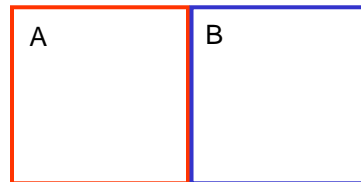


## VI. Entropy

A. Introduction (lesson 17 is an alternative way of looking at entropy)

1. Observation shows that isolated systems spontaneously change to a state of equilibrium.

a. Two blocks of iron with  $T_{A1} > T_{B1}$ . Bring the blocks together and allow the system to come to equilibrium.



The system is A + B.

$$Q_{in} = 0$$

$$W_{in} = 0$$

At equilibrium,  $T_{A2} = T_{B2}$ . The first law says nothing about the direction of change.

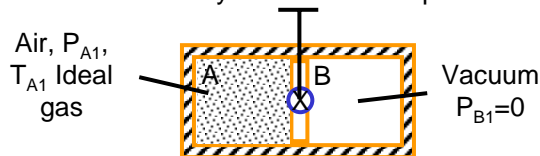
lesson 17

## VI. Entropy

A. Introduction

1. Observation shows that isolated systems spontaneously change to a state of equilibrium.

b. Two chambers connected by a valve. Open valve and allow system to reach equilibrium.



Air,  $P_{A1}$ ,  
 $T_{A1}$  Ideal  
gas

Vacuum  
 $P_{B1}=0$

The system is A + B.

$$Q_{in} = 0$$

$$W_{in} = 0$$

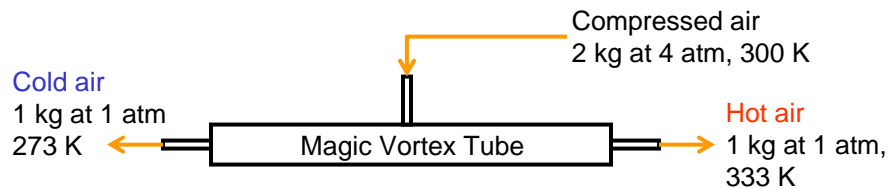
At equilibrium,  $T_{A2} = T_{B2}$  and  $P_{A2} = P_{B2}$ . The first law is silent on the direction of change.

lesson 17

## VI. Entropy

### A. Introduction

2. The magic vortex tube – will it work? The first law says yes, but will it work?



We need a **quantitative** answer. This suggests that we are looking for a property (like T, P, u, or volume). How can we find such a property?

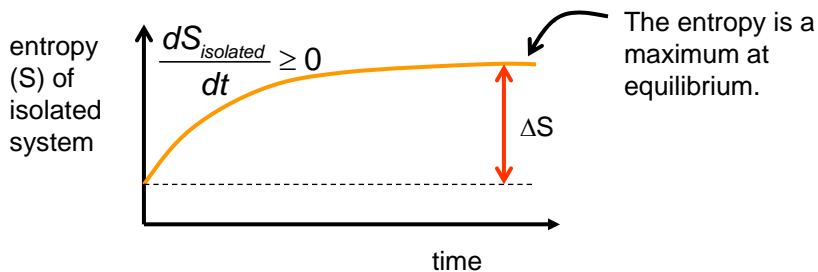
lesson 17

## VI. Entropy

### A. Introduction

#### 3. Entropy (use the symbol S)

- A property that in an isolated system either increases or is constant. The entropy is a maximum at equilibrium.
- Entropy is a measure of the **disorder** of a system on a **microscopic** scale. **There is only one form of entropy – internal entropy.**



lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

1. Four steps to a quantitative definition of S
  - a. Adding thermal energy, Q, to a closed system will increase its disorder.
  - b. Adding Q to a closed system at a low temperature will cause a larger increase in disorder than adding the same Q to a system at a high temperature.
  - c. Performing reversible work, under adiabatic conditions, on a closed system will not change its entropy.
  - d. To avoid the complexities associated with irreversibilities, use  $Q_{rev}$  to define  $\Delta S$ .

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 2. Definition of S for a closed system

$$dS = \frac{\delta Q_{rev}}{T}$$

Units on S are kJ/K

absolute temperature, kelvin

$$\frac{dS}{dt} = \frac{\dot{Q}_{rev}}{T}$$

rate form

$$\Delta S = \int_1^2 \frac{\delta Q_{rev}}{T}$$

integrated form

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 3. Application

- a. Two blocks of iron with  $T_{A1} > T_{B1}$ . What is  $\Delta S$  once equilibrium is reached?

A 0.5 kg, $T_{A1} = 373 \text{ K}$	B 0.5 kg, $T_{B1} = 293 \text{ K}$
---------------------------------------	---------------------------------------

The system is A + B.  
 $Q_{in} = 0$   
 $W_{in} = 0$   
 $T_{A2} = T_{B2}$  at equilibrium.  
 Iron is incompressible.

From first law:  $du = \delta q + \delta w$  or  $\delta q = du - \delta w$

For a reversible process with only P-V work:  $\delta q_{rev} = c_v dT + Pdv$

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 3. Application

- a. Two blocks of iron. What is  $\Delta S$  once equilibrium is reached?

Change in entropy:

$$ds = \frac{\delta q_{rev}}{T} = \frac{c_v dT}{T} + \frac{Pdv}{T}$$

0 since iron is incompressible

Integrate and recall  $c = c_v = c_p$  for solids:

$$\Delta S = \int_1^2 ds = c \int_1^2 \frac{dT}{T} = c \ln \frac{T_2}{T_1}$$

How to find  $T_2$ ?

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 3. Application

- a. Two blocks of iron. Find  $\Delta S$ .

Find  $T_2$  from first law.

$$\Delta U = \overset{0}{Q} + \overset{0}{W}$$

$$\Delta U = (U_{A2} - U_{A1}) + (U_{B2} - U_{B1}) = 0$$

Rewrite in terms of specific heat and temperature.

$$\Delta U = mc(T_2 - T_{A1}) + mc(T_2 - T_{B1}) = 0$$

Solve for  $T_2$ .

$$T_2 = \frac{T_{A1} + T_{B1}}{2}$$

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 3. Application

- a. Two blocks of iron. Find  $\Delta S$ .

Entropy balance.

$$\Delta S = (S_{A2} - S_{A1}) + (S_{B2} - S_{B1}) = S_{gen}$$

$$\Delta S = mc \left( \ln \frac{T_2}{T_{A1}} + \ln \frac{T_2}{T_{B1}} \right) = mc \ln \frac{T_{A1}^2 + 2T_{A1}T_{B1} + T_{B1}^2}{4T_{A1}T_{B1}}$$

$$\Delta S = 0.5 \text{ kg} \left( 450 \frac{\text{J}}{\text{kgK}} \right) \ln \frac{373^2 + 2(373)293 + 293^2}{4(373)(293)} = 3.27 \frac{\text{J}}{\text{K}}$$

$$\Delta S = S_{gen} = 3.27 \frac{\text{J}}{\text{K}}$$

$\Delta S > 0$  as advertised

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 3. Application

b. Unrestrained expansion of **ideal gas**. Open valve. Find  $\Delta S$ .

Air,  $m_{A1} = 1 \text{ kg}$ ,  
ideal gas,  $V_{A1} = 0.1 \text{ m}^3$ ,  
 $T_{A1} = 300 \text{ K}$

Vacuum  
 $P_{B1} = 0$ ,  
 $V_{B1} = 0.1 \text{ m}^3$

The system is A + B.  
 $Q_{in} = 0$   
 $W_{in} = 0$

$T_1 = T_2$  is also  
observed  
experimentally.

Energy balance.  $\Delta U = Q + W \Rightarrow T_1 = T_2 = 300 \text{ K}$

Ideal gas law.  $\frac{P_2}{P_1} = \frac{V_{A1}}{V_{A2}}$  and  $\frac{P}{T} = \frac{mR}{V}$

Change in entropy.  $ds = \frac{\delta q_{rev}}{T} = \frac{cdT}{T} + \frac{Pdv}{T}$

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 3. Application

b. Unrestrained expansion of **ideal gas**. Open valve. Find  $\Delta S$ .

Air,  $m_{A1} = 1 \text{ kg}$ ,  
ideal gas,  $V_{A1} = 0.1 \text{ m}^3$ ,  
 $T_{A1} = 300 \text{ K}$

Vacuum  
 $P_{B1} = 0$ ,  
 $V_{B1} = 0.1 \text{ m}^3$

The system is A + B.  
 $Q_{in} = 0$   
 $W_{in} = 0$

Change in entropy.  $dS = \frac{\delta Q_{rev}}{T} = \frac{PdV}{T} = mR \frac{dV}{V}$

Integrate.  $\Delta S = mR \int_1^2 \frac{dV}{V} = mR \ln \frac{V_2}{V_1}$

$\Delta S = S_{gen} = mR \ln \frac{V_2}{V_1} = 1 \text{ kg} \frac{8.314 \text{ kJ}}{29 \text{ kg}} \frac{\text{kmol K}}{\text{kmol}} \ln \frac{2}{1} = 199 \frac{\text{J}}{\text{K}}$   $\Delta S > 0$  as advertised

lesson 17

## VI. Entropy

### B. Calculation of changes in entropy

#### 3. Application

##### c. Conclusion

Our definition of entropy is consistent with all experiments that have been performed to test it. For a closed system, our entropy balance equations are

$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

$$\frac{dS_{\text{sys}}}{dt} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_{\text{gen}}$$

where

$S_{\text{gen}} > 0$  irreversible process

$S_{\text{gen}} = 0$  reversible process

$S_{\text{gen}} < 0$  impossible process