Problem One

QUESTION

Define the term ‘critical insulation radius’, both in words and with symbols. Explain what happens if $r_i < r_{cr}$ as well as what happens when $r_i > r_{cr}$. Also, compare the qualitative effects of the critical insulation radius for radial systems (cylinders and spheres) with those of plane walls.

ANSWER

The critical insulation radius is defined as the thermal conductivity divided by the convection heat transfer coefficient; this ratio allows for maximum heat transfer, symbolically seen below. (5 points)

$$r_{cr} \equiv \frac{k}{h}$$

[From Incropera, et.al. page 120-1]:
If $r_i < r_{cr}$, the total resistance decreases and the heat rate therefore increases with the addition of insulation. This trend continues until the outer radius of the insulation corresponds to the critical radius; desirable for electrical systems. (5 points)

If $r_i > r_{cr}$, any addition of insulation would increase the total resistance and therefore decrease the heat loss; desirable for steam pipes. (5 points)

The existence of a critical radius requires that the heat transfer area change in the direction of transfer, as for radial conduction in a cylinder (or a sphere). In a plane wall, the area perpendicular to the direction of heat flow is constant and there is no critical insulation thickness (the total resistance always increases with increasing insulation thickness). (5 points)

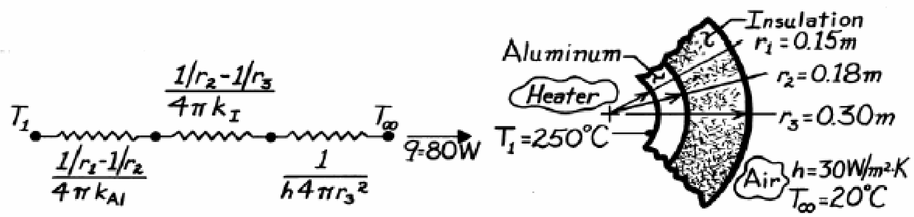
See Example 3.5 on page 119 for full description.

PROBLEM 3.57

KNOWN: Thickness of hollow aluminum sphere and insulation layer. Heat rate and inner surface temperature. Ambient air temperature and convection coefficient.

FIND: Thermal conductivity of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation exchange at outer surface.

PROPERTIES: *Table A-1*, Aluminum (523K): k ≈ 230 W/m·K.

ANALYSIS: From the thermal circuit,

$$q=\frac{T_1-T_\infty}{R_{tot}}=\frac{T_1-T_\infty}{\frac{1/r_1-1/r_2}{4\pi\, k_{A1}}+\frac{1/r_2-1/r_3}{4\pi\, k_I}+\frac{1}{h4\pi\, r_3^2}}$$

$$q=\frac{(250-20)^\circ\,C}{\left[\frac{1/0.15-1/0.18}{4\pi(230)}+\frac{1/0.18-1/0.30}{4\pi\, k_I}+\frac{1}{30(4\pi)(0.3)^2}\right]\frac{K}{W}}=80\ W$$

or

$$3.84\times10^{-4}+\frac{0.177}{k_I}+0.0029=\frac{230}{80}=2.875.$$

Solving for the unknown thermal conductivity, find

$$k_I = 0.062\ W/m\cdot K.$$ <

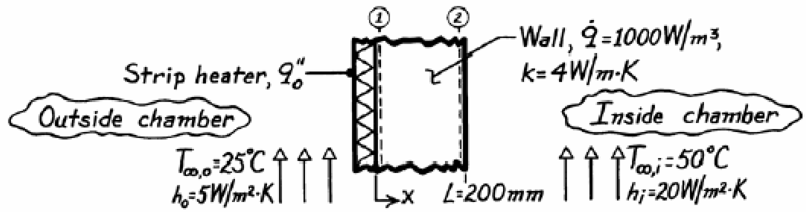
COMMENTS: The dominant contribution to the total thermal resistance is made by the insulation. Hence uncertainties in knowledge of h or k_{A1} have a negligible effect on the accuracy of the k_I measurement.

PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

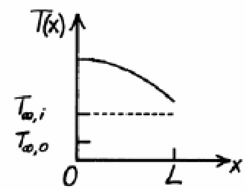
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}$=0 but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with T(L) > $T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1 x + C_2 \tag{1}$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \tag{2}$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow$ $T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \tag{3}$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \tag{4}$$

Continued …..

PROBLEM 3.79 (Cont.)

and from Eq. (3) with x = L and $T(L) = T_2$,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \qquad (5,6)$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ C + 1000\ W/m^3 \times 0.200\ m / 20\ W/m^2 \cdot K = 50^\circ C + 10^\circ C = 60^\circ C$$ <

$$T_1 = 60^\circ C + 1000\ W/m^3 \times (0.200\ m)^2 / 2 \times 4\ W/m \cdot K = 65^\circ C.$$ <

Approach No. 2: Using the boundary condition

$$-k\frac{dT}{dx}\bigg|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

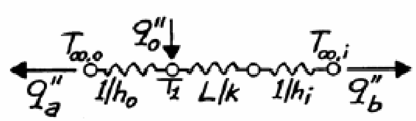
$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

(c) The value of q''_o when $T(0) = T_1 = 65°C$ follows from the circuit

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ W/m^2 \cdot K\,(65\text{-}25)^\circ C = 200\ W/m^2.$$ <

(d) With $\dot{q} = 0$, the situation is represented by the thermal circuit shown. Hence,



$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

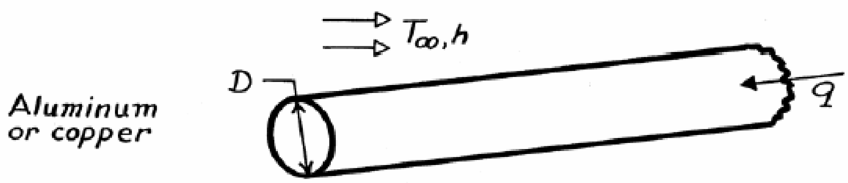
$$T_1 = 55^\circ C.$$ <

PROBLEM 3.119

KNOWN: Long, aluminum cylinder acts as an extended surface.

FIND: (a) Increase in heat transfer if diameter is tripled and (b) Increase in heat transfer if copper is used in place of aluminum.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Uniform convection coefficient, (5) Rod is infinitely long.

PROPERTIES: *Table A-1,* Aluminum (pure): k = 240 W/m·K; *Table A-1,* Copper (pure): k = 400 W/m·K.

ANALYSIS: (a) For an infinitely long fin, the fin heat rate from Table 3.4 is

$$q_f = M = \left(hPkA_c\right)^{1/2} \theta_b$$

$$q_f = \left(h\,\pi\,D\,k\,\pi\,D^2/4\right)^{1/2} \theta_b = \frac{\pi}{2}\left(hk\right)^{1/2} D^{3/2}\ \theta_b.$$

where $P = \pi D$ and $A_c = \pi D^2/4$ for the circular cross-section. Note that $q_f \propto D^{3/2}$. Hence, if the diameter is tripled,

$$\frac{q_{f(3D)}}{q_{f(D)}} = 3^{3/2} = 5.2$$

and there is a 420% increase in heat transfer. <

(b) In changing from aluminum to copper, since $q_f \propto k^{1/2}$, it follows that

$$\frac{q_f\,(Cu)}{q_f\,(Al)} = \left[\frac{k_{Cu}}{k_{Al}}\right]^{1/2} = \left[\frac{400}{240}\right]^{1/2} = 1.29$$

and there is a 29% increase in the heat transfer rate. <

COMMENTS: (1) Because fin effectiveness is enhanced by maximizing $P/A_c = 4/D$, the use of a larger number of small diameter fins is preferred to a single large diameter fin.

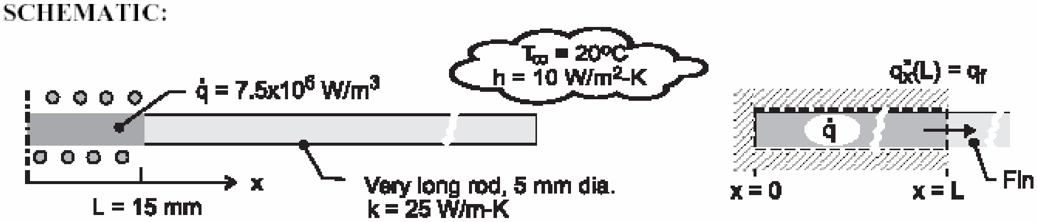
(2) From the standpoint of cost, weight and machinability, aluminum is preferred over copper.

PROBLEM 3.113

KNOWN: Very long rod (D, k) subjected to induction heating experiences uniform volumetric generation $(\dot{q})$ over the center, 30-mm long portion. The unheated portions experience convection (T_∞, h).

FIND: Calculate the temperature of the rod at the mid-point of the heated portion within the coil, T_o, and at the edge of the heated portion, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform $\dot{q}$ in portion of rod within the coil; no convection from lateral surface of rod, (3) Exposed portions of rod behave as infinitely long fins, and (4) Constant properties, (5) Neglect radiation.

ANALYSIS: The portion of the rod within the coil, $0 \le x \le +L$, experiences one-dimensional conduction with uniform generation. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \qquad (1)$$

The portion of the rod beyond the coil, $L \le x \le \infty$, behaves as an infinitely long fin for which the heat rate from Eq. 3.80 is

$$q_f = q_x(L) = (hPkA_c)^{1/2}(T_b - T_\infty) \qquad (2)$$

where $P = \pi D$ and $A_c = \pi D^2/4$. From an overall energy balance on the imbedded portion of the rod as illustrated in the schematic above, find the heat rate as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_f + \dot{q}A_c L = 0$$

$$q_f = \dot{q}A_c L \qquad (3)$$

Combining Eqs. (1-3),

$$T_b = T_\infty + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \qquad (4)$$

$$T_o = T_\infty + \frac{\dot{q}L^2}{2k} + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \qquad (5)$$

and substituting numerical values find

$$T_o = 305°C \qquad T_b = 272°C$$ <

COMMENT: Assuming $\varepsilon = 0.8$ and $T_{sur} = T_\infty = 20°C$, $h_{rad} = 14.6\ W/m^2 \cdot K$. Hence, radiation is significant and would serve to substantially reduce both T_o and T_b.

WIRE INSULATION PROBLEM.

→ The insulation will be hottest near the wire, so we want to first find the heat transferred through the insulation if the inner surface is 55°C and the outer surface is 20°C.

Let's take 1 meter of wire as a basis. From Eq. 3.27 we get

$$q = \frac{2\pi L k}{\ln(r_2/r_1)}\left(T_{s,1} - T_{s,2}\right)$$

$$= \frac{2\pi(1\ m)(0.24\ W/m.k)}{\ln(205/5)}\left(55 - 20\right)$$

$$= \underline{14.21\ \text{Watts}}$$

→ For the resistivity, we need to recall a little electrical engineering. We can use resistivity to determine the resistance of the wire:

$$R = \rho \frac{L}{A} \quad \text{where } L = \text{length } (1\ m), \ A = \text{cross-sectional area}$$

$$= (1.72 \times 10^{-8}\ \Omega\text{-m}) \frac{1\ m}{\pi (0.005/2\ m)^2}$$

$$= 8.76 \times 10^{-4}\ \text{Ohm}$$

Continued ...

WIRE INSULATION PROBLEM, CONT.

→ We have power (Watts) and resistance (Ohms), and we are asked for current.

These are related by

$$P = V I = I^2 R$$

↑ Power ↑ Voltage ↖ current ↖ resistance

$$I = \left(\frac{P}{R}\right)^{1/2}$$

$$= \left(\frac{14.21 \text{ W}}{8.76 \times 10^{-4}\ \Omega}\right)^{1/2}$$

$$= \boxed{127 \text{ Amps}}$$

RADIOACTIVE MATERIAL PROBLEM

→ For convenience, let's use 1 meter length as our basis.

Recognize that all the energy generated must flow through the shell to the water.

$$q = \dot{q}(\pi r_i^2 L) = \left(2\times10^5 \frac{W}{m^3}\right)\left[\pi(0.5\,m)^2(1\,m)\right]$$

$$= 157{,}100\ W.$$

So, $q = h(2\pi r_o L)(T_s - T_\infty)$

$$\Rightarrow T_s = T_\infty + \frac{q}{2\pi r_o L h}$$

$$= 25\,°C + \frac{157{,}100\ W}{2\pi(0.6\ m)(1\ m)(1000\ \frac{W}{m^2\cdot K})}$$

$$T_s = 67\,°C$$

Moving to the next step, we treat it as steady state conduction through the cylindrical shell.

$$q = \frac{2\pi L k}{\ln(r_o/r_i)}\left(T_{s,i} - T_{s,o}\right) \qquad \text{Eq. 3.27}$$

$$T_{s,i} = T_{s,o} + \frac{q \ln(r_o/r_i)}{2\pi L k}$$

$$= 67 + \frac{(157{,}100\ W)\ln(0.6/0.5)}{2\pi(1\,m)(15\ W/m\cdot K)} = 371\,°C$$

RADIOACTIVE MATERIAL, CONT.

→ For the final part, we can use Equation 3.53 evaluated at r = 0, which simplifies to:

$$T_o = \frac{\dot{q} r_o^2}{4k} + T_s$$

Note: r_o for this is our r_i

$$= \frac{(2 \times 10^5 \ W/m^3)(0.5 \ m)^2}{4(80 \ W/m \cdot k)} + 371°C$$

$$= \underline{527 °C}$$

a.) $T_{s, outer}$ = 67 °C

b.) $T_{s, inner}$ = 371 °C

c.) T_{center} = 527 °C