VI. Entropy

H. The Tds Relations and Changes in Entropy

4. Example

Two pieces of copper, A and B, with masses 1 and 3 kg, and initial temperatures 0 and 200 C are brought together and allowed to equilibrate while insulated from the surroundings. Determine (a) the entropy change of A and B and (b) the entropy generation (S_{gen}) for the process, in kJ/K.

Entropy change given by (7-28):

$$\Delta S = mC_{av} \ln \frac{T_2}{T_1}$$

VI. Entropy

4. Example

Final temperature, T_2, is obtained from energy balance.

$$\Delta U = Q - W = 0$$
$$[mC_{av}(T_2 - T_1)]_A + [mC_{av}(T_2 - T_1)]_B = 0$$
$$1(T_2 - 0) + 3(T_2 - 200) = 0$$
$$T_2 = 150\ C\ \ (151\ C\ \text{if use}\ C_{av}\ \text{for each})$$

Entropy changes for A and B are

$$\Delta S_A = 1(0.390)\frac{kJ}{kg}\ln\frac{423}{273} = 0.171\frac{kJ}{K}$$

$$\Delta S_B = 3(0.400)\frac{kJ}{kg}\ln\frac{423}{473} = -0.134\frac{kJ}{K}$$

VI. Entropy

4. Example

Entropy generation from overall entropy balance on composite system (7-9)

$$(S_2 - S_1)_{sys} = \int \frac{\delta Q}{T} + S_{gen}$$

There is no heat transfer ($\delta Q = 0$) so

$$S_{gen} = (S_2 - S_1)_{sys} = \Delta S_A + \Delta S_B = 0.171 + (-0.134) = 0.037 \frac{kJ}{K}$$

Because $S_{gen} > 0$, the process is irreversible.

lesson 19

VI. Entropy

5. Isentropic processes with ideal gases
   a. approximate treatment assuming constant specific heats
      i. first isentropic relation (applies to open and closed systems), recall

$$\Delta s = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \qquad \text{(ideal gas, 7-33)}$$

Reversible and adiabatic $\Rightarrow$ isentropic $\Rightarrow$ $\Delta s = 0$.
(7-33) becomes (remember $R = c_p - c_v$, $k = c_p/c_v$)

$$c_{v,av} \ln \frac{T_2}{T_1} = R \ln \frac{v_1}{v_2} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{R/C_v} = \left(\frac{v_1}{v_2}\right)^{(k-1)}$$

(ideal gas, isentropic, 7-42)

lesson 19

VI. Entropy

ii. second isentropic relation (applies to open and closed systems)

$$\Delta s = c_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$ (ideal gas, 7-34)

Reversible and adiabatic ⇒ isentropic ⇒ Δs = 0.
(6.23) becomes (remember $R = c_p - c_v$, $k = c_p/c_v$)

$$c_{p,av} \ln \frac{T_2}{T_1} = R \ln \frac{P_2}{P_1}$$ or $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{R/c_p} = \left(\frac{p_2}{p_1}\right)^{(k-1)/k}$$

(ideal gas, isentropic, 7-43)

lesson 19

VI. Entropy

iii. third isentropic relation (applies to open and closed systems)

From (7-42) and (7-43),

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^k$$ (ideal gas, isentropic, 7-44)

Equations 7-42 – 7-44 can be written as

$$Tv^{k-1} = C$$
$$TP^{(1-k)/k} = C$$
$$Pv^k = C$$

(ideal gas, isentropic, where C is a constant.)

lesson 19

VI. Entropy

b. exact treatment using variable specific heats

$$s_2 - s_1 = s_2^o - s_1^o - R\,ln\frac{P_2}{P_1}$$ (ideal gas, 7-39)

Reversible and adiabatic ⇒ isentropic ⇒ $\Delta s = 0$.
(7-39) becomes

$$\frac{P_2}{P_1} = exp\frac{s_2^o - s_1^o}{R} = \frac{exp(s_2^0 / R)}{exp(s_1^0 / R)} = \frac{P_{r2}}{P_{r1}}$$ (ideal gas, isentropic, 7-49)

where P_r is called the relative pressure (Table A-17).

We can also define v_r, the relative specific volume (Table A-17).

$$\left(\frac{v_2}{v_1}\right)_{s=const.} = \frac{v_{r2}}{v_{r1}}$$ (ideal gas, isentropic, 7-50)

lesson 19

VI. Entropy

I. Entropy Balance for a Closed System (Review)

$$\begin{pmatrix} \text{Time rate of change} \\ \text{of entropy within} \\ \text{system at time } t \end{pmatrix} = \begin{pmatrix} \text{net rate of entropy} \\ \text{transport to system} \\ \text{at time } t \end{pmatrix} + \begin{pmatrix} \text{rate of entropy} \\ \text{generation within} \\ \text{the system at time } t \end{pmatrix}$$

$$\frac{dS_{sys}}{dt} = \sum_{j=1}^{n} \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen}$$

$$dS_{sys} = \sum_{j=1}^{n} \frac{\delta Q_j}{T_j} + \delta S_{gen}$$

$$(S_2 - S_1)_{sys} = \int \frac{\delta Q}{T} + S_{gen}$$

lesson 19

VI. Entropy

J. Entropy Balance for an Open System

$$\frac{dS_{cv}}{dt} = \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \sum_{j=1}^{n} \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen\,cv} \quad (7\text{-}83)$$

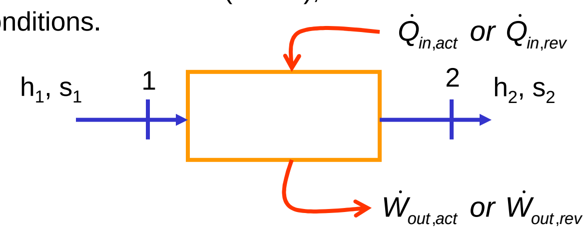
$$(S_2 - S_1)_{CV} = \sum_i m_i s_i - \sum_e m_e s_e + \int \frac{\delta Q}{T} + S_{gen\,cv} \quad (7\text{-}82)$$

This is the integrated form.

VI. Entropy

Reversible and Actual Work and the Generation of Entropy

Consider two, steady flow processes, one reversible and the other irreversible (actual), with the same inlet and exit conditions.



Energy balances: $\dot{m}(h_1 - h_2) = \dot{W}_{out,act} - \dot{Q}_{in,act} = \dot{W}_{out,rev} - \dot{Q}_{in,rev}$

Entropy balances: $\dot{Q}_{in,act} = \dot{m}T(s_2 - s_1) - T\dot{S}_{gen}$ and $\dot{Q}_{in,rev} = \dot{m}T(s_2 - s_1)$

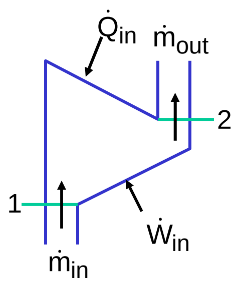
Conclusion: $\therefore \dot{W}_{out,rev} = \dot{W}_{out,act} + T\dot{S}_{gen}$

$T\dot{S}_{gen}$ represents the rate at which work is "lost" due to irreversibilities.

VI. Entropy

K. Reversible, Steady-Flow Work

1. Steady-flow balance equations (applied to a compressor)



$$0 = \dot{m}_1 - \dot{m}_2$$

$$\dot{Q}_{cv,in} + \dot{W}_{nonflow,in} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$\dot{m}_1 s_1 - \dot{m}_2 s_2 + \frac{\dot{Q}}{T} + \dot{S}_{gen} = 0$$

Divide the energy and entropy balances by $\dot{m}$:

$$q + w_{in} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

$$s_1 - s_2 + \frac{q_{rev}}{T} = 0$$

$$\text{where } \dot{Q} = \dot{m}q,\ \dot{W} = \dot{m}w,\text{and } \dot{S}_{gen} = 0.$$

lesson 19

VI. Entropy

In differential form

$$\delta q + \delta w_{in} = dh + d\left(\frac{V^2}{2}\right) + gdz \quad \text{and} \quad -ds + \frac{\delta q_{rev}}{T} = 0$$

Rearrange the later to give $\delta q_{rev} = Tds$

Recall that $Tds = dh - vdp$ (7-24)

Then $\delta w_{rev,in} = vdp + d(ke) + d(pe)$

lesson 19

VI. Entropy

Integrating from 1 to 2 gives

$$w_{rev,in} = \int_1^2 vdP + \Delta ke + \Delta pe \qquad (7\text{-}51)$$

For a compressor, $w_{rev,in}$ is a positive number.

For a turbine, $w_{rev,in}$ is a negative number.

Neglecting Δke and Δpe gives

$$w_{rev,in} = \int_1^2 vdP \qquad (7\text{-}52)$$

For a turbine, it is convenient to rewrite (7-51) as

$$w_{rev,out} = -\int_1^2 vdP - \Delta ke - \Delta pe$$