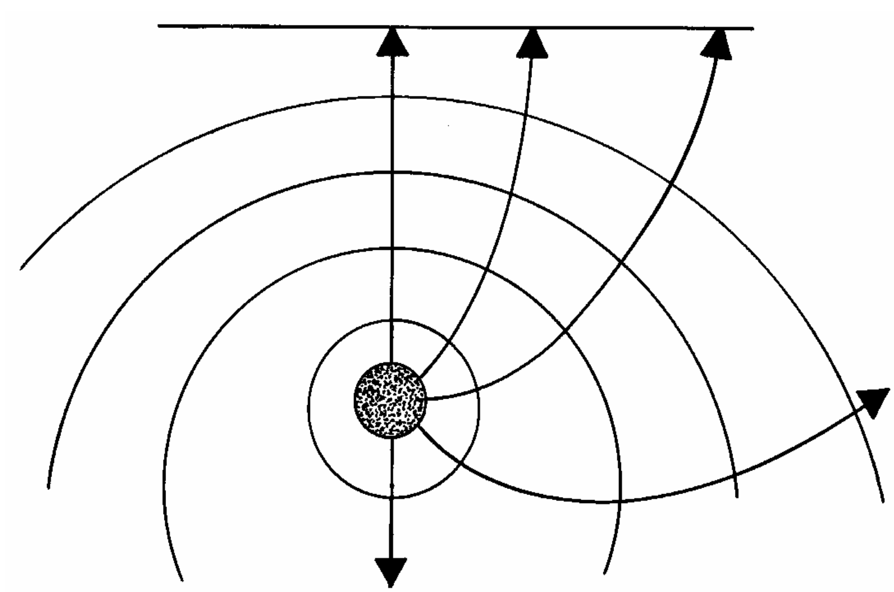
ANALYSIS: (a) From an energy balance on the container, $q = \dot{E}_g$ and from the first entry in Table 4.1,

$$q = \frac{2\pi D}{1 - D/4z} k\left(T_1 - T_2\right).$$

Hence,

$$T_1 = T_2 + \frac{q}{k}\frac{1 - D/4z}{2\pi D} = 20^\circ C + \frac{500\,W}{0.52\,\frac{W}{m \cdot K}} \frac{1 - 2m/40m}{2\pi\left(2m\right)} = 92.7^\circ C$$ <

(b) The isotherms may be viewed as spherical surfaces whose center moves downward with increasing radius. The surface of the soil is an isotherm for which the center is at z = ∞.

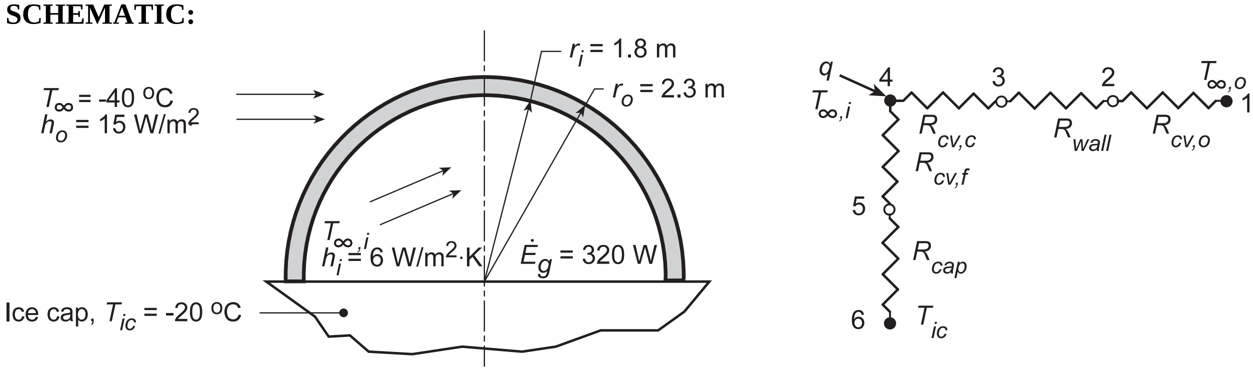


PROBLEM 4.25

KNOWN: Igloo constructed in hemispheric shape sits on ice cap; igloo wall thickness and inside/outside convection coefficients (h_i, h_o) are prescribed.

FIND: (a) Inside air temperature $T_{\infty,i}$ when outside air temperature is $T_{\infty,o}$ = -40°C assuming occupants provide 320 W within igloo, (b) Perform parameter sensitivity analysis to determine which variables have significant effect on T_i.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Convection coefficient is the same on floor and ceiling of igloo, (3) Floor and ceiling are at uniform temperatures, (4) Floor-ice cap resembles disk on semi-infinite medium, (5) One-dimensional conduction through igloo walls.

PROPERTIES: Ice and compacted snow (given): k = 0.15 W/m·K.

ANALYSIS: (a) The thermal circuit representing the heat loss from the igloo to the outside air and through the floor to the ice cap is shown above. The heat loss is

$$q = \frac{T_{\infty,i} - T_{\infty,o}}{R_{cv,c} + R_{wall} + R_{cv,o}} + \frac{T_{\infty,i} - T_{ic}}{R_{cv,f} + R_{cap}}.$$

Convection, ceiling: $$R_{cv,c} = \frac{2}{h_i\left(4\pi r_i^2\right)} = \frac{2}{6\,\text{W/m}^2\cdot\text{K}\times 4\pi(1.8\,\text{m})^2} = 0.00819\,\text{K/W}$$

Convection, outside: $$R_{cv,o} = \frac{2}{h_o\left(4\pi r_o^2\right)} = \frac{2}{15\,\text{W/m}^2\cdot\text{K}\times 4\pi(2.3\,\text{m})^2} = 0.00201\,\text{K/W}$$

Convection, floor: $$R_{cv,f} = \frac{1}{h_i\left(\pi r_i^2\right)} = \frac{1}{6\,\text{W/m}^2\cdot\text{K}\times \pi(1.8\,\text{m})^2} = 0.01637\,\text{K/W}$$

Conduction, wall: $$R_{wall} = 2\left[\frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_o}\right)\right] = 2\left[\frac{1}{4\pi\times 0.15\,\text{W/m}\cdot\text{K}}\left(\frac{1}{1.8} - \frac{1}{2.3}\right)\text{m}\right] = 0.1281\,\text{K/W}$$

Conduction, ice cap: $$R_{cap} = \frac{1}{kS} = \frac{1}{4kr_i} = \frac{1}{4\times 0.15\,\text{W/m}\cdot\text{K}\times 1.8\,\text{m}} = 0.9259\,\text{K/W}$$

where S was determined from the shape factor of Table 4.1. Hence,

$$q = 320\,\text{W} = \frac{T_{\infty,i} - (-40)^\circ\text{C}}{(0.00819 + 0.1281 + 0.00201)\,\text{K/W}} + \frac{T_{\infty,i} - (-20)^\circ\text{C}}{(0.01637 + 0.9259)\,\text{K/W}}$$

320 W = 7.231($T_{\infty,i}$ + 40) + 1.06($T_{\infty,i}$ + 20) $\qquad$ $T_{\infty,i}$ = 1.2°C. <

Continued...

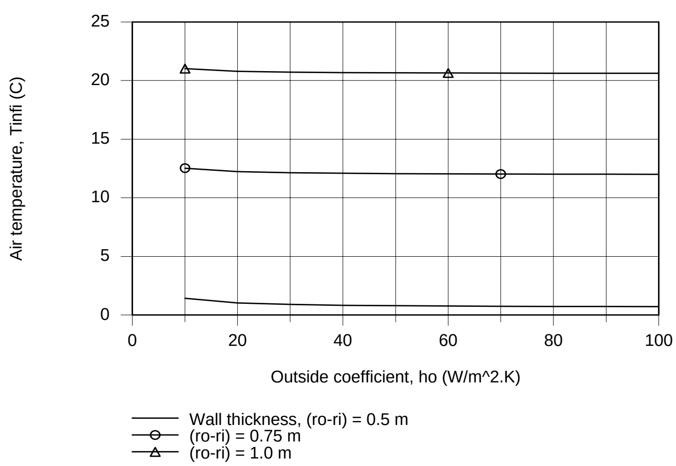
PROBLEM 4.25 (Cont.)

(b) Begin the parameter sensitivity analysis to determine important variables which have a significant influence on the inside air temperature by examining the thermal resistances associated with the processes present in the system and represented by the network.

Process	Symbols		Value (K/W)
Convection, outside	$R_{cv,o}$	R21	0.0020
Conduction, wall	R_{wall}	R32	0.1281
Convection, ceiling	$R_{cv,c}$	R43	0.0082
Convection, floor	$R_{cv,f}$	R54	0.0164
Conduction, ice cap	R_{cap}	R65	0.9259

It follows that the convection resistances are negligible relative to the conduction resistance across the igloo wall. As such, only changes to the wall thickness will have an appreciable effect on the inside air temperature relative to the outside ambient air conditions. We don't want to make the igloo walls thinner and thereby allow the air temperature to dip below freezing for the prescribed environmental conditions.

Using the *IHT Thermal Resistance Network Model*, we used the circuit builder to construct the network and perform the energy balances to obtain the inside air temperature as a function of the outside convection coefficient for selected increased thicknesses of the wall.



COMMENTS: (1) From the plot, we can see that the influence of the outside air velocity which controls the outside convection coefficient h_o is negligible.

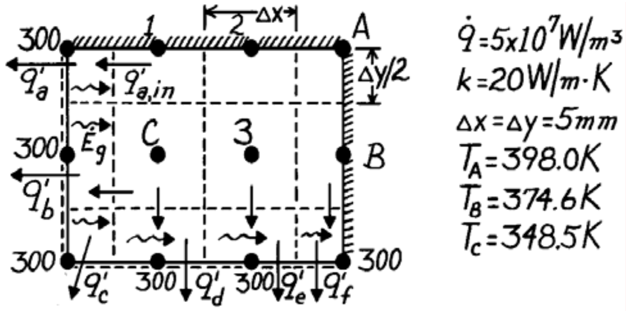
(2) The thickness of the igloo wall is the dominant thermal resistance controlling the inside air temperature.

PROBLEM 4.45

KNOWN: Steady-state temperatures (K) at three nodes of a long rectangular bar.

FIND: (a) Temperatures at remaining nodes and (b) heat transfer per unit length from the bar using nodal temperatures; compare with result calculated using knowledge of $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state, 2-D conduction, (2) Constant properties.

ANALYSIS: (a) The finite-difference equations for the nodes (1,2,3,A,B,C) can be written by inspection using Eq. 4.35 and recognizing that the adiabatic boundary can be represented by a symmetry plane.

$$\sum T_{neighbors} - 4T_i + \dot{q}\Delta x^2/k = 0 \quad \text{and} \quad \frac{\dot{q}\Delta x^2}{k} = \frac{5\times10^7 \text{ W/m}^3 (0.005\text{m})^2}{20 \text{ W/m}\cdot\text{K}} = 62.5\text{K}.$$

Node A (to find T_2*):* $\quad 2T_2 + 2T_B - 4T_A + \dot{q}\Delta x^2/k = 0$

$$T_2 = \frac{1}{2}(-2\times374.6 + 4\times398.0 - 62.5)\text{K} = 390.2\text{K} \quad <$$

Node 3 (to find T_3*):* $\quad T_c + T_2 + T_B + 300\text{K} - 4T_3 + \dot{q}\Delta x^2/k = 0$

$$T_3 = \frac{1}{4}(348.5 + 390.2 + 374.6 + 300 + 62.5)\text{K} = 369.0\text{K} \quad <$$

Node 1 (to find T_1*):* $\quad 300 + 2T_C + T_2 - 4T_1 + \dot{q}\Delta x^2/k = 0$

$$T_1 = \frac{1}{4}(300 + 2\times348.5 + 390.2 + 62.5) = 362.4\text{K} \quad <$$

(b) The heat rate out of the bar is determined by calculating the heat rate out of each control volume around the 300 K nodes. Consider the node in the upper left-hand corner; from an energy balance

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0 \quad \text{or} \quad q'_a = q'_{a,in} + \dot{E}_g \quad \text{where} \quad \dot{E}_g = \dot{q}V.$$

Hence, for the entire bar $\quad q'_{bar} = q'_a + q'_b + q'_c + q'_d + q'_e + q'_f$, or

$$q'_{bar} = \left[k\frac{\Delta y}{2}\frac{T_1 - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_a + \left[k\Delta y\frac{T_C - 300}{\Delta x} + \dot{q}\left[\frac{\Delta x}{2}\cdot\Delta y\right]\right]_b + \left[\dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_c +$$
$$\left[k\Delta x\frac{T_C - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_d + \left[k\Delta x\frac{T_3 - 300}{\Delta y} + \dot{q}\left[\Delta x\cdot\frac{\Delta y}{2}\right]\right]_e + \left[k\frac{\Delta x}{2}\frac{T_B - 300}{\Delta y} + \dot{q}\left[\frac{\Delta x}{2}\cdot\frac{\Delta y}{2}\right]\right]_f.$$

Substituting numerical values, find $q'_{bar} = 7,502.5$ W/m. From an overall energy balance on the bar,

$$q'_{bar} = \dot{E}'_g = \dot{q}V/\ell = \dot{q}(3\Delta x\cdot 2\Delta y) = 5\times10^7 \text{ W/m}^3 \times 6(0.005\text{m})^2 = 7,500 \text{ W/m}. \quad <$$

As expected, the results of the two methods agree. Why must that be?