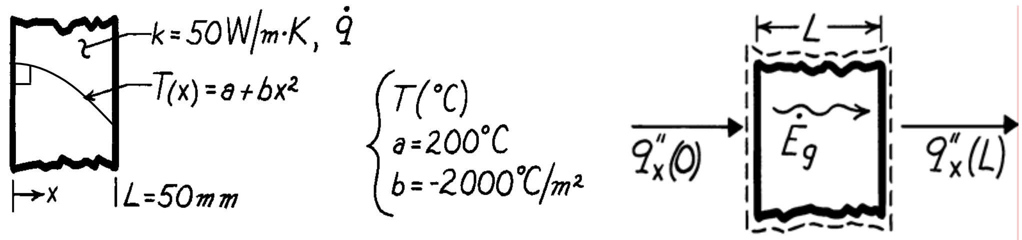
PROBLEM 2.23

KNOWN: Temperature distribution in a one-dimensional wall with prescribed thickness and thermal conductivity.

FIND: (a) The heat generation rate, $\dot{q}$, in the wall, (b) Heat fluxes at the wall faces and relation to $\dot{q}$.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat flow, (3) Constant properties.

ANALYSIS: (a) The appropriate form of the heat equation for steady-state, one-dimensional conditions with constant properties is Eq. 2.19 re-written as

$$\dot{q}=-k\frac{d}{dx}\left[\frac{dT}{dx}\right]$$

Substituting the prescribed temperature distribution,

$$\dot{q}=-k\frac{d}{dx}\left[\frac{d}{dx}\left(a+bx^2\right)\right]=-k\frac{d}{dx}\left[2bx\right]=-2bk$$

$$\dot{q}=-2\left(-2000^\circ C/m^2\right)\times 50\ W/m\cdot K=2.0\times 10^5\ W/m^3.$$ <

(b) The heat fluxes at the wall faces can be evaluated from Fourier's law,

$$q''_x\left(x\right)=-k\left.\frac{dT}{dx}\right]_x.$$

Using the temperature distribution T(x) to evaluate the gradient, find

$$q''_x\left(x\right)=-k\frac{d}{dx}\left[a+bx^2\right]=-2kbx.$$

The fluxes at x = 0 and x = L are then

$$q''_x\left(0\right)=0$$ <

$$q''_x\left(L\right)=-2kbL=-2\times 50W/m\cdot K\left(-2000^\circ C/m^2\right)\times 0.050m$$

$$q''_x\left(L\right)=10,000\ W/m^2.$$ <

COMMENTS: From an overall energy balance on the wall, it follows that, for a unit area,

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=0 \qquad q''_x\left(0\right)-q''_x\left(L\right)+\dot{q}L=0$$

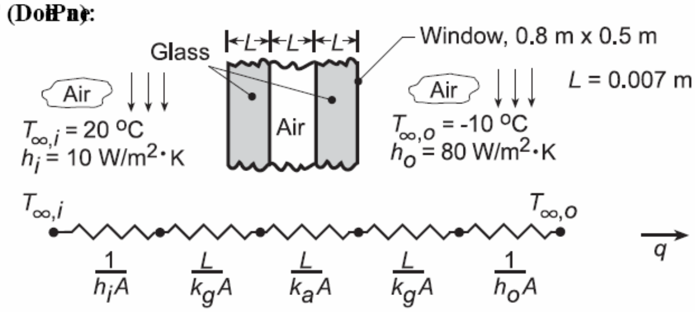
$$\dot{q}=\frac{q''_x\left(L\right)-q''_x\left(0\right)}{L}=\frac{10,000\ W/m^2-0}{0.050m}=2.0\times 10^5 W/m^3.$$

PROBLEM 3.8

KNOWN: Dimensions of a thermopane window. Room and ambient air conditions.

FIND: (a) Heat loss through window, (b) Effect of variation in outside convection coefficient for double and triple pane construction.

SCHEMATIC (Double Pane):



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional heat transfer, (3) Constant properties, (4) Neglect radiation effects, (5) Air between glass is stagnant.

PROPERTIES: *Table A-3*, Glass (300 K): k_g = 1.4 W/m·K; *Table A-4*, Air (T = 278 K): k_a = 0.0245 W/m·K.

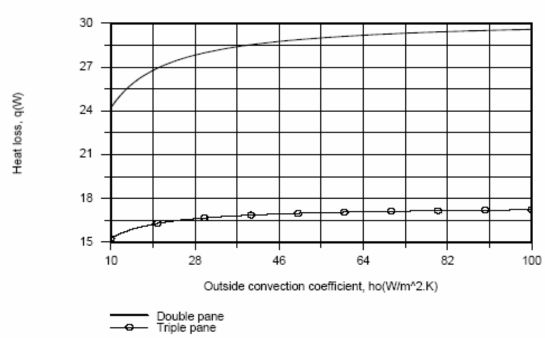
ANALYSIS: (a) From the thermal circuit, the heat loss is

$$q=\frac{T_{\infty,i}-T_{\infty,o}}{\frac{1}{A}\left(\frac{1}{h_i}+\frac{L}{k_g}+\frac{L}{k_a}+\frac{L}{k_g}+\frac{1}{h_o}\right)}$$

$$q=\frac{20^\circ C-\left(-10^\circ C\right)}{\left(\frac{1}{0.4\,m^2}\right)\left(\frac{1}{10\,W/m^2\cdot K}+\frac{0.007\,m}{1.4\,W/m\cdot K}+\frac{0.007\,m}{0.0245\,W/m\cdot K}+\frac{0.007\,m}{1.4\,W/m\cdot K}+\frac{1}{80\,W/m^2\cdot K}\right)}$$

$$q=\frac{30^\circ C}{\left(0.25+0.0125+0.715+0.0125+0.03125\right)K/W}=\frac{30^\circ C}{1.021\,K/W}=29.4\text{ W}$$ <

(b) For the triple pane window, the additional pane and airspace increase the total resistance from 1.021 K/W to 1.749 K/W, thereby reducing the heat loss from 29.4 to 17.2 W. The effect of h_o on the heat loss is plotted as follows.



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PROBLEM 3.8 (Cont.)

Changes in h_o influence the heat loss at small values of h_o, for which the outside convection resistance is not negligible relative to the total resistance. However, the resistance becomes negligible with increasing h_o, particularly for the triple pane window, and changes in h_o have little effect on the heat loss.

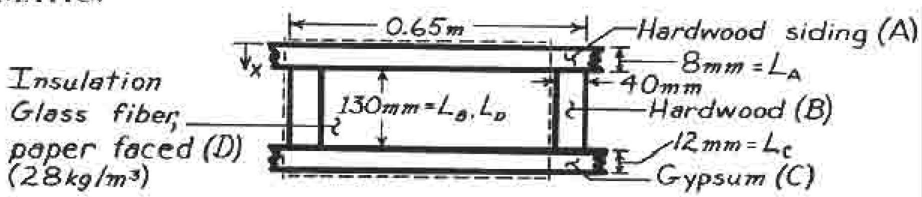
COMMENTS: (1) The largest contribution to the thermal resistance is due to conduction across the enclosed air. Note that this air could be in motion due to free convection currents. If the corresponding convection coefficient exceeded 3.5 $W/m^2 \cdot K$, the thermal resistance would be less than that predicted by assuming conduction across stagnant air, thereby increasing the heat loss.
(2) Determination of the radiation heat loss is complex and will be addressed in Chapters 12 and 13. Radiation would increase the heat loss between the room and outside air, but on a sunny day, solar radiation transmitted through the window would contribute to heating the room.

PROBLEM 3.15

KNOWN: Dimensions and materials associated with a composite wall (2.5m × 6.5m, 10 studs each 2.5m high).

FIND: Wall thermal resistance.

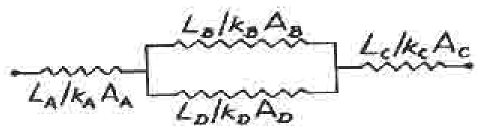
SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of composite depends only on x (surfaces normal to x are isothermal), (3) Constant properties, (4) Negligible contact resistance.

PROPERTIES: *Table A-3* ($T \approx 300K$): Hardwood siding, $k_A = 0.094$ W/m·K; Hardwood, $k_B = 0.16$ W/m·K; Gypsum, $k_C = 0.17$ W/m·K; Insulation (glass fiber paper faced, 28 kg/m^3), $k_D = 0.038$ W/m·K.

ANALYSIS: Using the isothermal surface assumption, the thermal circuit associated with a single unit (enclosed by dashed lines) of the wall is



$$\left(L_A/k_AA_A\right) = \frac{0.008\text{m}}{0.094 \text{ W/m}\cdot\text{K}\left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0524 \text{ K/W}$$

$$\left(L_B/k_BA_B\right) = \frac{0.13\text{m}}{0.16 \text{ W/m}\cdot\text{K}\left(0.04\text{m}\times 2.5\text{m}\right)} = 8.125 \text{ K/W}$$

$$\left(L_D/k_DA_D\right) = \frac{0.13\text{m}}{0.038 \text{ W/m}\cdot\text{K}\left(0.61\text{m}\times 2.5\text{m}\right)} = 2.243 \text{ K/W}$$

$$\left(L_C/k_CA_C\right) = \frac{0.012\text{m}}{0.17 \text{ W/m}\cdot\text{K}\left(0.65\text{m}\times 2.5\text{m}\right)} = 0.0434 \text{ K/W.}$$

The equivalent resistance of the core is

$$R_{eq} = \left(1/R_B + 1/R_D\right)^{-1} = \left(1/8.125 + 1/2.243\right)^{-1} = 1.758 \text{ K/W}$$

and the total unit resistance is

$$R_{tot,1} = R_A + R_{eq} + R_C = 1.854 \text{ K/W.}$$

With 10 such units in parallel, the total wall resistance is

$$R_{tot} = \left(10\times 1/R_{tot,1}\right)^{-1} = 0.1854 \text{ K/W.}$$ <

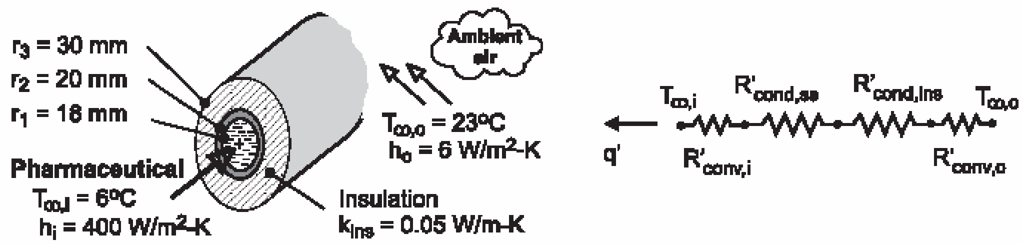
COMMENTS: If surfaces parallel to the heat flow direction are assumed adiabatic, the thermal circuit and the value of R_{tot} will differ.

PROBLEM 3.39

KNOWN: Wall thickness and diameter of stainless steel tube. Inner and outer fluid temperatures and convection coefficients.

FIND: (a) Heat gain per unit length of tube, (b) Effect of adding a 10 mm thick layer of insulation to outer surface of tube.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional radial conduction, (3) Constant properties, (4) Negligible contact resistance between tube and insulation, (5) Negligible effect of radiation.

PROPERTIES: *Table A-1*, Ss 304 (~280K): $k_{st} = 14.2$ W/m·K.

ANALYSIS: (a) Without the insulation, the total thermal resistance per unit length is

$$R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{conv,o} = \frac{1}{2\pi r_1 h_i} + \frac{\ln(r_2/r_i)}{2\pi k_{st}} + \frac{1}{2\pi r_2 h_o}$$

$$R'_{tot} = \frac{1}{2\pi(0.018\text{m})400\ \text{W/m}^2\cdot\text{K}} + \frac{\ln(20/18)}{2\pi(14.2\ \text{W/m}\cdot\text{K})} + \frac{1}{2\pi(0.020\text{m})6\ \text{W/m}^2\cdot\text{K}}$$

$$R'_{tot} = \left(0.0221 + 1.18\times10^{-3} + 1.33\right)\text{m}\cdot\text{K/W} = 1.35\ \text{m}\cdot\text{K/W}$$

The heat gain per unit length is then

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{(23-6)°\text{C}}{1.35\,\text{m}\cdot\text{K/W}} = 12.6\ \text{W/m} \qquad <$$

(b) With the insulation, the total resistance per unit length is now $R'_{tot} = R'_{conv,i} + R'_{cond,st} + R'_{cond,ins} + R'_{conv,o}$, where $R'_{conv,i}$ and $R'_{cond,st}$ remain the same. The thermal resistance of the insulation is

$$R'_{cond,ins} = \frac{\ln(r_3/r_2)}{2\pi k_{ins}} = \frac{\ln(30/20)}{2\pi(0.05\ \text{W/m}\cdot\text{K})} = 1.29\ \text{m}\cdot\text{K/W}$$

and the outer convection resistance is now

$$R'_{conv,o} = \frac{1}{2\pi r_3 h_o} = \frac{1}{2\pi(0.03\text{m})6\ \text{W/m}^2\cdot\text{K}} = 0.88\ \text{m}\cdot\text{K/W}$$

The total resistance is now

$$R'_{tot} = \left(0.0221 + 1.18\times10^{-3} + 1.29 + 0.88\right)\text{m}\cdot\text{K/W} = 2.20\,\text{m}\cdot\text{K/W}$$

Continued …..

PROBLEM 3.39 (Cont.)

and the heat gain per unit length is

$$q' = \frac{T_{\infty,o} - T_{\infty,i}}{R'_{tot}} = \frac{17°C}{2.20\ m\cdot K/W} = 7.7\ W/m$$

COMMENTS: (1) The validity of assuming negligible radiation may be assessed for the worst case condition corresponding to the bare tube. Assuming a tube outer surface temperature of $T_s = T_{\infty,i}$ = 279K, large surroundings at $T_{sur} = T_{\infty,o}$ = 296K, and an emissivity of $\varepsilon = 0.7$ (Table A-11), the heat gain due to net radiation exchange with the surroundings is $q'_{rad} = \varepsilon\sigma(2\pi r_2)\left(T_{sur}^4 - T_s^4\right) = 8.1\ W/m$. Hence, the net rate of heat transfer by radiation to the tube surface is comparable to that by convection, and the assumption of negligible radiation is inappropriate.

(2) If heat transfer from the air is by natural convection, the value of h_o with the insulation would actually be less than the value for the bare tube, thereby further reducing the heat gain. Use of the insulation would also increase the outer surface temperature, thereby reducing net radiation transfer from the surroundings.

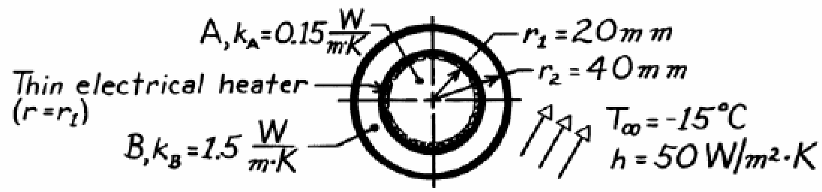
(3) The critical radius is $r_{cr} = k_{ins}/h \approx 8$ mm $< r_2$. Hence, as indicated by the calculations, heat transfer is reduced by the insulation.

PROBLEM 3.41

KNOWN: Thin electrical heater fitted between two concentric cylinders, the outer surface of which experiences convection.

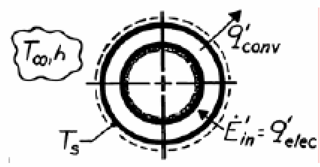
FIND: (a) Electrical power required to maintain outer surface at a specified temperature, (b) Temperature at the center.

SCHEMATIC:



ASSUMPTIONS: (1) One-dimensional, radial conduction, (2) Steady-state conditions, (3) Heater element has negligible thickness, (4) Negligible contact resistance between cylinders and heater, (5) Constant properties, (6) No generation.

ANALYSIS: (a) Perform an energy balance on the composite system to determine the power required to maintain $T(r_2) = T_s = 5°C$.



$$\dot{E}'_{in} - \dot{E}'_{out} + \dot{E}_{gen} = \dot{E}_{st}$$

$$+q'_{elec} - q'_{conv} = 0.$$

Using Newton's law of cooling,

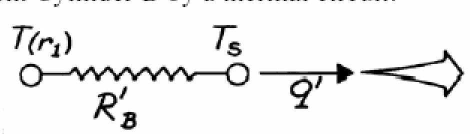
$$q'_{elec} = q'_{conv} = h \cdot 2\pi\ r_2 \left(T_s - T_\infty\right)$$

$$q'_{elec} = 50 \frac{W}{m^2 \cdot K} \times 2\pi (0.040 m)\ \left[5 - (-15)\right]^\circ C = 251\ W/m.$$ <

(b) From a control volume about Cylinder A, we recognize that the cylinder must be isothermal, that is,

$$T(0) = T(r_1).$$

Represent Cylinder B by a thermal circuit:



$$q' = \frac{T(r_1) - T_s}{R'_B}$$

For the cylinder, from Eq. 3.28,

$$R'_B = \ln r_2 / r_1 / 2\pi\ k_B$$

giving

$$T(r_1) = T_s + q'R'_B = 5^\circ C + 251 \frac{W}{m} \frac{\ln 40/20}{2\pi \times 1.5\ W/m \cdot K} = 23.5^\circ C$$

Hence, $T(0) = T(r_1) = 23.5°C$. <

Note that k_A has no influence on the temperature $T(0)$.