Turbulent Mixing

Discussion of mechanisms

Length Scales of the scalar field

Spectral Regimes of Scalar field

Mixing in Shear layers

Broadwell - Breidenthal Model

Mixing Models
(what do we mean by "mixing model"?

Gradient Transport
IEM
Mixing with application to reaction
Pdf Methods
Linear Eddy Modeling

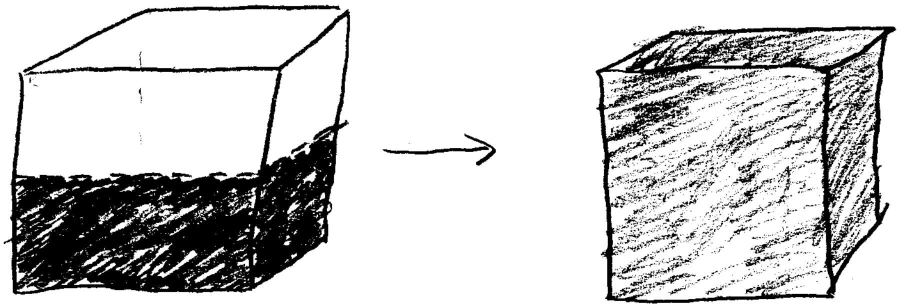
Applications & Examples

Turbulent Mixing

Mixing: The process by which 2 initially segregated constituents come into contact at the molecular level

Q?: By what mechanism does 0 + 1 mix to form ½ ?

Q?: How do we predict this process?



"light" + "dark" → "medium"

Process governed by convection-diffusion Equation

$$\frac{\partial c}{\partial t} + \frac{\partial uc}{\partial x} = D\frac{\partial^2 c}{\partial x^2} \qquad (\text{1-D, constant } D)$$

Two distinct physical mechanisms

1) Turbulent Stirring (Convection)
2) Molecular diffusion

Mixing at molecular level requires diffusion

Stirring increases gradients & surface area
→ thus promoting diffusion

Look at Reynolds averaged equations
(statistically steady)

$$\frac{\partial \bar{u}\,\bar{c}}{\partial x} + \frac{\partial \overline{u'c'}}{\partial x} = D\,\frac{\partial^2 c}{\partial x^2}$$

$\overline{u'c'}$ usually modeled with gradient diffusion approximation

$$\overline{u'c'} = D_t \frac{\partial \bar{c}}{\partial x}$$

so
$$\frac{\partial \bar{u}\,\bar{c}}{\partial x} = \frac{\partial}{\partial x}\left(D - D_t\right)\frac{\partial \bar{c}}{\partial x}$$

implies a certain physical process for the mixing

Good or not?

Often O.K. for bulk flux

But doesn't contain info on scalar fluctuations which are important in many mixing applications

chemical processing
combustion
rain formation
⋮

Length Scales of the Scalar Field

In turbulent flows, a large rang of length scales also characterize the scalar field

L_ϕ - integral scale of scalar field.

Largest correlation length
Can be fixed, or grow in time
(like velocity field)

l_ϕ - smallest scalar length scale (like a scalar equivalent to the Kolmogorov Scale)

1) If $l_\phi > \eta$ $\quad l_\phi \equiv l_c$ = Obukov - Corrsin Scale

2) $l_\phi < \eta$ $\quad l_\phi \equiv l_B \rightarrow$ Batchelor scale

(Some people use term "Batchelor scale" to always refer to smallest scalar scale - whether $>$ or $<$ than η - that's fine)

λ_ϕ - Taylor scalar scale

Defined through $\overline{\left(\frac{\partial \phi'}{\partial x}\right)^2} \equiv \frac{\overline{\phi'^2}}{\lambda_\phi^2}$

Other physics can result in other characteristic scales (e.g., reaction)

If $D < \nu$ expect $l_\phi < \eta$ (Batchelor scaling)

$D > \nu$ " $l_\phi > \eta$ (Obukov-Corrsin Scaling)

Scaling for Batchelor Scale:

$D < \nu$

l_B is a diffusion scale (l_B depends on D)

Diffusion scaling gives:

$$l_B^2 \sim Dt$$

What is appropriate time scaling?

Should be t_η (Kolmogorov time scale)

$l_B < \eta$ so fluctuation at l_B "feel" all strain rates

so $l_B \sim D\left(\frac{\nu}{\varepsilon}\right)^{1/2}$

using $\eta = \left(\nu^3/\varepsilon\right)^{1/4}$

$$\Rightarrow \frac{l_B}{\eta} \sim \left(\frac{D}{\nu}\right)^{1/2} = Sc^{-1/2}$$

Sc = Schmidt # $\equiv \nu/D$

This is appropriate for $Sc > 1$

Discuss length scales in high Sc fluids

Scaling for Obukov - Corrsin Scale

$D > \nu$

Again $l^2 \sim Dt$

If $D > \nu$, l_c can be in inertial subrange

Small scalar fluctuations aren't affected by fluctuations in velocity field at η

At $l > \eta$ parameters important to l_c are D, ε (ν not effective till near η)

So appropriate time scale is

$$t \sim \left(D/\varepsilon \right)^{1/2}$$

$$\Rightarrow l_c \sim \left(D^3/\varepsilon \right)^{1/4}$$

this gives $\dfrac{l_c}{\eta} \sim \left(\dfrac{D}{\nu}\right)^{3/4} = Sc^{3/4}$

Applies for $D > \eta$

Gasses — $Sc = O(1)$

Liquids $Sc = O(10^3)$ & larger

($Sc_{H_2O} \sim 800$)

Spectral Regimes of the Scalar Field

Batchelor - JFM Vol 5 134-139 1959
Lesieur & Herring JFM Vol 161 77-95 1985
Kerstein McMurtry Phys Rev E

1) Inertial - Convective

Velocity inertial $L >> l >> \eta$

Scalar convective $L_\phi >> l_\phi >> \eta$

Let K_D = diffusive wave # cutoff (scalar field)

K_V = viscous wave # cutoff (velocity field)

So in the inertial-convective convective range, we are in a wave # regime where

$$K << K_v, K_d$$

& $K >$ wave number of energy containing eddies

If Sc (or Pr) = 1 then $K_d = K_v$

2) Viscous - convective

This regime exists when Sc, Pr > 1, that is, when scalar field length scales are < Kolmogorov scale

This is a wave number regime, K, where

$$K_v < K < K_D$$

or, in terms of length scale

$$\eta > l_\phi > l_B$$

Below Kolmogorov scale

Scalar field not yet strongly affected by diffusion.

3) Inertial- Diffusive For $Sc < 1$

Velocity inertial - $L \gg \ell \gg \eta$

Scalar diffusive - $\ell_c > \eta$

This is a regime in the scalar field wher

$K_v < K < K_d$

$K_d \quad k \quad K_\eta$

or length scale regime where we are below the Obukov- corrsin scale (scalar length scales not really defined below ℓ_c) but greater than Kolmogorov scale

Form of the scalar "energy" spectrum
(Wave number distribution of scalar fluctuations)

In inertial - convective regime

$$E_\phi(k) \sim k^{-5/3}$$

Viscous convective

$$E_\phi(k) \sim k^{-1}$$

Inertia Diffusive

$$E_\phi(k) \sim k^{-17/3}$$

Spectrum of the Scalar field