7 LENGTH AND TIME SCALES IN TURBULENT FLOWS

Turbulent motions occur over a wide range of length and time scales. For example, consider the growth of a cumulus cloud. The large scale of the cloud can be of the order of kilometers and may grow or persist over long periods of time. Within the cloud, mixing of dry "external" air and moist air within the cloud may occur over scales on the order of millimeters. In more down to earth applications consider a utility boiler. Large eddies transport fuel and oxidizer throughout the combustion chamber, while mixing and chemical reaction ultimately occur at the very small scales with very short time scales compared with the large scale motions. This variation in length and time scales is an important characteristic of turbulent flows, and a characteristic that is in part responsible for the difficulty encountered in the numerical and theoretical analysis of turbulent flows. Here we take some time to discuss some of the features associated with the length and time scales of turbulence.

We have already seen that scaling laws are an important tool for describing turbulence. Much of what we say and conclude about turbulence is based on order of magnitude estimates that follow from logical applications of scaling laws and dimensional analysis. Also identifying the appropriate scalings using scale analysis along with an understanding of how different lengths and times scale has very practical uses. Within the class lectures a lot of more particular examples and uses will be discussed.

7.1 Turbulent Length Scales

First let us consider the range of length scales (eddy sizes) that one may expect to encounter in turbulent flows. The size of the largest eddies in the flow will be given by L, the smallest eddies by η. As previously discussed, the largest eddies in the flow account for most of the transport of momentum and energy. The size of these eddies is only constrained by the physical boundaries of the flow. We will refer to L as the *integral* length scale. The size of the smallest scales of the flow will be determined by viscosity. We have already discussed the idea that as we approach smaller and smaller length scales, the effects of viscosity become more important. The smallest length scales existing in a turbulent flow are those where the kinetic energy is dissipated into heat. For very high Reynolds number flows, the viscous forces become increasingly small with respect to the inertial forces. Smaller scale motions are then necessarily generated until the effects of viscosity become important and energy is dissipated.

For a statistically steady turbulent flow, the energy dissipated at the small scales must equal the energy supplied by the large scales. From the arguments leading to

Kolmogorov's 1st similarity hypothesis, the only factors influencing the behavior of the small scale motions are the overall kinetic energy production rate (which equals the dissipation rate) and the viscosity. The dissipation rate will be independent of viscosity, *but the scales at which this energy is dissipated will depend on both the dissipation rate and viscosity.*

To arrive at an estimate for the scales at which the energy is dissipated we must then form a length scale based only on the dissipation rate and viscosity. The dissipation rate per unit mass (ϵ) has dimensions (m^2/sec^3) and viscosity, ν has dimension (m^2/sec). The length scale formed from these quantities is:

$$\eta = \left(\frac{\nu^3}{\epsilon}\right)^{1/4} \tag{7.1}$$

This length scale is called the *Kolmogorov* length scale and is the smallest hydrodynamic scale in turbulent flows.

To relate this length scale to the largest length scales in the flow we need an estimation for the dissipation rate in terms of the large scale flow features. Since the dissipation rate is equal to the kinetic energy production rate, we need to obtain an approximation for the rate at which kinetic energy is supplied to the small scales. The kinetic energy of the flow is proportional to U^2. The time scale of the large eddies (commonly referred to as the large eddy "turnover" time) can be estimated as L/U. It is reasonable to assume that the kinetic energy supply rate will be related to the inverse of this time scale. The dissipation rate can now be estimated by the relation

$$\epsilon \sim \frac{UU}{L/U} \sim \frac{U^3}{L} \tag{7.2}$$

With this estimate for ϵ, Eq. 7.1 becomes:

$$\eta = \left(\frac{\nu^3 L}{U^3}\right)^{1/4} \tag{7.3}$$

To repeat what we said earlier, we can see from Eq. 7.2 and 7.3 that the dissipation does not depend on the viscosity. Viscosity serves only to determine at what length scale the dissipation occurs. This now immediately gives an estimate for the ratio of the largest to smallest length scales in the flow:

$$\frac{L}{\eta} \sim \left(\frac{UL}{\nu}\right)^{3/4} = Re^{3/4} \tag{7.4}$$

where Re is the Reynolds number based on the large scale flow features. As we should intuitively expect, the separation of the largest and smallest length scales increases as the Reynolds number is increased. This is a widely used relationship.

Another commonly encountered length scale in turbulence is the *Taylor* microscale. This length scale does not have the same easily understood physical significance as the Kolmogorov or integral length scales but provides a convenient estimate for the fluctuating strain rate field. The Taylor microscale, λ is defined through the relation:

$$\left(\frac{\partial u'}{\partial x}\right)^2 = \frac{u'^2}{\lambda^2} \tag{7.5}$$

u' is the rms of the fluctuating velocity field. We will later talk about some features of turbulence where the Taylor Microscale is the appropriate length scale.

Since the Taylor microscale is related to the turbulence fluctuations, it is sometimes called the *turbulence length scale*. A turbulence Reynolds number can be computed based on the Taylor microscale and the rms velocity fluctuations:

$$Re_\lambda = \frac{u'\lambda}{\nu} \tag{7.6}$$

The Taylor microscale, λ, has a historical significance as well as it was the first length scale derived to describe the turbulence.

7.2 Time Scales

In the previous subsection we have already referred to the "large eddy turnover" time defined by

$$t_L = \frac{L}{U} \tag{7.7}$$

From the information we have already presented, we can also generate a time scale for the small eddies using the viscosity and the dissipation:

$$t_\eta = \left(\frac{\nu}{\epsilon}\right)^{1/2} \tag{7.8}$$

Using our previous estimate for the dissipation rate we obtain

$$t_\eta = \left(\frac{\nu L}{U^3}\right) \tag{7.9}$$

The ratio of time scales is therefore:

$$\frac{t_L}{t_\eta} = \left(\frac{UL}{\nu}\right)^{1/2} = Re_L^{1/2} \tag{7.10}$$

The large scale structures in the flow are seen to have a much larger time scale (duration) than the smallest energy dissipating eddies. As the Reynolds number of the flow increases, the magnitude of the separation between both time and length scales increases.

It is worth taking some time to look at the definition of the large eddy turnover time in a little more detail and how it relates to some physical processes. The eddy turnover time of a size l eddy (τ_l) can be related to the time it takes for that size eddy to traverse the inertial range (t_l) (i.e., be reduced to the Kolmogorov scale). From dimensional analysis we get

$$\frac{dl}{dt} \sim -\frac{l}{t_l} \tag{7.11}$$

where the characteristic time scale of a size l eddy can be related to the large eddy turnover time, τ_L using Kolmogorov scalings applicable to the inertial range:

$$t_l \sim \left(\frac{l}{L}\right)^{2/3} \tau_L \tag{7.12}$$

Now substituting the value of t_l in Eq. 7.12 into Eq. 7.11 and then integrating from the integral scale to the Kolmogorov scale, η gives the time it takes for an integral scale eddy to be reduced to the Kolmogorov scale:

$$\frac{t_L}{\tau_L} = 1 - \left(\frac{\eta}{L}\right)^{2/3} = 1 - Re^{-1/2} \tag{7.13}$$

where the final equality is obtained using the relationship $L/\eta \sim Re^{3/4}$ for inertial range turbulence. Note from Eq. 7.13 that the large eddy turnover time is the time scale for an eddy to traverse the inertial range for high Re flows. Equation 7.13 illustrates the Re dependence of this process, which vanishes for high-Re flows.

In the previous subsection it was shown how the separation of length scales effects the ability to numerically predict a turbulent flow. A similar argument holds for the separation of time scales. In a numerical calculation of the time development of the velocity field, the time step must be small enough to resolve the fast time scales of the small scale motion. Implicit algorithms could be used to avoid this time stepping restriction, but time accurate solutions require the small time steps.

6.3 Length and Time Scales of the Scalar Field

A scalar in a turbulent flow field will experience stretching and straining due to the turbulent motions, as well as diffuse under the action of molecular diffusivity. We have shown previously that the effects of the turbulence on the transport of the scalar field dominate over the effects of diffusion. However, molecular diffusion plays an important role in turbulent transport, mixing and reaction. For example, mixing between two species can only occur under the action of molecular diffusivity. If the molecular diffusivity was absent, the two constituents could be well stirred and distributed by the turbulence, but they would never see each other at a molecular level. Under this condition the scalar length scale will be reduced to a molecular scale. In reality, diffusion acts over the scalar field, most effective at the smallest length scales, so that the scalar field is homogenized at length scales much larger than the molecular scales. This smallest length scale of the scalar field where this homogenization occurs is called the *Batchelor* scale if it is smaller than the Kolmogorov scale, and the Obukov-Corrsin scale if it is larger than the Kolmogorov scale. The relative size of the smallest scalar length scale to the Kolmogorov scale depends on the relative magnitudes of the kinematic viscosity and molecular diffusivity. A major focus of this course is to study the details of the overall turbulent mixing process. A derivation of the scalar length scales and a discussion of the scalar energy spectrum will be deferred until we specifically treat the mixing process.

7.3 Implications on the Numerical Solution of Turbulent Flows

The ratio of largest to smallest length scales in the flow has just been shown to be proportional the Reynolds number raised to the three-quarters power. This result has important implications concerning the numerical solution of a turbulent flow. For example, in a flow with a Reynolds number of 10^5, the ratio L/l is proportional to $10^{15/4}$. However, turbulence is three-dimensional and to resolve the entire range of length scales in a turbulent flow simulation we would need a computational domain that consisted of at least 10^{10} grid points. Considering that this resolution would be needed for each of the dependent variables, it is easy to see that this amount of information far exceeds the capacity of any existing computer. This becomes even clearer when the unsteady, transient nature of turbulence is considered. In an accurate numerical solution, time must be discretized and the governing equations must be integrated in time using small time steps. Even if computer memory was large enough to contain all the information needed to describe the turbulent flow field in detail, the time needed to integrate the equations in both space and time would be completely unpractical. Even though 40 something years have passed since this type of argument was first made and computing power has grown by orders of magnitude, the argument is still valid today.

Since full numerical simulations of turbulent flow are impossible, what implications does this have regarding our ability to predict the behavior of turbulent flows in applications of practical importance? Since the flow field cannot be predicted exactly, the general approach is to use approximate methods and statistical methods that give us the desired information about the flow. In the absence of the needed technology to solve the exact governing equations, much of the effort in turbulence research is directed at developing *models* that can be used in conventional solution procedures. These models take on a wide variety of forms, ranging from the very empirical to some that are very complex and physically based. At some level, however, all models rely on empirical data. The goal of the models is to provide as good as description as possible of the turbulence within a framework that allows for solutions of practical problems. As we will see, there have been many successes along with many failures in this venture. The current and continued research in the turbulence modeling area indicates the difficulty in describing the turbulence. The next several sections will discuss some of the modeling approaches used to treat the effects of turbulence.