## VI. Entropy

L. Adiabatic or Isentropic Efficiencies of Steady Flow Devices

1. Turbines ($\eta_T$ = 80-90%)

$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \qquad (7\text{-}60)$$

Isentropic work, $w_s$, is calculated assuming same inlet condition and same outlet pressure as actual turbine.

Recall $$0 = w_{in} + (h_1 - h_2) + \left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) + g(z_1 - z_2)$$

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} \qquad (7\text{-}61)$$



## VI. Entropy

2. Compressors and Pumps ($\eta_C$ = 75-85%)

$$\eta_C = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

Isentropic work, $w_s$, is calculated assuming same inlet condition and same outlet pressure as actual compressor.

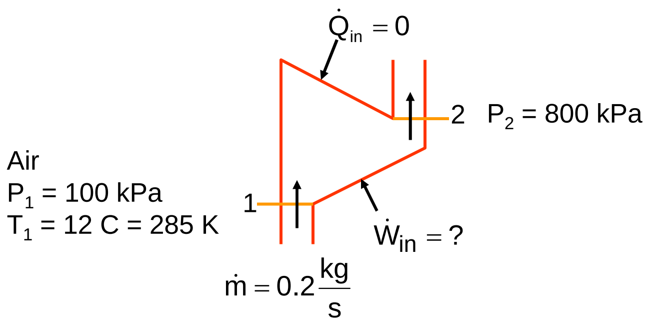
Recall $$0 = w_{in} + (h_1 - h_2) + \left(\frac{V_1^2}{2} - \frac{V_2^2}{2}\right) + g(z_1 - z_2)$$

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \qquad (7\text{-}63)$$

## VI. Entropy

3. Compressor Example (Ex. 7-15, p. 374 of text)

Given an adiabatic efficiency, $\eta_C$ = 0.80, find required power input.

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## VI. Entropy

3. Compressor Example (cont.)

Approach: calculate the isentropic work, $w_s$, and then find the actual work, $w_a$, using the adiabatic efficiency. Do this (1) assuming ideal gas with constant specific heats and then (2) with value obtained using ideal gas with variable specific heats.

First approach: $w_s = h_{2s} - h_1 = c_p (T_{2s} - T_1) = c_p T_1 \left( \frac{T_{2s}}{T_1} - 1 \right)$

$$\dot{W}_s = \dot{m} C_p T_1 \left[ \left( \frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

$$\dot{W}_s = 0.2(1.005)285 \left[ \left( \frac{800}{100} \right)^{(1.4-1)/1.4} - 1 \right] = 46.5\ kW$$

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## VI. Entropy

3. Compressor Example (cont.)

Second approach: $w_s = h_{2s} - h_1$

Find $h_1$ from Table A-17 at 285 K. Find $h_{2s}$ by first finding $p_{r2}$.

$$s_2 - s_1 = \int_{T_1}^{T_2} c_p \frac{dT}{T} - R\ln\frac{P_2}{P_1} = 0 \longrightarrow s_2^o - s_1^o - R\ln\frac{P_2}{P_1} = 0$$

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} \longrightarrow P_{r2} = P_{r1}\frac{P_2}{P_1} = 1.1584\,\frac{800}{100} = 9.2672$$

Using $P_{r2}$, interpolate in Table A-17 to give $h_{2s}$ = 517.05 kJ/kg

$$\dot{W}_s = \dot{m}\left(h_{2s} - h_1\right) = 0.2\left(517.05 - 285.14\right) = 46.4\ kW$$



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3. Compressor Example (cont.)

$$\eta_C = \frac{Isentropic\ compressor\ work}{Actual\ compressor\ work} = \frac{w_s}{w_a} = \frac{\dot{W}_s}{\dot{W}_a}$$

$$\dot{W}_a = \frac{\dot{W}_s}{\eta_C} = \frac{46.4\ kW}{0.8} = 58.0\ kW$$

## VI. Entropy

4. Another Compressor Example

On p. 565 of Metcalf & Eddy, Wastewater Engineering, 3rd ed., Chapter 10, on design of activated sludge process, recommends the use of the following equation to calculate the power requirements for blowers to aerate sludge.

$$P_w = \frac{wRT_1}{29.7ne}\frac{[(p_2)^{0.283}}{p_1} - 1\,]$$

(SI units, typographical errors exactly as they appear)

Our equation for a reversible process with an ideal gas is

$$\dot{W}_{in} = \frac{\dot{m}kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right] \qquad (7\text{-}57a)$$

Is the Metcalf & Eddy equation correct?



## VI. Entropy

4. Another Compressor Example (cont.)

$$P_w = \frac{wRT_1}{29.7ne}\frac{[(p_2)^{0.283}}{p_1} - 1\,]$$

Table provided by Metcalf & Eddy for their formula.

| | |
|---|---|
| $P_w$ | = power requirement, kW |
| w | = mass flow rate of air, kg/s |
| R | = universal gas constant, 8.314 kJ/(kmol K) |
| $T_1$ | = absolute inlet temperature, K |
| $p_1$ | = absolute inlet pressure, atm |
| $p_2$ | = absolute outlet pressure, atm |
| n | = (k-1)/k = 0.283 for air |
| k | = 1.395 for air |
| 29.7 | = constant for SI units conversion |
| e | = efficiency (usual range for compressors is 0.70 to 0.90) |

## VI. Entropy

4. Another Compressor Example (cont.)

After correcting the obvious typographical errors, the Metcalf & Eddy equation becomes

$$P_w = \frac{wRT_1}{29.7ne}\left[\left(\frac{p_2}{p_1}\right)^{0.283} - 1\right]$$

We need to check two things:

1) Based on Table A-1, is 0.283 = $\frac{k-1}{k} = \frac{1.400-1}{1.400} = 0.2857$

Yes, pretty close.

2) Is $R_u/29.7 = R$? $R = \frac{R_u}{MW_{air}} = \frac{8.314\ kJ/(kmol\ K)}{28.97\ kg/kmol}$

Yes, pretty close.

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## VI. Entropy

K. Entropy Balance for a Closed System (Review)

$$\begin{pmatrix}\text{Time rate of change}\\ \text{of entropy within}\\ \text{system at time } t\end{pmatrix} = \begin{pmatrix}\text{net rate of entropy}\\ \text{transport to system}\\ \text{at time } t\end{pmatrix} + \begin{pmatrix}\text{rate of entropy}\\ \text{generation within}\\ \text{the system at time } t\end{pmatrix}$$

$$\frac{dS_{sys}}{dt} = \sum_{j=1}^{n}\frac{\dot{Q}_j}{T_j} + \dot{S}_{gen}$$

$$dS_{sys} = \sum_{j=1}^{n}\frac{\delta Q_j}{T_j} + \delta S_{gen}$$

$$\left(S_2 - S_1\right)_{sys} = \int\frac{\delta Q}{T} + S_{gen}$$

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## VI. Entropy

H. Entropy Balance for an Open System (Review)

$$\frac{dS_{cv}}{dt} = \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum_{j=1}^{n} \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen\,cv} \quad (7\text{-}83)$$

$$(S_2 - S_1)_{CV} = \sum_i m_i s_i - \sum_e m_e s_e + \int \frac{\delta Q}{T} + S_{gen\,cv} \quad (7\text{-}82)$$

This is the integrated form.

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## VI. Entropy

I. Closing Thoughts on Entropy and the Second Law

1. High temperature thermal energy is more available for doing work than low temperature thermal energy.

$$\eta_{th,int\,rev} = \frac{\dot{W}_{out}}{\dot{Q}_H} = 1 - \frac{T_L}{T_H}$$

2. Entropy is not conserved. $S_{gen} > 0$ for all real processes.
3. Isolated systems will only change in a direction that results in an increase in entropy.
4. Irreversibilities result in the generation of entropy. Highly irreversible processes have high rates of entropy generation. High rates of entropy generation usually degrade the performance of a process.

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