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K. Reversible, Steady Flow Work

2. Special cases

a. incompressible fluid

$v = v_f$ and (7-53) becomes

$$w_{rev,in} = v_f\left(P_2 - P_1\right) + \Delta ke + \Delta pe \qquad \text{(7-54)}$$

If there is no shaft work

$$v_f\left(P_2 - P_1\right) + \frac{V_2^2 - V_1^2}{2} + g\left(z_2 - z_1\right) = 0 \qquad \text{(7-55)}$$

Bernoulli's equation

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b. reversible polytropic process ($Pv^n = c,\ n \neq 1$) with negligible Δke and Δpe

i. closed system

$$w_{in} = \int_1^2 -Pdv = \frac{Pv_2 - P_1v_1}{n-1}$$

ii. steady flow compression

$$w_{in} = \int_1^2 vdP = \int_1^2 c^{1/n}P^{-(1/n)}dp = \frac{n\left(P_2v_2 - P_1v_1\right)}{n-1}$$

$$\text{where } c = P_1v_1^n = P_2v_2^n$$

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iii. steady flow compression, ideal gas (Pv = RT)

$$w_{in} = \int_1^2 vdP = \frac{nR(T_2 - T_1)}{n-1} = \frac{nRT_1}{n-1}\left(\frac{T_2}{T_1} - 1\right) \qquad \text{(7-57b)}$$

But for a polytropic process with an ideal gas

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(n-1)/n} \quad \text{and} \quad w_{in} = \frac{nRT_1}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{(n-1)/n} - 1\right]$$

(7-57b)

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c. isentropic process, ideal gas ($pv^k = c$) with negligible Δke and Δpe using constant or average c_p, c_v.

i. closed system

$$w_{in} = \int_1^2 -Pdv = \frac{P_2v_2 - P_1v_1}{k-1} = \frac{R}{k-1}(T_2 - T_1)$$

ii. steady flow compression

$$w_{in} = \int_1^2 vdP = \frac{kRT_1}{k-1}\left(\frac{T_2}{T_1} - 1\right) \quad \text{and} \quad w_{in} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

(7-57a) (7-57a)

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d. isothermal, ideal gas, steady flow with negligible Δke and Δpe

$$w_{in} = \int_1^2 v dP = RT \ln \frac{P_2}{P_1} \qquad \text{(7-57c)}$$

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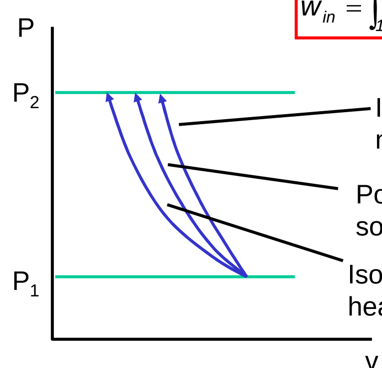
3. P-v diagram of isentropic, polytropic, and isothermal <u>compression</u> processes between the same pressure limits for an ideal gas.

$$w_{in} = \int_1^2 v dP$$

Keeping v small will reduce w_{in} required for compression.



Isentropic (n = k), maximum w_{in}, no heat transfer

Polytropic (1 < n < k), intermediate w_{in}, some heat transfer

Isothermal (n = 1), minimum w_{in}, heavy cooling

see Fig 7-45 in text

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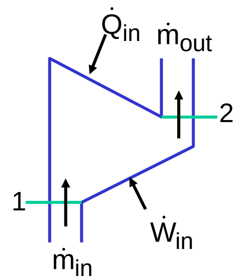
4. Example

Air enters a steady flow compressor at 17 C and is compressed through a ratio of 8.6:1. If the process is modeled as adiabatic and internally reversible, calculate the steady flow work required (kJ/kg), using room temperature specific heat data.



Apply (7-57a) with T_1 = 290 K.

$$w_{in} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

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4. Example

Data for air from Table A-1, A-2.

R = 0.2870 kJ/(kg K)
k = 1.40

$$w_{in} = \frac{kRT_1}{k-1}\left[\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right]$$

$$= \frac{1.4(0.2870)290}{1.4-1}\left[(8.6)^{0.4/1.4} - 1\right] = 247\,\frac{kJ}{kg}$$

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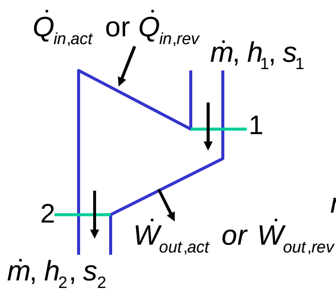
5. Reversible and actual steady flow work



Consider an actual and reversible turbine with inlet and exit conditions the same for both cases. The steady flow energy and entropy balances are

$$\dot{m}(h_1 - h_2) = \dot{W}_{out,act} - \dot{Q}_{in,act} = \dot{W}_{out,rev} - \dot{Q}_{in,rev}$$

$$\dot{Q}_{in,act} = T\dot{m}(s_2 - s_1) - T\dot{S}_{gen}$$

$$\dot{Q}_{in,rev} = T\dot{m}(s_2 - s_1)$$

Combine the energy and entropy balances to eliminate the rates of heat transfer. We conclude that

$$\dot{W}_{out,rev} = \dot{W}_{out,act} + T\dot{S}_{gen}$$

and

$$\dot{W}_{out,rev} > \dot{W}_{out,act}$$