

Name Key

Do not plug in any numerical values until they are required to carry the analysis further.

#1 (20 Points) Consider an insulated room that has a uniform temperature,  $T$ . At some time, a 100 W light bulb and a 1500 W hair dryer are turned on. How much has the energy content of the room changed one minute after the light and hair dryer have been turned on? State any assumptions.

$$\Delta E = Q_{in, net} + W_{out, net}$$

$$\dot{E} = \dot{Q} + \dot{W} \quad \text{rate form}$$

$$\dot{Q} = 100 \text{ J/sec} + 1500 \text{ J/sec}$$

$$= 1600 \text{ J/sec} \quad \dot{W} = 0$$

$$\Delta E = Q_{in}, \quad Q_{in} = \dot{Q} \Delta t \quad (\text{Eq 2.37})$$

$$\text{So } \Delta E = \dot{Q} \Delta t$$

$$= 1600 \frac{\text{J}}{\text{sec}} \times 60 \text{ sec} =$$

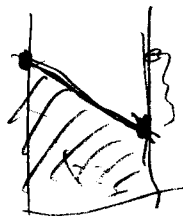
$$960 \text{ kJ}$$

Write 1<sup>st</sup> law 5

$\dot{Q}$  correct 5

$\Delta E = \dot{Q} \Delta t$  5

All correct 5



#2 (20 Points) The force required to push in a piston increases with distance as

$$F = Cx^2$$

C is an appropriately dimensioned constant. What is the total work done in moving the piston from  $x = 0$  to  $x = x_1$ ? Express your answer in terms of C,  $x_1$ , and any other constants or values as appropriate.

$$W = F \cdot x \text{ for constant force}$$

$$W = \int_{x_i}^{x_f} F dx \text{ for variable force}$$

$$= \int_0^{x_1} Cx^2 dx = \left. \frac{Cx^3}{3} \right|_0^{x_1} = \underline{\underline{\frac{Cx_1^3}{3}}}$$

$$W = F \times \text{distance}$$

$$\text{Variable Force } W = \int F dx$$

Setting up integral correctly

All correct

5  
5  
07 integrand  
limits  
dx  
3

#3 (30 points) Water at 25°C flows out of a 10 cm diameter pipe at a velocity of 10 m/sec. If a device is designed to convert all the kinetic energy of the exiting water into work, what is the maximum power output of the device?

$$D = 10 \text{ cm} \quad V = 10 \text{ m/sec}, \quad \text{Water @ } 25^\circ\text{C} \\ \Rightarrow \rho = 997 \text{ Kg/m}^3$$

App 3



$$W = F \cdot d$$

$$\text{Power} = \text{Work rate} = F \cdot \frac{d}{dt} = FV$$

~~Kinetic energy~~

$$K.E. = \frac{1}{2} m V^2$$

$$\text{Power} = \frac{1}{2} \dot{m} V^2$$

$$\dot{m} = \rho V A$$

$$\text{So max power} = \frac{1}{2} \rho A V^3$$

$$A = \pi R^2 = \pi \frac{D^2}{4} = \pi \frac{.1^2}{4} = .0079$$

$$\text{So max power} = \frac{1}{2} 997 \frac{\text{Kg}}{\text{m}^3} (.0079) \text{m}^2 10^3 \frac{\text{m}^3}{\text{sec}^3} \cdot \frac{\text{Kg m}}{\text{sec}^2} \frac{\text{m}}{\text{sec}}$$

$$= 39.78 \text{ KJ/sec}$$

*for mixed mass*

$$K.E. = \frac{1}{2} m V^2 : 5$$

$$\text{Power} = \left. \begin{aligned} &K.E. / \text{time} \\ &= \frac{1}{2} \dot{m} V^2 \end{aligned} \right\} 10$$

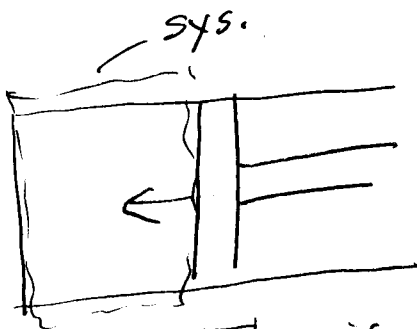
$$\dot{m} = \rho A V : 7$$

$$\rho \text{ correct : } 3$$

$$A \text{ correct : } 3$$

$$\text{All correct : } 2$$

#4 (30 points) 1 kg of air in a cylinder is compressed by a piston from  $1 \text{ m}^3$  in volume to  $0.4 \text{ m}^3$ . The initial temperature is 300 K. During the compression, heat is transferred out of the cylinder so that the final temperature is 120 K. how much work is done during this process.



$$V_1 = 1 \text{ m}^3 \quad T_1 = 300 \text{ K}$$

$$V_2 = 0.4 \text{ m}^3 \quad T_2 = 120 \text{ K}$$

This is boundary work

$$W = \int_{V_1}^{V_2} P dV$$

But we don't know relationship between  $P$  &  $V$ .

So let's compute  $P_1$  &  $P_2$  & see what we get:

$$P_1 = \frac{mRT}{V_1} \quad m = 1 \text{ kg} \quad R = .287 \frac{\text{kJ}}{\text{kg K}}$$

$$= \frac{(1 \text{ kg}) \cdot .287 \left( \frac{\text{kJ}}{\text{kg K}} \right) 300 \text{ K}}{1 \text{ m}^3} = 86.1 \text{ kPa}$$

$$P_2 = \frac{1 (\text{kg}) \left( .287 \frac{\text{kJ}}{\text{kg K}} \right) (120) \text{ K}}{0.4 \text{ m}^3} = 86.1$$

So  $P_1 = P_2$  & since uniform compression & heating (given  $m$  &  $c_{\text{loss}}$ )  
this is const pressure process

$$\text{So } W = P \int_{V_1}^{V_2} dV = P(V_2 - V_1) = 86.1 \text{ kPa} (-0.6 \text{ m}^3)$$

$$= -51.6 \text{ kJ on system}$$

$$W = \int P dV \quad 5 \checkmark$$

compute  $P_1$  6  $\checkmark$   
(use Eos only 4)  $\checkmark$   
compute  $P_2$  6  $\checkmark$

recognize constant  $P$  10  $\checkmark$   
(but must show not assume)  
All correct 3  $\checkmark$