QUESTION

An understanding of the basic laws governing heat transfer is imperative to everything you will learn this semester. Write the equation for and explain the following laws governing the three basic modes of heat transfer. Do not forget to explain each symbol or constant along with any pertinent units.

a. Fourier's Law
b. Newton's Law of Cooling
c. The Stefan-Boltzmann Law

ANSWER

a. **Fourier's law pertains to conductive heat transfer**. A one-dimensional form of this law is below. Units are given in brackets.

$$q_x'' = -k\frac{dT}{dx}$$

This equation allows heat flux (q_x'') $[W/m^2]$ to be put in a rate form with temperature (dT) $[K]$ and spatial (dx) $[m]$ differentials along with thermal conductivity (k) $[W/m \cdot K]$. The thermal conductivity is a transport property of the medium through which the heat travels. Because heat is transferred via a thermal gradient, a negative sign is present to signify a heat loss. A more in-depth discussion of Fourier's law is given on page 4 of the text, Fundamentals of Heat Transfer by Incropera, sixth edition.

b. **Newton's law of cooling pertains to convective heat transfer**; it has the following form.

$$q'' = h(T_s - T_\infty)$$

This equation is commonly referred to Newton's law of cooling, but it is simply another rate equation. Here, q'' $[W/m^2]$ represents the heat flux, but is not specific to one direction as in Fourier's law. Heat flux is proportional to the temperature difference between surface $(T_s)[K]$ and fluid $(T_\infty)[K]$. The symbol h $[W/m^2 \cdot K]$ is the convection heat transfer coefficient. As described on page 8 of the text, the convection heat transfer coefficient "depends on conditions in the boundary layer, which are influenced by surface geometry, the nature of the fluid motion, and an assortment of fluid thermodynamic and transport properties". A more in-depth discussion of Newton's law of cooling is given on page 8 of the text, Fundamentals of Heat Transfer by Incropera, sixth edition.

c. **The Stefan-Boltzmann law pertains to radiative heat transfer**. The ideal case where emissivity is at a maximum is known as blackbody radiation; given by the equation below.

$$E_b = \sigma T_s^4$$

Here, emissive power is denoted as E $[W/m^2]$ and subscript b signifying ‘blackbody’ radiation. Again, T_s $[K]$ is termed as the surface temperature from which radiation is the source. The Stefan-Boltzmann constant is denoted as σ $[W/m^2 \cdot K^4]$ and has a value of 5.67 E -8 $W/m^2 \cdot K^4$. For a real surface, and not a blackbody, the radiative property of emissivity, ε, is added to the equation to become the following.

$$E = \varepsilon\sigma T_s^4$$

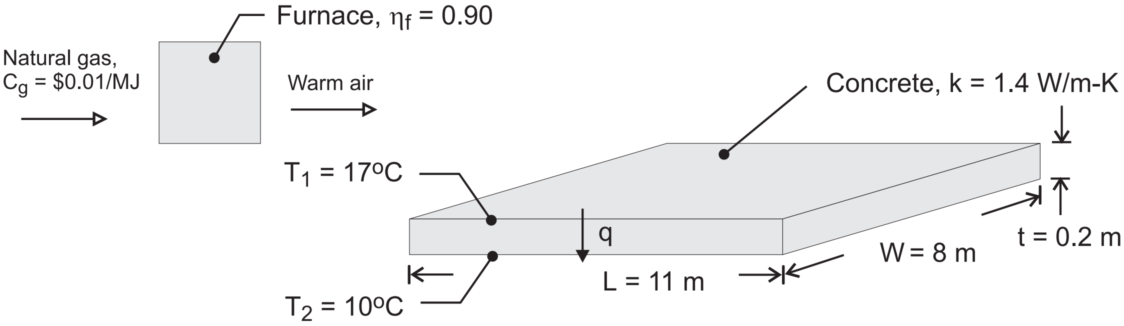
Emissivity is a property of the surface from which heat is transferred. The value of emissivity is always between zero and one and is unitless; when $\varepsilon = 1$, blackbody radiation is present. A more in-depth discussion of the Stefan-Boltzmann law is given on page 9 of the text, Fundamentals of Heat Transfer by Incropera, sixth edition.

PROBLEM 2

KNOWN: Dimensions, thermal conductivity and surface temperatures of a concrete slab. Efficiency of gas furnace and cost of natural gas.

FIND: Daily cost of heat loss.

SCHEMATIC:



ASSUMPTIONS: (1) Steady state, (2) One-dimensional conduction, (3) Constant properties.

ANALYSIS: The rate of heat loss by conduction through the slab is

$$q = k\,(LW)\frac{T_1 - T_2}{t} = 1.4\,W/m\cdot K\,(11\,m \times 8\,m)\frac{7°C}{0.20\,m} = 4312\ W \qquad <$$

The daily cost of natural gas that must be combusted to compensate for the heat loss is

$$C_d = \frac{q\,C_g}{\eta_f}(\Delta t) = \frac{4312\,W \times \$0.01/MJ}{0.9 \times 10^6\,J/MJ}(24\,h/d \times 3600\,s/h) = \$4.14/d \qquad <$$

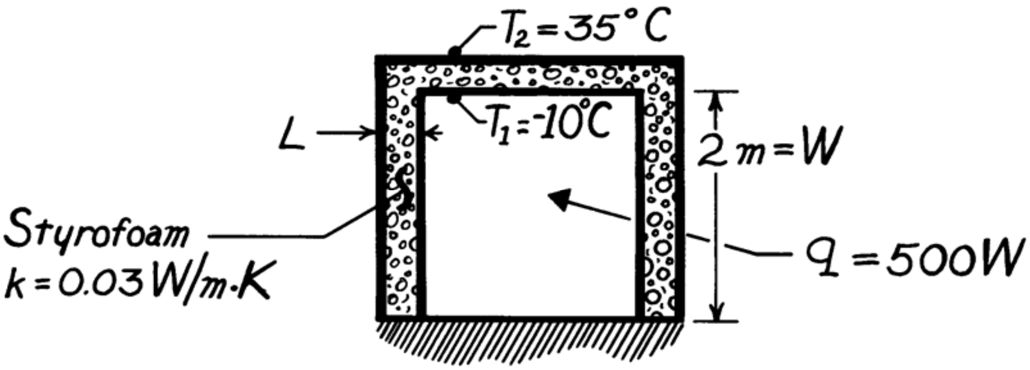
COMMENTS: The loss could be reduced by installing a floor covering with a layer of insulation between it and the concrete.

PROBLEM 3

KNOWN: Dimensions of freezer compartment. Inner and outer surface temperatures.

FIND: Thickness of styrofoam insulation needed to maintain heat load below prescribed value.

SCHEMATIC:



ASSUMPTIONS: (1) Perfectly insulated bottom, (2) One-dimensional conduction through 5 walls of area A = $4m^2$, (3) Steady-state conditions, (4) Constant properties.

ANALYSIS: Using Fourier's law, Eq. 1.2, the heat rate is

$$q = q'' \cdot A = k \frac{\Delta T}{L} A_{total}$$

Solving for L and recognizing that $A_{total} = 5 \times W^2$, find

$$L = \frac{5 \, k \, \Delta T \, W^2}{q}$$

$$L = \frac{5 \times 0.03 \text{ W/m} \cdot \text{K} \left[35 - (-10)\right]^\circ \text{C} \left(4\text{m}^2\right)}{500 \text{ W}}$$

$$L = 0.054\text{m} = 54\text{mm}.$$ <

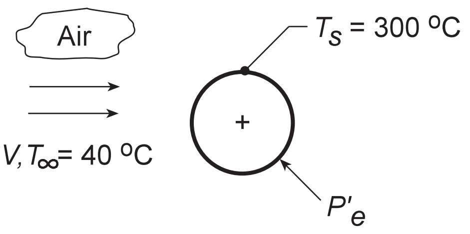
COMMENTS: The corners will cause local departures from one-dimensional conduction and a slightly larger heat loss.

PROBLEM 4

KNOWN: Power required to maintain the surface temperature of a long, 25-mm diameter cylinder with an imbedded electrical heater for different air velocities.

FIND: (a) Determine the convection coefficient for each of the air velocity conditions and display the results graphically, and (b) Assuming that the convection coefficient depends upon air velocity as h = CV^n, determine the parameters C and n.

SCHEMATIC:



V(m/s)	1	2	4	8	12
P'_e (W/m)	450	658	983	1507	1963
h ($W/m^2 \cdot K$)	22.0	32.2	48.1	73.8	96.1

ASSUMPTIONS: (1) Temperature is uniform over the cylinder surface, (2) Negligible radiation exchange between the cylinder surface and the surroundings, (3) Steady-state conditions.

ANALYSIS: (a) From an overall energy balance on the cylinder, the power dissipated by the electrical heater is transferred by convection to the air stream. Using Newton's law of cooling on a per unit length basis,

$$P'_e = h(\pi D)(T_s - T_\infty)$$

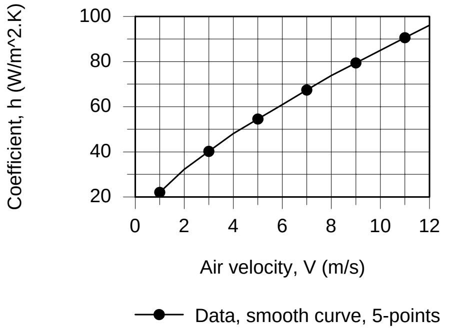
where P'_e is the electrical power dissipated per unit length of the cylinder. For the V = 1 m/s condition, using the data from the table above, find

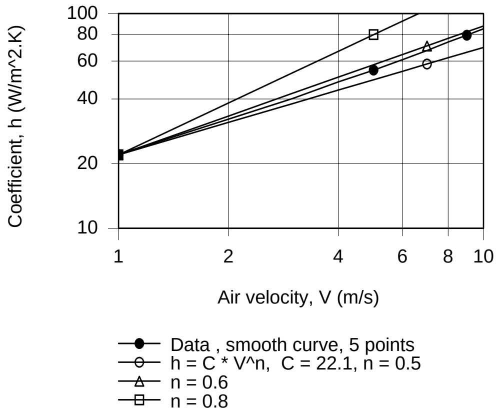
$$h = 450\,W/m \Big/ \pi \times 0.025\,m\,(300-40)^\circ C = 22.0\,W\Big/m^2 \cdot K$$ <

Repeating the calculations, find the convection coefficients for the remaining conditions which are tabulated above and plotted below. Note that h is not linear with respect to the air velocity.

(b) To determine the (C,n) parameters, we plotted h vs. V on log-log coordinates. Choosing C = 22.12 $W/m^2 \cdot K(s/m)^n$, assuring a match at V = 1, we can readily find the exponent n from the slope of the h vs. V curve. From the trials with n = 0.8, 0.6 and 0.5, we recognize that n = 0.6 is a reasonable choice.

Hence, C = 22.12 and n = 0.6. <





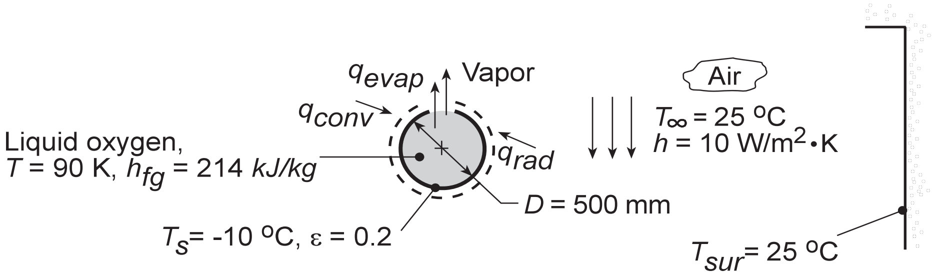
COMMENTS: Radiation may not be negligible, depending on surface emissivity.

PROBLEM 5

KNOWN: Boiling point and latent heat of liquid oxygen. Diameter and emissivity of container. Free convection coefficient and temperature of surrounding air and walls.

FIND: Mass evaporation rate.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Temperature of container outer surface equals boiling point of oxygen.

ANALYSIS: (a) Applying mass and energy balances to a control surface about the container, it follows that, at any instant,

$$\frac{dm_{st}}{dt} = -\dot{m}_{out} = -\dot{m}_{evap} \qquad \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} = q_{conv} + q_{rad} - q_{evap}. \tag{1a,b}$$

With h_f as the enthalpy of liquid oxygen and h_g as the enthalpy of oxygen vapor, we have

$$E_{st} = m_{st} h_f \qquad q_{evap} = \dot{m}_{out} h_g \tag{2a,b}$$

Combining Equations (1a) and (2a,b), Equation (1b) becomes (with $h_{fg} = h_g - h_f$)

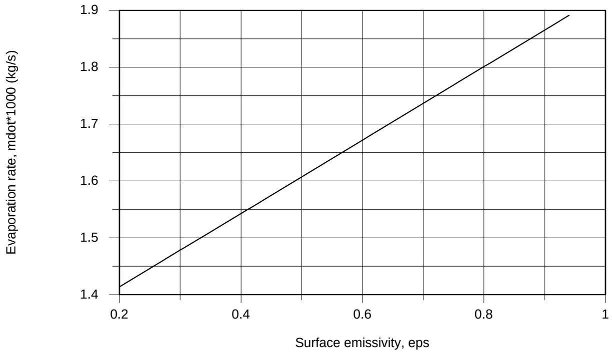
$$\dot{m}_{out} h_{fg} = q_{conv} + q_{rad}$$

$$\dot{m}_{evap} = \left(q_{conv} + q_{rad}\right)/h_{fg} = \left[h\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right]\pi D^2 \Big/ h_{fg} \tag{3}$$

$$\dot{m}_{evap} = \frac{\left[10\,\mathrm{W/m^2\cdot K}\left(298-263\right)\mathrm{K} + 0.2\times5.67\times10^{-8}\,\mathrm{W/m^2\cdot K^4}\left(298^4-263^4\right)\mathrm{K^4}\right]\pi\left(0.5\,\mathrm{m}\right)^2}{214\,\mathrm{kJ/kg}}$$

$$\dot{m}_{evap} = \left(350+35.2\right)\mathrm{W/m^2}\left(0.785\ \mathrm{m^2}\right)\Big/214\,\mathrm{kJ/kg} = 1.41\times10^{-3}\ \mathrm{kg/s}. \qquad <$$

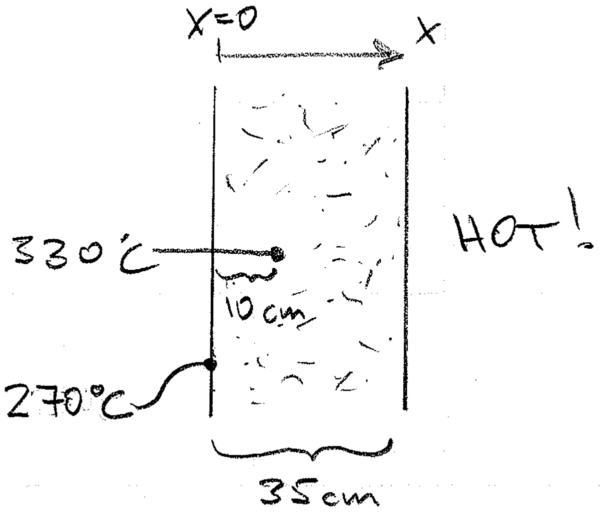
(b) Using Equation (3), the mass rate of vapor production can be determined for the range of emissivity 0.2 to 0.94. The effect of increasing emissivity is to increase the heat rate into the container and, hence, increase the vapor production rate.

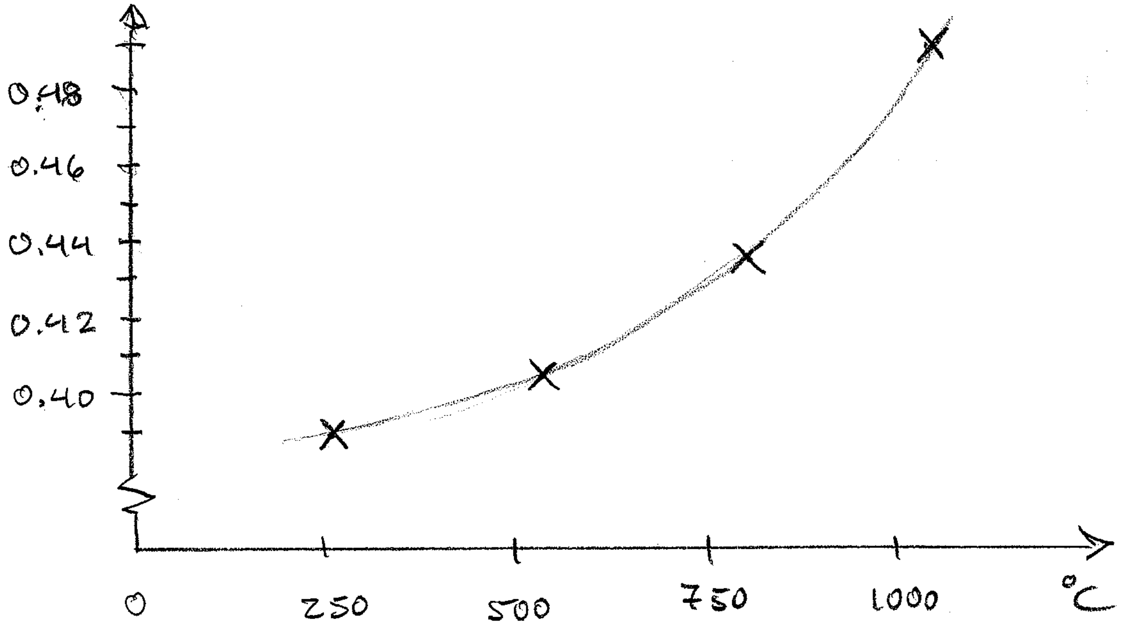


COMMENTS: To reduce the loss of oxygen due to vapor production, insulation should be applied to the outer surface of the container, in order to reduce q_{conv} and q_{rad}. Note from the calculations in part (a), that heat transfer by convection is greater than by radiation exchange.

HEATED REFRACTORY BRICK PROBLEM

First, we need to develop some mathematical relationship between the temperature and thermal conductivity of the refractory:





→ While there are many forms of equations to fit this data, I find that a second-order polynomial gives a good fit:

$$k\left(\frac{W}{m\cdot K}\right) = 1.35\times10^{-7}\,T^{2} - 6.37\times10^{-5}\,T + 0.397$$

... with T in Celcius

REFRACTORY WALL PROBLEM, CONT.

Now, let's consider heat transfer when thermal conductivity is not constant.

$$q'' = -k \, dT/dx$$

$$= -\left(1.35\times10^{-7}T^2 - 6.37\times10^{-5}T + 0.397\right)\frac{dT}{dx}$$

→ We can use this expression, considering the known temperatures:

$$q''\int_0^{0.1\,m} dx = \int_{270}^{330}\left(-1.35\times10^{-7}T^2 + 6.37\times10^{-5}T - 0.397\right)dT$$

$$0.1\,q'' = \left[\frac{-1.35\times10^{-7}}{3}T^3 + \frac{6.37\times10^{-5}}{2}T^2 - 0.397\,T\right]_{270}^{330}$$

$$= -129.2 - (-105.8) = -23.4$$

$$q'' = 23.4 / 0.1 = \boxed{234\ W/m^2}$$

↑ PART B ANSWER

Note: I reversed the negative here because I chose $x = 0$ as the cold side while I should be the hot side for q to be positive

REFRACTORY WALL PROBLEM, CONT.

Since we now know that $q'' = 234\ W/m^2$ and that it is constant through the wall, we can work out the hot face temperature

$$q''\int_0^{0.35} dx = \int_{270}^{T}\left(-1.35\times10^{-7}T^2 + 6.37\times10^{-5}T - 0.397\right)dT$$

$$\underbrace{0.35(234)}_{81.9} = \left[\frac{-1.35\times10^{-7}}{3}T^3 + \frac{6.37\times10^{-5}}{2}T^2 - 0.397T\right]_{270}^{T}$$

$$-4.5\times10^{-8}T^3 + 3.19\times10^{-5}T^2 - 0.397T$$

$$-(-105.8) + 81.9 = 0$$

→ Solving for all of this gives $\boxed{T = 479°C}$

↑

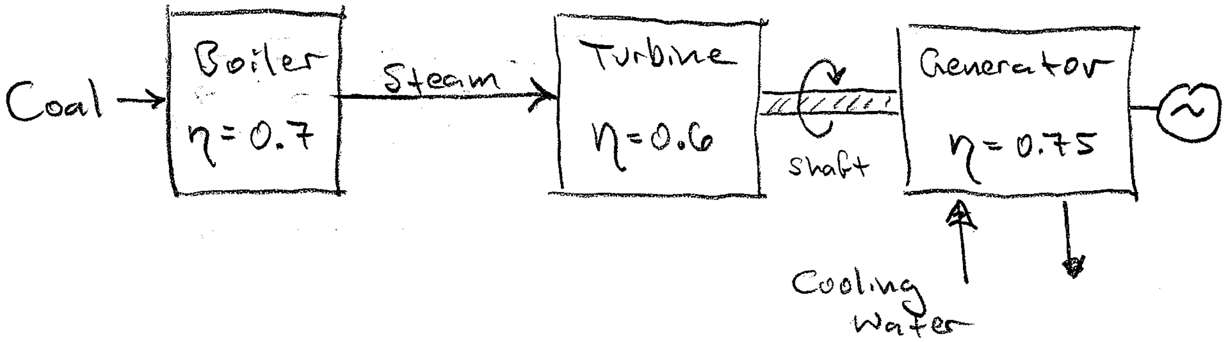
PART A

SOLUTION

Total energy in from coal:

$$\left(2000\,\frac{\text{ton}}{\text{day}}\right)\left(\frac{\text{day}}{24\,\text{h}}\right)\left(\frac{\text{h}}{3600\,\text{s}}\right)\left(1000\,\frac{\text{kg}}{\text{ton}}\right)\left(20\,\frac{\text{MJ}}{\text{kg}}\right)\left(10^6\,\frac{\text{J}}{\text{MJ}}\right)\left(1\,\frac{\text{W}\cdot\text{s}}{\text{J}}\right)$$

$$= 4.63 \times 10^8 \text{ Watts}$$



Energy in to generator:

$$(4.63 \times 10^8 \text{ W})(0.7)(0.6) = \underline{1.94 \times 10^8 \text{ Watts}}$$

Heat loss to be absorbed by cooling water:

$$(1.94 \times 10^8 \text{ W}) \times 0.25 = \underline{4.86 \times 10^7 \text{ W}}$$

This value corresponds to q

Continued...

SOLUTION, CONT.

$$q = hA\Delta T$$

$$4.86 \times 10^7\ W = \left(14{,}000\ \frac{W}{m^2\cdot K}\right) \underline{\underline{A}}(120-20)$$

→ Note that although the units of h are Watts per m^2 · <u>Kelvin</u> we can use °C in the equation above because it is a temperature <u>difference</u>.

$$A = \left(4.86 \times 10^7\ W\right)\left(\frac{m^2\cdot K}{14000\ W}\right)\left(\frac{1}{100\ K}\right)$$

$$= 34.7\ m^2$$

$$= \underset{\nwarrow L}{\text{Length}} \times \underset{\uparrow}{\text{Circumference}}$$

$$\qquad\qquad\qquad = \pi D$$

$$\qquad\qquad\qquad = \pi(0.015\ m)$$

$$L = \left(34.7\ m^2\right) \big/ \pi(0.015\ m)$$

$$= \boxed{737\ \text{meters}}$$