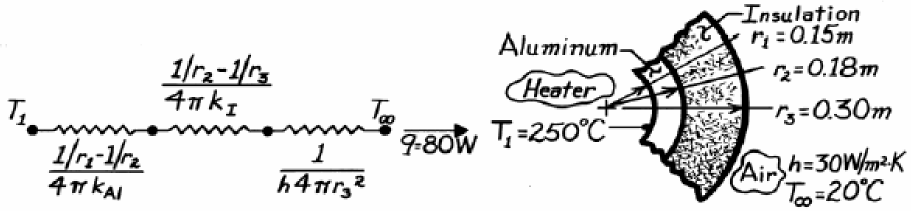
PROBLEM 3.57

KNOWN: Thickness of hollow aluminum sphere and insulation layer. Heat rate and inner surface temperature. Ambient air temperature and convection coefficient.

FIND: Thermal conductivity of insulation.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation exchange at outer surface.

PROPERTIES: *Table A-1*, Aluminum (523K): k ≈ 230 W/m·K.

ANALYSIS: From the thermal circuit,

$$q=\frac{T_1-T_\infty}{R_{tot}}=\frac{T_1-T_\infty}{\frac{1/r_1-1/r_2}{4\pi k_{A1}}+\frac{1/r_2-1/r_3}{4\pi k_I}+\frac{1}{h4\pi r_3^2}}$$

$$q=\frac{(250-20)^\circ C}{\left[\frac{1/0.15-1/0.18}{4\pi(230)}+\frac{1/0.18-1/0.30}{4\pi k_I}+\frac{1}{30(4\pi)(0.3)^2}\right]\frac{K}{W}}=80\text{ W}$$

or

$$3.84\times10^{-4}+\frac{0.177}{k_I}+0.0029=\frac{230}{80}=2.875.$$

Solving for the unknown thermal conductivity, find

$$k_I = 0.062\text{ W/m·K}. \qquad <$$

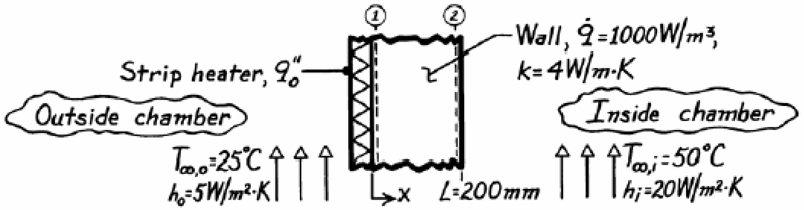
COMMENTS: The dominant contribution to the total thermal resistance is made by the insulation. Hence uncertainties in knowledge of h or k_{A1} have a negligible effect on the accuracy of the k_I measurement.

PROBLEM 3.79

KNOWN: Wall of thermal conductivity k and thickness L with uniform generation $\dot{q}$; strip heater with uniform heat flux q''_o; prescribed inside and outside air conditions (h_i, $T_{\infty,i}$, h_o, $T_{\infty,o}$).

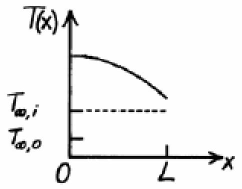
FIND: (a) Sketch temperature distribution in wall if none of the heat generated within the wall is lost to the outside air, (b) Temperatures at the wall boundaries T(0) and T(L) for the prescribed condition, (c) Value of q''_o required to maintain this condition, (d) Temperature of the outer surface, T(L), if $\dot{q}$=0 but q''_o corresponds to the value calculated in (c).

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Uniform volumetric generation, (4) Constant properties.

ANALYSIS: (a) If none of the heat generated within the wall is lost to the *outside* of the chamber, the gradient at x = 0 must be zero. Since $\dot{q}$ is uniform, the temperature distribution is parabolic, with $T(L) > T_{\infty,i}$.



(b) To find temperatures at the boundaries of wall, begin with the general solution to the appropriate form of the heat equation (Eq.3.40).

$$T(x) = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \qquad (1)$$

From the first boundary condition,

$$\left.\frac{dT}{dx}\right|_{x=o} = 0 \quad \rightarrow \quad C_1 = 0. \qquad (2)$$

Two approaches are possible using different forms for the second boundary condition.

Approach No. 1: With boundary condition $\rightarrow T(0) = T_1$

$$T(x) = -\frac{\dot{q}}{2k}x^2 + T_1 \qquad (3)$$

To find T_1, perform an overall energy balance on the wall

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = 0$$

$$-h\left[T(L) - T_{\infty,i}\right] + \dot{q}L = 0 \qquad T(L) = T_2 = T_{\infty,i} + \frac{\dot{q}L}{h} \qquad (4)$$

Continued …..

PROBLEM 3.79 (Cont.)

and from Eq. (3) with x = L and $T(L) = T_2$,

$$T(L) = -\frac{\dot{q}}{2k}L^2 + T_1 \quad \text{or} \quad T_1 = T_2 + \frac{\dot{q}}{2k}L^2 = T_{\infty,i} + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k} \qquad (5,6)$$

Substituting numerical values into Eqs. (4) and (6), find

$$T_2 = 50^\circ\text{C} + 1000\ \text{W/m}^3 \times 0.200\ \text{m}/20\ \text{W/m}^2\cdot\text{K} = 50^\circ\text{C} + 10^\circ\text{C} = 60^\circ\text{C}$$ <

$$T_1 = 60^\circ\text{C} + 1000\ \text{W/m}^3 \times (0.200\ \text{m})^2 / 2 \times 4\ \text{W/m}\cdot\text{K} = 65^\circ\text{C}.$$ <

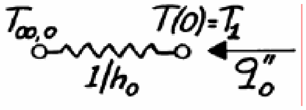
Approach No. 2: Using the boundary condition

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h\left[T(L) - T_{\infty,i}\right]$$

yields the following temperature distribution which can be evaluated at x = 0,L for the required temperatures,

$$T(x) = -\frac{\dot{q}}{2k}\left(x^2 - L^2\right) + \frac{\dot{q}L}{h} + T_{\infty,i}.$$

(c) The value of q''_o when $T(0) = T_1 = 65^\circ$C follows from the circuit



$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o}$$

$$q''_o = 5\ \text{W/m}^2\cdot\text{K}\,(65\text{-}25)^\circ\text{C} = 200\ \text{W/m}^2.$$ <

(d) With $\dot{q}=0$, the situation is represented by the thermal circuit shown. Hence,

$$q''_o = q''_a + q''_b$$

$$q''_o = \frac{T_1 - T_{\infty,o}}{1/h_o} + \frac{T_1 - T_{\infty,i}}{L/k + 1/h_i}$$

which yields

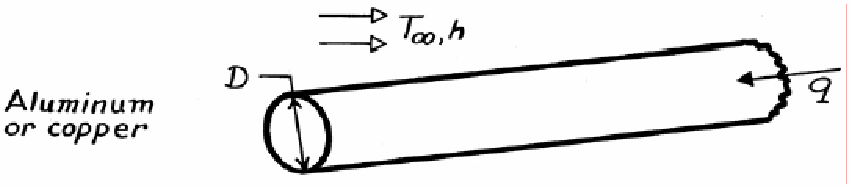
$$T_1 = 55^\circ\text{C}.$$ <

PROBLEM 3.119

KNOWN: Long, aluminum cylinder acts as an extended surface.

FIND: (a) Increase in heat transfer if diameter is tripled and (b) Increase in heat transfer if copper is used in place of aluminum.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Uniform convection coefficient, (5) Rod is infinitely long.

PROPERTIES: *Table A-1,* Aluminum (pure): k = 240 W/m·K; *Table A-1,* Copper (pure): k = 400 W/m·K.

ANALYSIS: (a) For an infinitely long fin, the fin heat rate from Table 3.4 is

$$q_f = M = \left(hPkA_c\right)^{1/2}\theta_b$$

$$q_f = \left(h\,\pi\,D\,k\,\pi\,D^2/4\right)^{1/2}\theta_b = \frac{\pi}{2}\left(hk\right)^{1/2} D^{3/2}\;\theta_b.$$

where $P = \pi D$ and $A_c = \pi D^2/4$ for the circular cross-section. Note that $q_f \, \alpha \, D^{3/2}$. Hence, if the diameter is tripled,

$$\frac{q_{f(3D)}}{q_{f(D)}} = 3^{3/2} = 5.2$$

and there is a 420% increase in heat transfer. <

(b) In changing from aluminum to copper, since $q_f \, \alpha \, k^{1/2}$, it follows that

$$\frac{q_f\left(Cu\right)}{q_f\left(A1\right)} = \left[\frac{k_{Cu}}{k_{A1}}\right]^{1/2} = \left[\frac{400}{240}\right]^{1/2} = 1.29$$

and there is a 29% increase in the heat transfer rate. <

COMMENTS: (1) Because fin effectiveness is enhanced by maximizing $P/A_c = 4/D$, the use of a larger number of small diameter fins is preferred to a single large diameter fin.

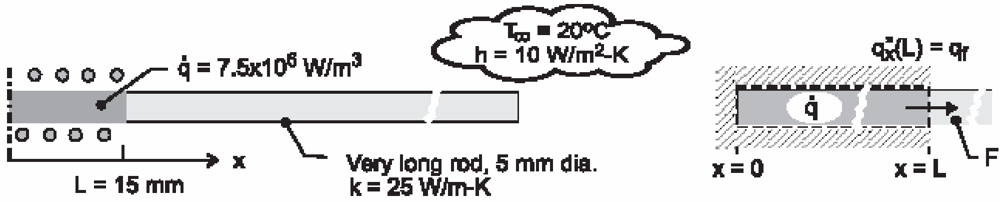
(2) From the standpoint of cost, weight and machinability, aluminum is preferred over copper.

PROBLEM 3.113

KNOWN: Very long rod (D, k) subjected to induction heating experiences uniform volumetric generation $(\dot{q})$ over the center, 30-mm long portion. The unheated portions experience convection (T_∞, h).

FIND: Calculate the temperature of the rod at the mid-point of the heated portion within the coil, T_o, and at the edge of the heated portion, T_b.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction with uniform $\dot{q}$ in portion of rod within the coil; no convection from lateral surface of rod, (3) Exposed portions of rod behave as infinitely long fins, and (4) Constant properties, (5) Neglect radiation.

ANALYSIS: The portion of the rod within the coil, $0 \le x \le +L$, experiences one-dimensional conduction with uniform generation. From Eq. 3.43,

$$T_o = \frac{\dot{q}L^2}{2k} + T_b \tag{1}$$

The portion of the rod beyond the coil, $L \le x \le \infty$, behaves as an infinitely long fin for which the heat rate from Eq. 3.80 is

$$q_f = q_x(L) = (hPkA_c)^{1/2}(T_b - T_\infty) \tag{2}$$

where $P = \pi D$ and $A_c = \pi D^2/4$. From an overall energy balance on the imbedded portion of the rod as illustrated in the schematic above, find the heat rate as

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = 0$$

$$-q_f + \dot{q}A_cL = 0$$

$$q_f = \dot{q}A_cL \tag{3}$$

Combining Eqs. (1-3),

$$T_b = T_\infty + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \tag{4}$$

$$T_o = T_\infty + \frac{\dot{q}L^2}{2k} + \dot{q}A_c^{1/2}L(hPk)^{-1/2} \tag{5}$$

and substituting numerical values find

$$T_o = 305°C \qquad T_b = 272°C$$ <

COMMENT: Assuming $\varepsilon = 0.8$ and $T_{sur} = T_\infty = 20°C$, $h_{rad} = 14.6$ W/m²·K. Hence, radiation is significant and would serve to substantially reduce both T_o and T_b.