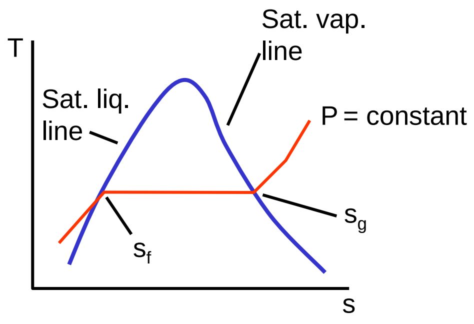
VI. Entropy

G. Entropy Change of a Pure Substance

1. T-s diagram for a pure substance



$$\boxed{s_{fg} = s_g - s_f}$$

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2. Tabular entropy data
   - a. Superheated vapor: s from table
   - b. Saturated liquid: $s = s_f$
   - c. Saturated mixture: $s = s_f + xs_{fg} = (1 - x)s_f + xs_g$
   - d. Compressed liquid: $s(T,p) \cong s_f$ *at T*
3. Entropy change: $\Delta S = m\Delta s = m(s_2 - s_1)$ (7-12)

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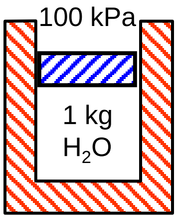
4. Example. The piston-cylinder device shown below contains 1.00 kg of water at 150 C and 100 kPa. The piston is frictionless and weightless. We now add 50 kJ of heat to the water. Find the increase in u, h, and s for the water. How much work is done by the water? Sketch the process on a T-s diagram.



1.00 kg H_2O *(system)*
$T_1 = 150\ C$
$T_2 = ?\ C$
$P_1 = P_2 = 100\ kPa$
$Q_{in} = 50\ kJ$

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a. assume heat capacity of piston and cylinder are negligible, ΔKE and ΔPE are negligible, and that expansion is quasi-equilibrium.

b. data from Table A-6 (superheated water) at 150 C and 0.10 MPa.

$u_1 = 2582.8\ kJ/kg$
$h_1 = 2776.4\ kJ/kg$
$s_1 = 7.6134\ kJ/(kg\ K)$

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c. first law for a closed system

$$\Delta U = Q_{in} - W_{out}$$
$$\Delta U = Q - P\Delta V \quad (for\ a\ reversible\ process)$$
$$\Delta H = Q$$
$$m(h_2 - h_1) = Q$$

$$\boxed{h_2 = Q + h_1 = 50.0\ kJ + 2776.4\ kJ = 2826.4\ kJ}$$

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d. interpolate in Table A-6, using h_2, to find T_2, u_2, and s_2

$T_2 = 175.3\ C$
$u_2 = 2620.9\ kJ/kg$
$s_2 = 7.725\ kJ/(kg\ K)$

$$\boxed{\begin{aligned}\Delta u &= u_2 - u_1 = 38.1\ kJ/kg \\ \Delta h &= h_2 - h_1 = 50.0\ kJ/kg \\ \Delta s &= s_2 - s_1 = 0.1117\ kJ/(kg\ K)\end{aligned}}$$

$$\boxed{W = Q - \Delta u = 50.0 - 38.1 = 11.9\ kJ}$$

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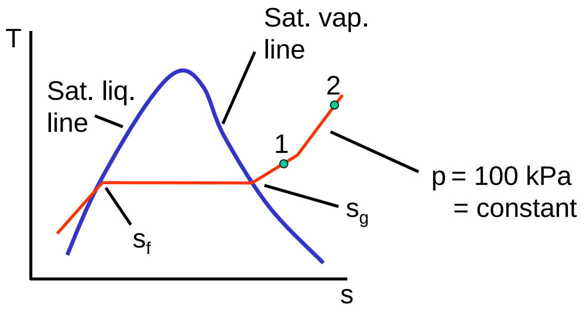
e. Sketch on T-s diagram



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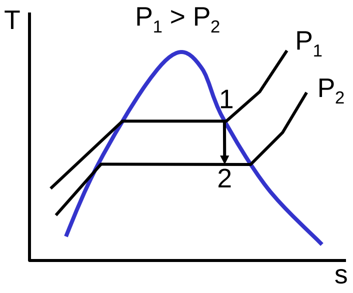
H. Isentropic processes

$$\boxed{\Delta S = 0 \text{ or } s_2 = s_1} \quad (7\text{-}13)$$

Good approximation for processes that are nearly adiabatic and reversible.

Example: isentropic expansion of sat. vapor in a piston-cylinder device or turbine.



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I. The TdS Relations and Changes in Entropy

1. First and second laws for a closed system

$$dU = \delta Q - \delta W \qquad dS = \sum_{j=1}^{n} \frac{\delta Q_j}{T_j} + \delta S_{gen}$$

a. First Tds equation for simple, compressible substance

$$dU = \delta Q_{rev} - \delta W_{rev} \qquad dS = \frac{\delta Q_{rev}}{T} \qquad \delta W_{rev} = PdV$$

$$\boxed{Tds = du + Pdv} \quad \text{or} \quad \boxed{ds = \frac{du}{T} + \frac{Pdv}{T}} \qquad \text{(7-23, 25)}$$

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b. Second Tds equation for simple, compressible substance

Recall that $h = u + Pv$ or $dh = du + Pdv + vdP$

$$Tds = du + Pdv$$

$$\boxed{Tds = dh - vdP} \quad \text{or} \quad \boxed{ds = \frac{dh}{T} - \frac{vdP}{T}} \qquad \text{(7-24, 26)}$$

7-25 and 7-26 are valid for reversible and irreversible processes.

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2. ΔS for solids and liquids (incompressible substances)

$$ds = \frac{du}{T} + \frac{pdv}{T}$$ But dv = 0

Then $$\boxed{ds = \frac{du}{T} = \frac{cdT}{T}}$$ and $c = c_p = c_v$.

Integration gives

$$\boxed{\Delta s = s_2 - s_1 = \int_1^2 c(T)\frac{dT}{T} \cong c_{av}\ln\frac{T_2}{T_1}} \qquad (7\text{-}28)$$

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3. ΔS for ideal gases

$$ds = \frac{du}{T} + \frac{Pdv}{T} = c_v\frac{dT}{T} + R\frac{dv}{v}$$

$$ds = \frac{dh}{T} - \frac{vdP}{T} = c_p\frac{dT}{T} - R\frac{dP}{P}$$

$$\boxed{\Delta s = \int_1^2 c_v(T)\frac{dT}{T} + R\ln\frac{v_2}{v_1}}$$ (ideal gas, 7-31)

$$\boxed{\Delta s = \int_1^2 c_p(T)\frac{dT}{T} - R\ln\frac{P_2}{P_1}}$$ (ideal gas, 7-32)

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a. Approximate analysis using averaged specific heats

$$\boxed{\Delta s = c_{v,av} \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1}}$$ (ideal gas, 7-33)

$$\boxed{\Delta s = c_{p,av} \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}}$$ (ideal gas, 7-34)

where

$$T_{av} = \frac{T_1 + T_2}{2} \quad and \quad c_{av} = c(T_{av})$$

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b. Analysis for ideal gases using integrated specific heats

Let $s^o = \int_0^T c_p(T)\frac{dT}{T}$ (see Equation 7-37 and Table A-17)

Then $\int_{T_1}^{T_2} c_p(T)\frac{dT}{T} = \int_0^{T_2} c_p(T)\frac{dT}{T} - \int_0^{T_1} c_p(T)\frac{dT}{T} = s_2^o - s_1^o$

$$\boxed{s_2 - s_1 = s^o_2 - s^o_1 - R\ln\frac{P_2}{P_1}}$$ (ideal gas, 7-39)

$$\boxed{\bar{s}_2 - \bar{s}_1 = \bar{s}^o_2 - \bar{s}^o_1 - \bar{R}\ln\frac{P_2}{p_1}}$$ (ideal gas, 7-40)

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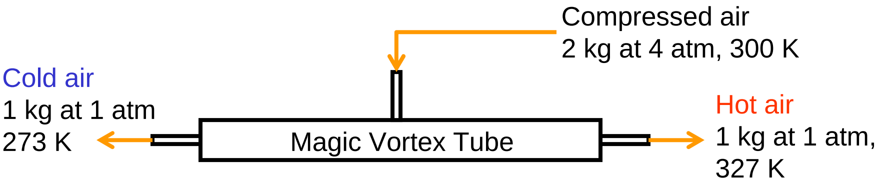
c. Example - Is it possible? Is $\Delta S > 0$?



Compressed air
2 kg at 4 atm, 300 K

Cold air
1 kg at 1 atm
273 K

Magic Vortex Tube

Hot air
1 kg at 1 atm,
327 K

Assume ideal gas with constant specific heat evaluated at 300 K.

From (7-34), $\Delta s = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$

From Table A-1, 2, c_p = 1.005 kJ/(kg K) and R = 0.2870 kJ/(kg K)

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c. Example - Is $\Delta S > 0$?

For the hot stream

$$\Delta s_{hot} = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = 1.005 \ln \frac{327}{300} - 0.2870 \ln \frac{1}{4} = 0.4845 \frac{kJ}{K}$$

For the cold stream

$$\Delta s_{cold} = 1.005 \ln \frac{273}{300} - 0.2870 \ln \frac{1}{4} = 0.3031 \frac{kJ}{K}$$

The total change in entropy is

$$\Delta S_{tot} = 0.4845 + 0.3031 = 0.7876 \frac{kJ}{K}$$

We conclude that the process is possible (although we don't know how it works) because $\Delta S > 0$.