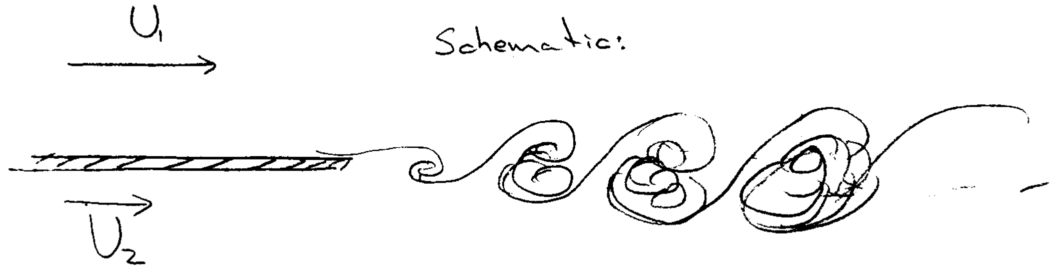
The mixing layer

Generic configuration for mixing between two initially separated fluids.

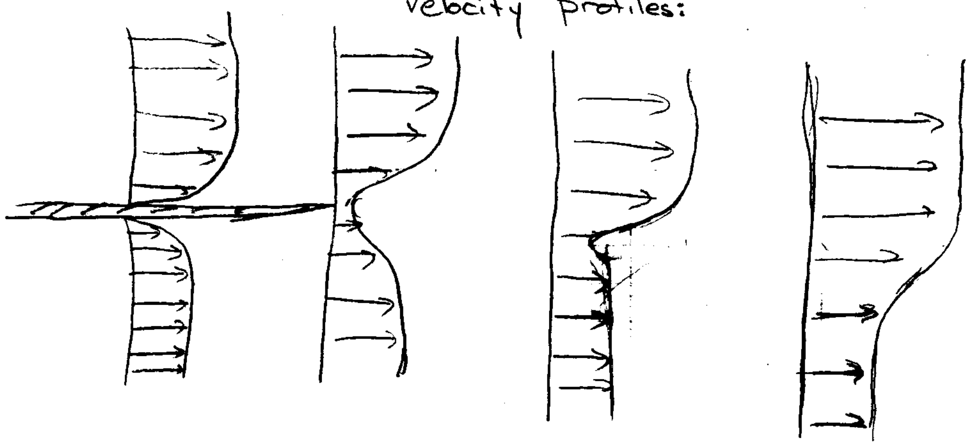
Clearly exhibits large-scale "coherent" structures

Spatial mixing (shear) layer:

Schematic:



Velocity profiles:

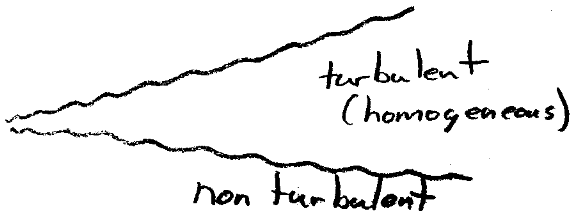


Temporal Mixing Layer

Shear Layer Mixing

Two stream shear layer – generic flow for studying turbulent mixing processes. First – how non-turbulent fluid becomes turbulent

Early Ideas:



interface separates turbulent from non-turbulent fluid

"Entrainment" envisioned as flux of nonturbulent fluid across interface

Corsin, Kistler (1955)

Talk about entrainment

More recent (Dimotakis, AIAA late early 80's?)

"Gulping"

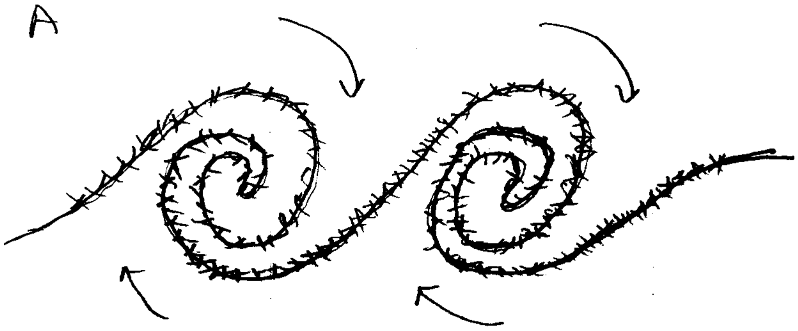
a) fluid set in motion "by" nearby vorticity –

This fluid will participate in large scale motion before it obtains vorticity of its own

b) Stretching of fluid elements untill length scales small enough for diffusion to dominate

(irrotational fluid becomes contaminated with vorticity)

c) Some Mixing (diffusion) can occur across interface at all times



Fluid is entrained and transported in mixing region (arrows)

Molecular diffusion causes mixing (xxxxx)

Now if turbulence dominates mixing process & mixing "occurs" when fluid parcels "reach" Kolmogorov scale, then mixing would be independent of molecular diffusion

<u>Idealize representation</u>

$$L \longrightarrow \eta$$

no diffusion ↑ — ↑ instant diffusion

But this isn't verified by experiment

Experiments on mixing in H_2O (Koochesfahani & Dimotakis, 1986) and in air (Mungal & Dimotakis, 1984) illustrated importance of distinguish and accounting for molecular diffusion effects.

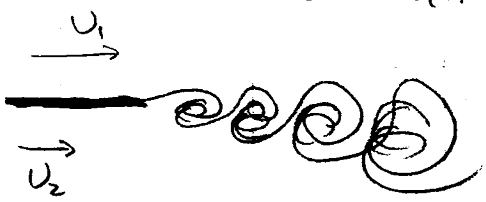
Broadwell-Breidenthal Model of Shear Layer Mixing

Nice use of scaling analysis to illustrate molecular diffusion effects.

Consider a 2-step entrainment & mixing process:

1) Fluid brought into layer by process characterized by large scale structures, L

$$T_L \sim L/(U_1 - U_2)$$

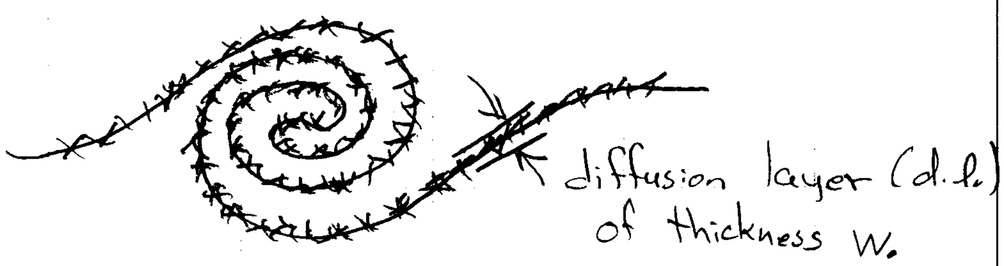


T_L is characteristic time of eddy breakdown
In this time $L \to \eta$ with little mixing

2) Once at η, diffusion occurs fast (negligible compared to T_L)

If the above is true, mixing depends only on T_L — but this is not in agreement with experiments

Why not? During vortex breakdown, diffusion layers form. Thickness of layer depends on $\mathcal{D}$.



Thickness of diffusion layer, w

$$w \sim (Dt)^{1/2}$$

where appropriate time is T_L

$$w \sim (D T_L)^{1/2}$$

(We are trying to find out how much mixing has occurred across diffusion layers by T_L)

Amount of mixing from diffusion layers

$$\sim \frac{\text{surface area}}{\text{Volume}} \times w$$

$$\sim \frac{w}{L} \sim \frac{(D T_L)^{1/2}}{L} \sim \left(\frac{D}{UL}\right)^{-1/2} = Pe^{-1/2}$$

$$Pe = \text{Peclet \#} = Re\, Sc$$

$$\left(\frac{UL}{\nu}\frac{\nu}{D}\right)$$

So contribution from diffusion sheets $\sim Re^{-1/2} Sc^{-1/2}$

Often expressed as

$$\delta_p = \phi_d\, Re^{-1/2} Sc^{-1/2} + \phi_m$$

fluid mixed at Kolmogorov scale

Contribution of mixed fluid from diffusion sheets

Some scaled measure of mixing

Mixing Models:

What do we mean by "mixing" model

Model that provides information on behavior, evolution, statistics of scalar field in a turbulent flow (temp., concentration, species,)

Range from conceptual models, scalings to provide insight into observed behavior (not really a predictive tool) e.g. Broad-Breid.

to sophisticated closures to solve mean scalar equations.

In many mixing models, velocity field statistics are <u>assumed</u> known

<u>Difficulty</u> Mixing is ultimately a small-scale process & a description of small scale structure is important. Particularly when chemical reactions (or other micro-physical processes) are involved.

<u>Reynolds Average</u>

Convection-Diffusion

$$\frac{\partial \bar{C}}{\partial t} + \frac{\partial \bar{u}_i \bar{C}}{\partial x_i} = -\frac{\partial \overline{u_i' c'}}{\partial x_i} + D\frac{\partial^2 \bar{C}}{\partial x_i \partial x_i}$$

model using gradient transport

$$\frac{\partial \overline{u_i' c'}}{\partial x_i} = \frac{\nu_t}{\sigma_t}\frac{\partial \bar{C}}{\partial x_i}$$

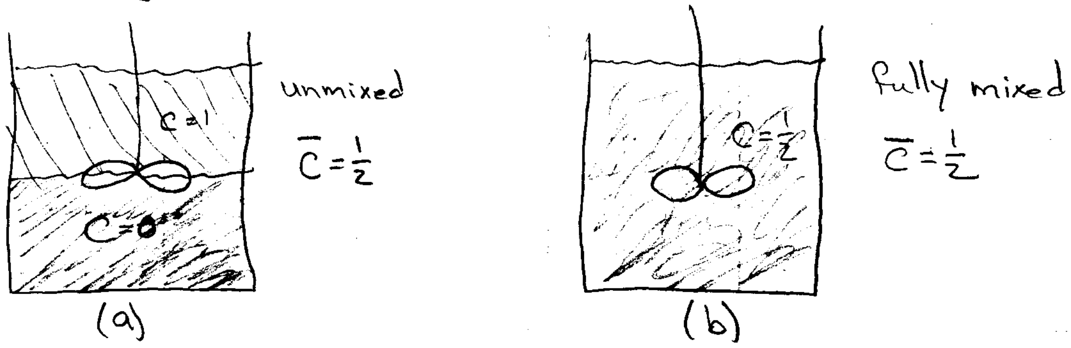
ν_t - turbulent viscosity

$\sigma_t \sim$ ratio of turb viscosity to turb diffusivity $O(1)$

This gives info on mean $\overline{C}$, may or may not be accurate.

But does not give information on scalar fluctuations which are important in many applications

Mixing Tank



Amount of mixing characterized by $\overline{C'^2}$

(a) $\overline{C'^2} = .5$

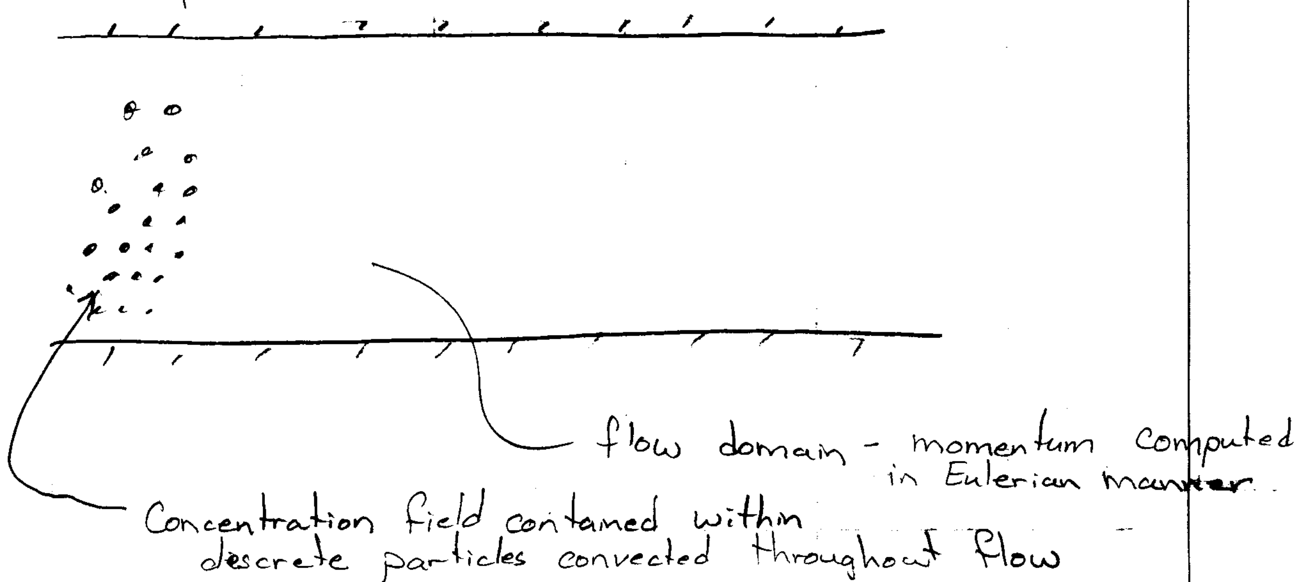
(b) $\overline{C'^2} = 0$

So what is of key interest in mixing applications is evolution (decay) of scalar varience. We will come back to this later in much detail.

Lagrangian Mixing

We will discuss next some simple (but still widely used) mixing models. They can be formulated clearly mathematically, but in numerical simulations are usually implemented with Lagrangian particle methods.

Example:



Mixing models describe how individual particles change concentration

Interaction by Exchange with Mean (IEM)

In Lagrangian applications, fluid elements change concentrations through interactions with mean

In Eulerian applications, diffusion (turbulent & molecular) is replaced by an exchange model

$$\text{Let } \bar{c} = \frac{1}{N}\sum_{i=1}^{N} c_i$$

where scalar field is described by N elements (Lagrangian) or grid points (Eulerian)

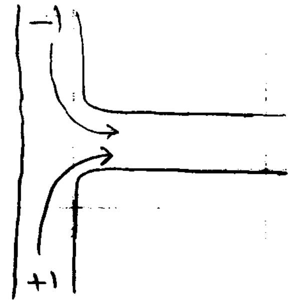
The model can be stated mathematically as:

$$\frac{\partial c_i}{\partial t} = h(\bar{c} - c_i)$$

h = exchange coefficient or mixing frequency

For initial field of $c = -1$ or $+1$, $\bar{c} = 0$ always

e.g. pipe mixing



$$\frac{dc_i}{dt} = -\frac{1}{\tau_m} c_i$$

for τ_m = constant

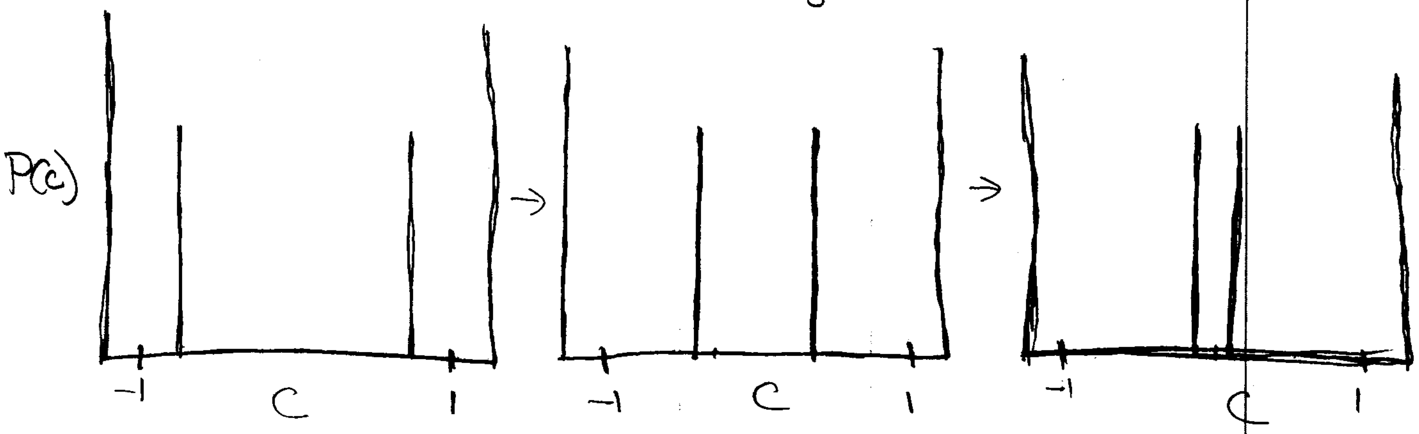
$$\boxed{c_i = c_0 e^{-t\tau_m}}$$

Issues with this?

Start with a configuration where $\tau_m = c$ throughout domain. (τ_m is a mixing frequency and contains the information about the turbulent flow field. – e.g. $\tau_m \sim k/\varepsilon$).

Also start with equally distributed initial scalar field as described by following pdf:

PDF of Scalar field during IEM evolution:



Maintains a "double-delta" distribution

But real mixing will give a continuous distribution.

Can achieve this by adding randomness to process.

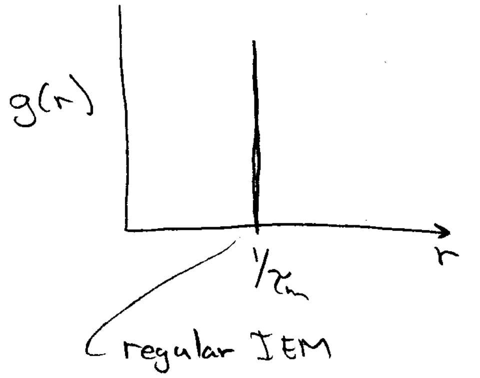
Let each particle be updated at random τ_m

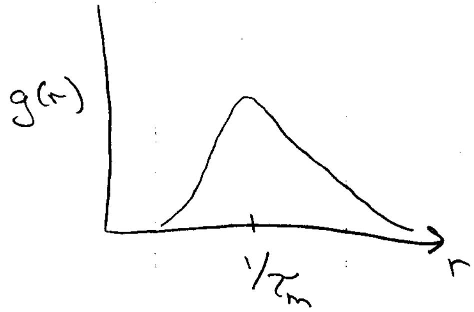
For each c_i, let evolution equation take form:

$$\frac{dc_i}{dt} = -r c_i$$

where r is a random # selected from an appropriated pdf, $g(r)$

if $g(r) = \delta\left(r - \frac{1}{\tau_m}\right)$, regular IEM is obtained.





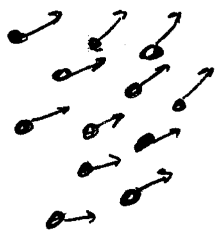
other possible choice (out of infinite #

Problems: higher order moments diverge.

Coalescense- Dispersion (C-D)

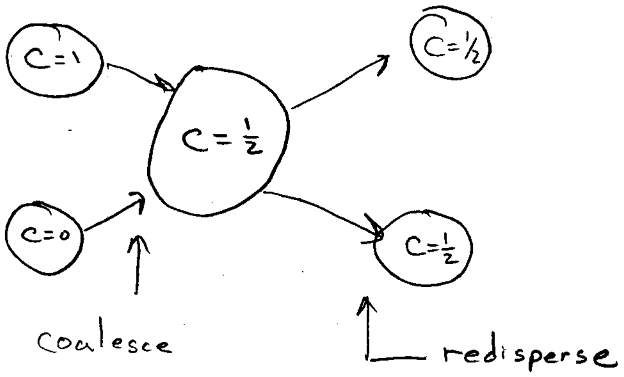
This model is implemented in a Lagrangian manner

Scalar field described by a large # of particles. Each particle is assumed to have a uniform concentration and is convected by the velocity field



Interactions

Mixing occurs in the C-D formulation by interactions between two fluid particles



Particles are selected based on a mixing frequency - then instantaneously mix.

What about length scale information? Explicitely lacking
(Also expliqitly lacking in IEM)

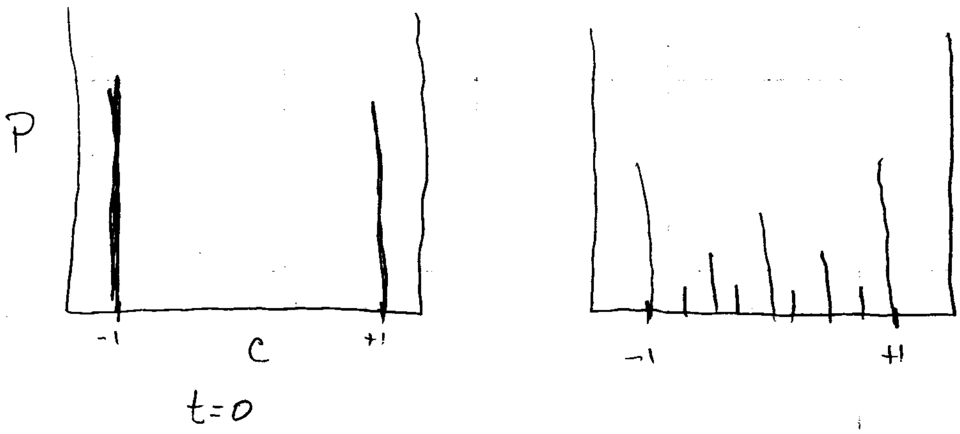
Mixing between elements a & b occurs as

$$\underbrace{C_a^* = C_b^*}_{\text{Concentration of a, b after mixing event}} = \frac{1}{2}\left(\underbrace{C_a + C_b}_{\text{Concentration of element a, b before mixing event}}\right)$$

This is Curl's original CD model.

Mixing frequency given by $\omega = C\,\varepsilon/k$

Problem with model is in the evolution of the pdf obtained from this process: it does not give a uniform distribution



To generate continuous pdf's, can control the extent of mixing,

$$C_a^* = (1-\alpha)\,C_a + \tfrac{1}{2}\alpha\,(C_a + C_b)$$
$$C_b^* = (1-\alpha)\,C_b + \tfrac{1}{2}\alpha\,(C_a + C_b)$$

$\alpha = 0 \Rightarrow$ no mixing
$\alpha = 1 \Rightarrow$ Curl's model

other α, changes location of δ's in pdf, but does not give continuous pdf

To obtain continuous pdf, let α be a random variable, selected from continuous pdf, $A(\alpha)$

Then model will generate continuous pdf

Various $A(\alpha)$ distinguish among different C-D models

$A(\alpha) = \delta(1-\alpha)$ recovers Curls original model

Model still shows problem with higher order moments

This can be fixed by biasing selection of mixing events based on "age"

(See Pope, Comb. Sci Tech 1982, Vol. 28)

Also, these models can be explicitely written in terms of an evolution equation for the pdf. We will get back to this point in a short while.

In the mean time, let's go back to the evolution of the scalar variance & some stuff on the scalar power spectrum.

Predicting & understanding the evolution of fluctuations in the scalar field is important

Degree of mixing characterized by $\overline{c'^2}$

You previously described this in a homework

Derivation

Start with exact scalar equation

$$\frac{\partial c}{\partial t} + \frac{\partial u_i c}{\partial x_i} = D \frac{\partial^2 c}{\partial x_i \partial x_i}$$

decompose: $c = \overline{c} + c'$
$U_i = \overline{U_i} + u_i'$

plug into equation
multiply by c'
average equation

$$\overline{c'\left(\frac{\partial(\overline{C}+c')}{\partial t} + \frac{\partial(\overline{U_i}+u_i')(\overline{C}+c')}{\partial x_i} \right) = D c' \frac{\partial^2(\overline{C}+c')}{\partial x_i \partial x_i}}$$

The result is:

$$\frac{D\overline{c'^2}}{Dt} = \underset{①}{-\frac{\partial}{\partial x_i}\overline{u_i' c'^2}} \;\underset{②}{- 2\,\overline{c' u_i'}\,\frac{\partial \overline{C}}{\partial x_i}} \;\underset{③}{- 2D\overline{\left(\frac{\partial c'}{\partial x_i}\right)\left(\frac{\partial c'}{\partial x_i}\right)}}$$

Homework: Propose a modeled form for this equation that leaves it in closed form.

Interpretation of terms:

① Turbulent diffusion

② Production of variance by mean scalar gradients

③ Destruction, or dissipation of variance (ε_c)

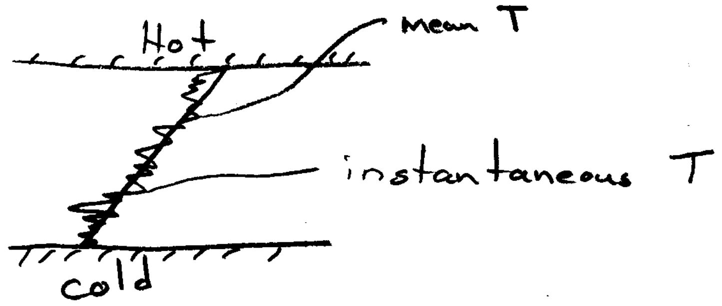
Special Cases:

a) Decaying (mixing) scalar in a homogeneous case with no mean scalar gradient

End up with (Heated grid)

$$\frac{\partial \overline{c'^2}}{\partial t} = -\varepsilon_c$$

b) Steady-state, homogeneous — scalar fluctuations maintained by mean gradient



$$2\,\overline{c'u_i'}\,\frac{\partial \bar{c}}{\partial x_i} = -\varepsilon_c$$

Lets look at scalings for decay of scalar fluctuations for case a)

$$\frac{\partial \overline{c'^2}}{\partial t} = -D\overline{\left(\frac{\partial c'}{\partial x_i}\right)\left(\frac{\partial c'}{\partial x_i}\right)}$$

$$\frac{\partial \overline{c'^2}}{\partial t} \sim -D\,\frac{\overline{c'^2}}{L^2}$$

Note that L^2/D is a time scale

If $L^2 = C$ (L is characteristic length of scalar fluctuations)

Then $\dfrac{\partial \overline{c'^2}}{\partial t} \sim \dfrac{\overline{c'^2}}{\tau_m}$

$$\Rightarrow \ln \overline{c'^2} \sim -t/\tau_m \quad \text{or}$$

$$\overline{c'^2} \sim e^{-t/\tau_m} \qquad \star$$

For a constant $L \Rightarrow$ constant τ_m

e.g., mixing in a box

What if L changes with time? Say $L \sim t^{1/2}$

Then $$\frac{\partial \overline{c'^2}}{\partial t} \sim -\frac{\overline{c'^2}}{t}$$

$$\& \ \ln \overline{c'^2} \sim A \ln t$$

or $\overline{c'^2} \sim t^A \leftarrow$ power law decay $\star\star$

Behavior of $\star$ very different from $\star\star$

We will talk about examples later

Scalar Power spectrum:

We already discussed in some detail. But we can make close analogy to energy spectrum.

Define Scalar Spatial Autocorrelation:

$$R_c(\vec{r}) = \overline{c'(\vec{x},t)\,c'(\vec{x}+\vec{r},t)}$$

Spectrum

$$R_c(\vec{r}) = \int \Phi_c(\vec{k})\, e^{i\vec{k}\cdot\vec{r}}\, d\vec{k}$$

&

$$\Phi_c(\vec{k}) = \frac{1}{(2\pi)^3}\int R_c(\vec{r})\, e^{i\vec{k}\cdot\vec{r}}\, d\vec{r}$$

$$E_c(k) = \underbrace{\int \tfrac{1}{2}\Phi_c\, d\sigma}_{\text{spectral density of waves of same magnitude}}$$ shell of radius k in wave # space ★

Let $\vec{r}=0$

Then $\frac{1}{2}\overline{c'^2} = \int_0^\infty E_c(k)\,dk$

can compute $\overline{c'^2}$ directly

or compute spectrum & integrate to obtain $\overline{c'^2}$

For an isotropic field, in 3-D

$$E_c(k) = 2\pi k^2 \Phi_c(k) \quad (\text{from } ★)$$

In 2-D, $E_c(k) = \pi k\, \Phi_c(k)$

Form of scalar in different regimes:

In general, $E_c(k)$ will scale with parameters governing velocity field & scalar field:

$$\varepsilon, \nu, \varepsilon_c, D$$

In inertial-convective subrange, D, ν not important

$$\Rightarrow E_c(k) = f(\varepsilon, \varepsilon_c)$$

$E_c(k)$ has dimensions $\frac{c'^2}{k}$

ε_c has dimensions $\frac{c'^2}{t}$

ε " " $\frac{k.e}{t} \sim \frac{L^2}{t^3} \sim \frac{1}{t^3 k^2}$

$$E_c(k) \sim \varepsilon_c^x \varepsilon^y k^z = \left(\frac{c'^2}{t}\right)^x \left(\frac{1}{t^3 k^2}\right)^y k^z$$

so $x = 1$

$y = -1/3 \quad \Rightarrow z = -5/3$

so $E_c(k) \approx \varepsilon_c \varepsilon^{1/3} k^{-5/3}$ in inertial convective regime.

Viscous-convective

D not yet important

but we are below η so η not important

Scalar field subject to strain field $(\varepsilon/\nu)^{1/2}$

So: $E_c(k) = \varepsilon_c (\nu/\varepsilon)^{1/2} k^{-1}$