

Advanced Finite Elements

ME EN 7540

Hyperelastic Circular Plate

Spring 2006

This handout shows an example of hyperelastic problem using Neo-Hookean Hyperelastic material constants. The strain energy potential is given by

$$W = \frac{\mu}{2}(\bar{I}_1 - 3) + \frac{1}{d}(J - 1)^2$$

where μ is the initial shear modulus,
 d is material incompressibility,
 \bar{I}_1 is the first deviatoric strain invariant,
 J is the determinant of the elastic deformation gradient.

The Neo-Hookean option (TB,HYPER,,,NEO) represents the simplest form of strain energy potential, and has an applicable strain range of 20-30%. An example input listing showing a typical use of the Neo-Hookean option is presented below.

```
TB,HYPER,1,,,NEO      !Activate Neo-Hookean data table
TBDATA,1,0.577148     !Define  $\mu$  shear modulus
TBDATA,2,7.0e-5       !Define incompressibility parameter
                      !(as  $2/K$ ,  $K$  is the bulk modulus)
```

Example: Hyperelastic Circular Plate

A flat circular membrane made of a rubber material is subjected to uniform water pressure. The edges of the membrane are fixed. Determine the response as pressure is increased to 50 psi.

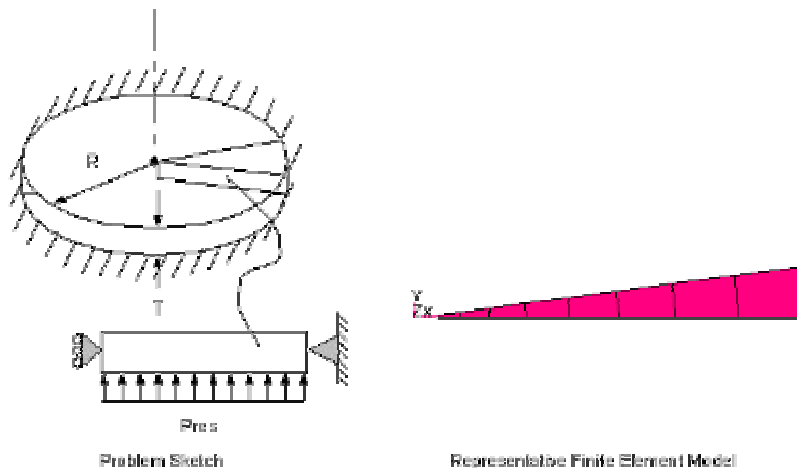


Figure 1. Hyperelastic Circular Plate Project Sketch

Table 1 Material, geometric and loading properties

Material Properties $m = 200$ psi	Geometric Properties $R = 7.5$ in $T = 0.5$ in	Loading Pres = 25 psi
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The full circular plate is reduced to a 7.5 degree sector for analysis. The mid-plane of the outer edge of the circle is considered to be fixed. A pressure of 25 psi is applied to the bottom surface of the shell sector. SHELL181, a finite strain layered shell, is used in this simulation. Consult *ANSYS Element Manual* for detailed description of this element.

Table 2 Results comparison

	Target	ANSYS	Ratio
U_z at 4 psi	2.25	2.275	0.989
U_z at 9.8 psi	3.2	3.379	0.947
U_z at 15.3 psi	4.1	4.357	0.941
U_z at 18.8 psi	4.9	5.095	0.962
U_z at 24 psi	6.2	6.989	0.887

Input Listing

```

/REP7
ET,1,SHELL181
R,1,0.5,
N , 1, 0., 0.
N , 2, 0.17143, 0.
N , 3, 0.47143, 0.
N , 4, 0.90000, 0.
N , 5, 1.4571, 0.
N , 6, 2.1429, 0.
N , 7, 2.9571, 0.
N , 8, 3.9000, 0.
N , 9, 4.9714, 0.
N , 10, 6.1714, 0.
N , 11, 7.5000, 0.
N , 101, 0., 0.
N , 102, 0.16996, 2.23759E-02
N , 103, 0.46740, 6.15338E-02
N , 104, 0.89230, 0.11747
N , 105, 1.4447, 0.19020
N , 106, 2.1245, 0.27970
N , 107, 2.9318, 0.38598
N , 108, 3.8666, 0.50905
N , 109, 4.9289, 0.64890
N , 110, 6.1186, 0.80553
N , 111, 7.4358, 0.97895
E, 1, 2, 102, 102
E, 2, 3, 103, 102
E, 3, 4, 104, 103
E, 4, 5, 105, 104
E, 5, 6, 106, 105
E, 6, 7, 107, 106
E, 7, 8, 108, 107
E, 8, 9, 109, 108
E, 9, 10, 110, 109
E, 10, 11, 111, 110
TB,HYPER,1,,NEO
TBDATA,1,200,0
D, 1,UY,0.0,,11,1 ,ROTX,ROTZ
D,102,UY,0.0,,111,1 ,ROTX,ROTZ
D, 11,UX,0.0,,111,100,UY ,UZ
D, 1,UX,0.0,, , ,UY ,ROTX,ROTY,ROTZ

```

LOCAL, 11, 0, 0.0,0.0,0.0, 7.5,0.0,0.0
NROTAT,102,111,1

AUTOTS,ON
NSUBST, 400, 1200,25
NLGEOM,ON
NROPT,FULL,,OFF
OUTRES, ALL, ALL,
SF,ALL,PRES,25
NEQITR,20
/AUTO,1
/VIEW,1,,,1
/ANG,1
/ESHAPE,1
EPLT
FINISH

/SOLUTION
SOLVE
FINISH

/POST1
/NOPR !SUPPRESS GRAPHING DATA
/VIEW,1,,,-1
/ANG,1
/USER
/FOCUS,1,4,,8.0 !SET UP CENTER OF GRAPHICS SCREEN FOR DISPLACEMENT PLOT
/DIST,,12 !SET DISTANCE TO ZOOM OUT
/TRIAD,OFF
SET,FIRST !SET DISPLACEMENT DATA FOR FIRST SUBSTEP
PLDISP,0 !PLOT DISPLACEMENT DATA
/NOERASE !SET DISPLAY TO OVERLAY PLOTS
SET,,10
PLDISP,0
SET,,20
PLDISP,0
SET,,25
PLDISP,0
SET, LAST
PLDISP,1 !PLOT FINAL DISPLACEMENT WITH ORIGINAL POSITION
/ERASE
/TRIAD,ON
/GOPR
/ESHAPE,0
FINISH
/POST26
/XRANGE,0,3.0
/YRANGE,0,1
/AXLAB,X,UZ OF CENTER/R-INITIAL
/AXLAB,Y,THICKNESS/ORIGINAL THICKNESS
NSOL,2,1,U,Z,UZ_1
ESOL,3,1,,SMIS,17,TH_1
ADD,4,2,, ,UZRATIO,, ,0.13333333,0,0,
ADD,5,3,, ,SH.181,, ,2,0,0,
/COLOR,CURVE,MRED
XVAR,4
PLVAR,5
/XRANGE,0,10
/YRANGE,0,60
/AXLAB,X,UZ OF CENTER (IN)
/AXLAB,Y,PRESSURE (LB/SQ IN)
/COLOR,CURVE,YGRE
NSOL,2,1,U,Z,UZ_1
PROD,7,1,, ,SH.181,, ,25,0,0, !MULTIPLY SOLUTION BY 25
/COLOR,CURVE,MRED
XVAR,2 !SPECIFY X VARIABLE TO BE DISPLAYED
PLVAR,7 !DISPLAY SOLUTION IN GRPH FILE
PRVAR,7,2 !LIST VARIABLE 7 VERSUS VARIABLE 2
FINISH