3 OBSERVATIONS ABOUT TURBULENT FLOW

Fluid mechanics has been recognized as a topic of engineering importance for most of recorded history. Around 200 B.C. Archimedes formulated principles of buoyancy and floatation that remain essentially unchanged today. Even earlier there were sophisticated aqueduct systems that archaeologists have now discovered all over the world. These aqueduct builders obviously had some working knowledge of fluid mechanics and also probably had some description for the phenomena we now call turbulence.

One of the first descriptions of a turbulent flow by an engineer was by Leonardo da Vinci. Among his many accomplishments in art and science were his detailed studies of fluid motion. As an astute observer of nature and strong proponent of experimental methods, Leonardo studied and recorded flow patterns in many different configurations. As a result of these studies he formulated some principles of fluid motion including the first laws of mass conservation for incompressible flows. It is interesting to note that in his early descriptions, Leonardo identified two types of "eddying" motions: one of which is ordered, and one of which is random. It is now recognized that coherent motions exist even in very high intensity turbulence. This organized structure plays an important role in turbulent mixing and has been the subject of much recent turbulence research.

As already pointed out, turbulence is usually defined in ways that express it as a random, unpredictable fluid motion. Turbulence is also a dissipative process. That is, by the action of viscosity, the fluctuations in a fluid will tend to be damped out. This means that for a flow to remain turbulent, there must be some external source of energy and some mechanisms for that energy to be fed into the flow. The mechanisms of energy transfer within a turbulent flow is a subject we will later treat in some detail.

3.1 Laminar vs. Turbulent Flow

Most fluid flows can be conveniently placed into one of two different categories: Laminar or Turbulent. The qualitative difference between the two is usually (although not always) obvious. This distinction is important as the characteristics of a flow change dramatically as a flow transitions from a laminar to a turbulent state. As already mentioned, mixing of mass, momentum, and energy occur much more rapidly in turbulent flows. Many applications related to such diverse areas as aerodynamics, sediment transport, combustion, acoustics, and the weather are all significantly affected by turbulence.

By laminar flow we are generally referring to a smooth, steady fluid motion, in which any induced perturbations are damped out due to the relatively strong viscous forces. In turbulent flows, other forces may be acting that counteract the action of viscosity. If such forces are large enough, the equilibrium of the flow is upset and the fluid cannot respond rapidly enough to viscosity. As a result, a complex, rapidly changing flow structure may develop. The forces that upset this equilibrium can include buoyancy, inertia, or rotation to mention only a few. To illustrate this idea consider fluid flowing in a channel. The viscous and inertial forces acting on the fluid are proportional to:

$$F_v \propto \nu L \tag{3.1}$$

$$F_i \propto V L^2 \tag{3.2}$$

where ν is the fluid viscosity, and L and V are characteristic velocity and length scales. (This material can be found in any introductory fluid mechanics text book.) If the viscous forces on the fluid are large compared with others, any disturbances introduced in the flow will tend to be damped out. If, on the other hand, the inertial forces become large, the fluid will tend to break up into eddies. (**NOTE***: We will be using the term "eddy" often in this course. More than anything else, this terminology is used to express a concept rather than a distinct entity in the flow. We want to think of an eddy as an entity associated with a given length and time scale. We often refer to a coherent structure in a flow as an "eddy," and this terminology is OK as long as you keep in mind what your definition is. On the other hand, there are many flows in which the distinction of coherent motions is unclear, but which clearly exhibit a range of length and time scales. The motions associated with these length and time scales can be conceptualized as eddies.*) For greater inertial forces, the eddies will break up into even smaller eddies. This will continue until we reach a small enough length scale (eddy size) on which the viscous forces dominate. From 3.1 and 3.2 we see that the inertial forces are proportional to L^2 while the viscous forces are proportional to L. Therefore, for a small enough L, the viscous forces will eventually dominate and damp out all perturbations. Another way of looking at this is to recognize that this process results in a large distribution of eddy sizes in the flow. The largest of these eddies will be constrained by the physical size constraints on the flow (like channel diameter). The smallest eddies will be constrained by the viscous forces which act strongest at the smallest length scales. One of the difficulties associated with the prediction of turbulent flow is that the range of length scales (eddies) present can be very large. We will quantify this later.

Although I am no fan of cute poems, there is a famous little rhyme that well describes this breakdown process:

> Big whorls have little whorls,
> That feed on their velocity.
> And little whorls have lesser whorls,
> And so on to viscosity.
> (in the molecular sense).
>
> *Richardson, 1922*[1]

Again we have the qualitative description of inertial forces breaking up a flow into eddies of various sizes, with the limiting size of the smallest eddies being determined by the viscosity. Richardson was the first to qualitatively express the idea of an *energy cascade* in which the turbulent kinetic energy passes through the wavenumber spectrum before it is converted to heat by friction. Later, Kolmogorov extended this concept into his two famous hypotheses about the equilibrium range in turbulent flows. We will come back to this many times.

The ratio between the inertial and viscous forces can be cautiously considered to give a numerical estimate of the relative strengths of the two forces. The nondimensional number that results is the Reynolds number, one of the most important parameters in fluid dynamics. For turbulence to develop, the inertial forces must be much larger than the viscous forces:

$$Re = \frac{UL}{\nu} \gg 1 \tag{3.3}$$

This interpretation of the Reynolds number as a ratio of forces is neither unique or specifically useful. Therefore, this interpretation of the Reynolds number must be treated cautiously. For many flows of practical importance, the Reynolds number can be on the order of $Re \sim 10^6$. This does not mean that viscous forces are not important. It is just that the scales over which molecular effects are important are much smaller than in the definition of the length scales given above. As we will see in later sections, it is the viscous decay at the small scales that provides the necessary sink of kinetic energy that makes a statistically steady universal range possible. For high Reynolds number flows, we can assume there are scales over which molecular forces are not significant. However, in any turbulent flow the molecular viscosity is always important at some scale. Turbulence is always a dissipative process, and if there is no source of energy to sustain the turbulence, it will decay as a result of the turbulent motions. Putting this in other wording we can say that the inertial forces result in the spreading of the turbulent kinetic energy over progressively wider ranges of length scales (wave numbers). This process is only stopped when viscous damping becomes important at small length scales (large wavenumbers).

This idea can be related to other fluid mechanical situations. Consider a flow across a flat surface with a no-slip condition at the wall. The viscous forces are confined to a small region near the wall (the *boundary layer*). As the flow Reynolds number is increased, this region decreases in thickness and the velocity of the flow changes very rapidly from zero at the surface to the free-stream velocity at the outer edges of the boundary layer. Again we see the tendency of the nonlinear inertial terms to generate discontinuities in the flow at high Re, and the viscous damping effect to smooth out these discontinuities at high wavenumbers. It is notable that in general turbulent flows these processes occur within the flow.

Other Reynolds Number Interpretations

Since the Reynolds number is a dimensionless number, there are many other interpretations. To be useful, these interpretations must, of course, have physical significance. We will see in our subsequent analysis that interpretations in terms of length and time scale ratios will be more useful. For example, consider fluid flowing in a duct of width L, with a mean velocity U. A fluid particle with a transverse velocity u' would cross the duct in a time ("inertial" time) $T_i \sim L/u'$. However, this motion will also be acted on by viscous forces, which have a time scale, $T_v \sim L^2/\nu$. In a turbulent flow, the inertial time-scale will be much less than the diffusive time-scale, giving

$$\frac{T_v}{T_{in}} = \frac{u'L}{\nu} > 1 \tag{3.4}$$

In terms of length scale interpretations, we will see that the Reynolds number can also be used to describe the range of length scales we can expect in a turbulent flow. We will explore these ideas more carefully in future lectures.

3.2 Turbulent Diffusivity

One of the most significant features of turbulence is the effect it has on the transport of mass, momentum, and energy. Turbulent flows all exhibit much higher mixing or diffusion rates than could be accomplished by molecular diffusion alone. This may or may not be a desirable feature, depending on the application. In many applications, such as combustion, where an efficient process depends on thorough mixing of fuel and oxidizer, turbulence is a desired feature. We often speak of the *turbulent diffusivity.* Of course, we must be able to describe and characterize the effects of turbulence.

To illustrate the idea of turbulent diffusivity, consider a "marked" fluid in which a region of the fluid is marked with some contaminant. Assume also that the fluid is not in relative motion with itself (in other words, we are considering a pure diffusion process). Under these conditions, the marked fluid satisfies the diffusion equation

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x_j x_j} \tag{3.5}$$

D is the molecular diffusivity of the fluid. Dimensionally, Eq. 3.5 can be interpreted as

$$\frac{\Delta C}{t_m} \sim D\frac{\Delta C}{\Delta L^2} \tag{3.6}$$

or

$$t_m \sim \frac{L^2}{D} \tag{3.7}$$

where ΔC is an initial concentration difference, L is a characteristic length scale of the problem, t_m is the time scale for the molecular diffusion, and $\sim$ can be read as "scales as."

Now assume the marked fluid is transported not only by molecular diffusion, but by turbulent velocity fluctuations as well. In this case, the marked fluid satisfies the convection-diffusion equation

$$\frac{\partial C}{\partial t} + \frac{\partial u_j C}{\partial x_j} = D \frac{\partial^2 C}{\partial x_j x_j} \tag{3.8}$$

Performing a scale analysis of the convection and diffusion terms gives

$$\frac{\partial u_j C}{\partial x_j} \sim \frac{u \Delta C}{L} \tag{3.9}$$

$$D \frac{\partial^2 C}{\partial x_j x_j} \sim D \frac{\Delta C}{L^2} \tag{3.10}$$

The ratio of convection to diffusion in Eq. 3.8 can be then expressed as

$$\frac{\text{convection}}{\text{diffusion}} \sim \frac{uL}{D} \tag{3.11}$$

The quantity $\frac{uL}{D}$ is the Peclet number (Pe) and as we will show, usually takes on a large value for turbulent flows. Using our rules for scale analysis, we can express Eq. 3.8 dimensionally as (assuming turbulent flows):

$$t_T \sim \frac{L}{u} \tag{3.12}$$

Here t_T is the turbulent time-scale. It is instructive to plug in some reasonable numbers and look at the ratio of these two time scales expressed in Eq. 3.7 and 3.12.

It will turn out useful to express Eq. 3.12 in terms of a *turbulent diffusivity.* Following the form of Eq. 3.7:

$$t_T \sim \frac{L^2}{D_T} \tag{3.13}$$

D_T is the turbulent diffusivity. Comparison of Eqs. 3.12 and 3.13 gives

$$D_T \sim uL \tag{3.14}$$

Eq. 3.14 is an important relationship that expresses the turbulent diffusivity of the fluid in terms of characteristic turbulent velocity and length scales.

There is another way to look at this issue, and since the idea of a turbulent diffusivity is so important to the material we will be covering and so fundamental to the description of turbulence in general, let us explore this idea from a different point of view. The classical definition of a diffusion coefficient is as a flux divided by a gradient. That is, the diffusion flux vector is driven by local gradients. And this definition is clear in applications in which molecules execute a random walk (Brownian motion) with mean free path much smaller than the distance over which the bulk properties vary. Now looking at turbulence from a macroscopic level, the idea that fluid elements experience random motions over distances much smaller than distances over which bulk properties vary is not necessarily the case. To define a turbulent diffusion coefficient, we must therefore go back to the micro-picture and reconsider the underlying random processes.

To do this, let us refer to the analysis of one-dimensional Brownian motion. In this case, we are considering the displacement of a particle after n random steps of length l. A one dimensional analysis of the random walk problem (see Chandrasekar[2] for a detailed analysis) gives the following for the probability of having a displacement between some value x and $x + dx$ after n steps:

$$p(x,n)dx = (2\pi nl^2)^{-1/2} \exp\left[\frac{-x^2}{2nl^2}\right] dx \tag{3.15}$$

Where the number of steps taken is proportional to time, $n = Kt$.

Now consider the diffusion of a concentration field, c where $c = c_0$ at $t = 0$ and $x = 0$, and $C = 0$ for all $x \neq 0$ at $t = 0$. The concentration at any time t can be given by

$$C(x,t) = C_0 p(x,t) \tag{3.16}$$

Since the process we are describing satisfies the diffusion equation

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial x^2}, \tag{3.17}$$

we can substitute Eq. 3.15 and 3.16 into 3.17. This is satisfied if $K = 2D/l^2$, giving

$$C = \frac{C_0}{2\sqrt{\pi Dt}} \exp\left[\frac{-x^2}{4Dt}\right] \tag{3.18}$$

From Eq. 3.18 it is possible to relate the diffusivity to the mean square displacement of fluid particles. Note that finding a particle between x and $x + dx$ can be obtained by multiplying the solution for C/C_0 by the element of area dx:

$$p(x,t)dx = \frac{1}{2\sqrt{\pi Dt}} \exp\left[-x^2/4Dt\right] dx \tag{3.19}$$

The mean square displacement is given by multiplying $p(x,t)dx$ by x^2 and integrating over all possible values of x:

$$\langle x^2 \rangle = \int_0^\infty x^2 2P(x,t)dx \tag{3.20}$$

Substituting Eq. 3.19 into 3.20 and integrating gives:

$$\langle x^2 \rangle = 2Dt \tag{3.21}$$

A similar analysis can be carried out for three-dimensional configurations. The analysis for a three dimensional random walk gives [2]:

$$p(r,n) = \frac{1}{(2\pi nl^2/3)^{3/2}} \exp(-3r^2/2nl^2) \tag{3.22}$$

Continuing with the analysis outlined above for the one dimensional problem yields

$$\langle x^2 \rangle = 6Dt \tag{3.23}$$

Note that there is no prefered direction or biasing of the molecular random walks. The flux arises due to the inhomogeneity of the number density of the diffusing species. The molecular diffusivity D obtained from the classical statistical mechanics/kinetic theory arguments ($\langle x^2 \rangle = 6Dt$) is exactly the bulk property (flux)/(gradient).

To relate this all to turbulence, consider a flow seeded with fluid particles with no inertia of their own so that they follow fluid path lines exactly. These particles experience random fluctuations, so as time progresses, the particles will wander from their initial positions. At some later time, t_2, assume that we can determine the particle displacements from their initial positions, and that the mean square displacement is given as $\langle x^2 \rangle$. In this case, the turbulent diffusivity can be defined from

$$\langle x^2 \rangle = 6D_T t \tag{3.24}$$

Note that the above does not imply any particular mechanism for the transport. It provides a definition of turbulent diffusivity in terms of displacements without any specification about how that displacement occurs. As mentioned above, the molecular diffusivity D obtained from the classical statistical mechanics/kinetic theory arguments ($\langle x^2 \rangle = 6Dt$) is exactly the bulk property (flux)/(gradient). The statistical mechanics definition is the more fundamental, but the (flux)/(gradient) interpretation is more accessible experimentally so is the more widely used in engineering.

In turbulence, the random walk analysis should always give the correct answer for the turbulent diffusivity, while the gradient transport ideas may or may not be applicable.

3.3 3-D Nature of Turbulence

Turbulence is rotational and an inherently three-dimensional phenomena. It is characterized by large fluctuations in vorticity. Most of the important characteristics of turbulent flow, such as vortex stretching and length scale reduction (to be discussed later) are identically zero in two dimensions. This contributes to the difficulties in describing turbulence both analytically and numerically. You will, however, find reference made to "two-dimensional turbulence." There are situations in which the "turbulent" velocity field is, for some length scales confined to two dimensions. Flow in the atmosphere is often treated as two-dimensional as it is confined to a thin layer over the surface of a planet. Although some important dynamical mechanisms are absent in two-dimensional flows, they can be highly complex and nonlinear, and often provide a reasonable approximation to study the features of interest.

Later in the course we will be spending some time discussing different types of "free-shear flows" in which a lot of the statistics are dominated by the influence of large-scale 2-D structures. These structures can play a dominant role in the *transport* of scalar material. However, the smaller-scale, three-dimensional motions come into play when mixing at molecular scales is important (for example, in combustion problems). The distinction between scalar transport and molecular diffusion will be critical to understanding and predicting turbulent mixing processes.

3.4 Order and Randomness

The idea that turbulence consists of completely random motions must be modified to take into account the coherent motions that are now recognized to be inherent aspects of many turbulent flows. For example, turbulent boundary layers and homogeneous turbulent shear flows exhibit horseshoe, or hairpin vortices that appear to be inherent characteristics. Free shear flows like the mixing layer exhibit coherent vortex structures very clearly, again even for very high turbulence intensities. This realization that coherent structure is a part of turbulence has led to some important developments in the numerical simulations of turbulent flows. These developments include Direct Numerical Simulation (DNS) and Large Eddy Simulation (LES). These methods, and some of what they have told us about the structure and dynamics of turbulent flow will be discussed later.

The concepts of order and randomness have also led to some new analytic approaches and new interpretations in the study of turbulence. The names of these disciplines are familiar to most of us: Chaos, Bifurcation Theory, and Dynamical Systems. Most of the impact these theories have had on the study of turbulence have been in the area of hydrodynamic stability and the transition from laminar flow to turbulence. These approaches have come to the attention of mathematicians, physicists, and engineers as a result of observations that very simple (limited degrees of freedom) nonlinear systems obeying deterministic equations can exhibit very complex, unpredictable behavior. The solution of two problems obeying the same nonlinear equations with only slight differences in initial conditions, will diverge rapidly. This "sensitivity to initial conditions" is a characteristic of the nonlinear systems. Turbulence is a complex nonlinear system with many degrees of freedom. Much simpler systems that have been studied, however, display many of the characteristics of real turbulence. It is hoped that by studying these simpler systems, a better understanding, and possibly some fundamentally different approaches to treating turbulence in engineering applications, can be achieved.

Related to this is the use of fractals to describe turbulence. Here the idea is that uniform scaling laws exist that should allow us to describe a wide class of flows. Unfortunately, these approaches have not had any impact on engineering calculations of the prediction of turbulence.

References

[1] L.F. Richardson. *Weather Prediction by Numerical Process.* Cambridge University Press, Cambridge, 1922.

[2] S. Chandrasekar. Stochastic problems in physics and astronomy. *Reviews of Modern Physics*, 15, 1943.