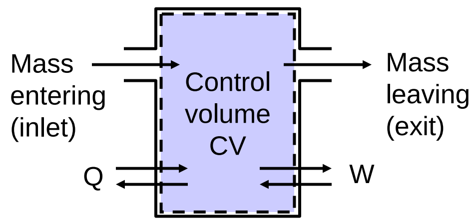
IV. First Law of Thermodynamics

A. Introduction to Open Systems

1. Analysis of flow processes begins with the selection of an open system.
2. An open system is a region of space called a control volume (CV).



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3. Example

We can write a balance equation for the conservation of any extensive property of the open system. As an example, consider the mass of water in the Great Salt Lake. We might be interested in (1) the instantaneous rate of change of mass or (2) the change that occurs over a period of time.

$$\begin{pmatrix}\text{rate of change}\\ \text{of mass of water}\\ \text{in lake}\end{pmatrix} = \begin{pmatrix}\text{rate at which}\\ \text{water enters}\\ \text{lake}\end{pmatrix} - \begin{pmatrix}\text{rate at which}\\ \text{water leaves}\\ \text{lake}\end{pmatrix} \quad (1)$$

$$\begin{pmatrix}\text{change in mass}\\ \text{of water in lake}\\ \text{during February}\end{pmatrix} = \begin{pmatrix}\text{amount of water}\\ \text{which enters lake}\\ \text{during February}\end{pmatrix} - \begin{pmatrix}\text{amount of water}\\ \text{which leaves lake}\\ \text{during February}\end{pmatrix} \quad (2)$$

We obtain Form (2) by integrating Form (1). We use both forms for mass, energy, and entropy.

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B. Conservation of Mass for a Control Volume

1. For a closed system, m_{sys} = constant and $dm_{sys}/dt = 0$
2. For an open system

$$\begin{pmatrix}\text{Time rate of change of}\\ \text{mass within a control}\\ \text{volume at time t}\end{pmatrix} = \begin{pmatrix}\text{total rate of mass}\\ \text{entering a control}\\ \text{volume at time t}\end{pmatrix} - \begin{pmatrix}\text{total rate of mass}\\ \text{leaving a control}\\ \text{volume at time t}\end{pmatrix}$$

$$\frac{dm_{CV}}{dt} = \sum_{in} \dot{m} - \sum_{out} \dot{m} \qquad \text{(kg/s)} \qquad \text{(5-9)}$$

(5-9) can be integrated with respect to time to give

$$\Delta m_{CV} = \sum_{in} m - \sum_{out} m \qquad \text{(kg)} \qquad \text{(5-8)}$$

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3. Simplified cases

a. Steady flow

$$\frac{dm_{cv}}{dt} = 0 = \sum_{in} \dot{m} - \sum_{out} \dot{m} \quad (kg/s) \qquad \text{(5-18)}$$

b. One-dimensional flow (velocity and density are assumed constant or averaged over cross-sectional area of inlet or exit).

$$\dot{m} = \rho VA = \frac{VA}{v} = \frac{\dot{V}}{v} \quad (kg/s) \qquad \text{(5-19)}$$

where $\dot{V}$ is the volumetric flow rate (m^3/s) of the fluid.

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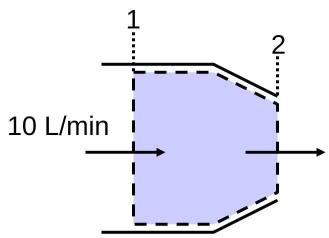
c. Example - liquid water with a constant density of 1000 kg/m^3 enters a nozzle at the rate 10 L/min. The inlet of the nozzle has a diameter of 1.50 cm, the diameter of the exit is 0.75 cm. Find the velocities of the water at the inlet and exit.



Given: Area = πr^2
r_1 = 0.0075 m
r_2 = 0.00375 m
ρ = 1000kg/m^3
Find: V_1 and V_2
Model: one-dimensional, steady flow

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c. Example

Analysis

Because we are at steady state,

$$\frac{dm_{cv}}{dt} = 0 = \dot{m}_1 - \dot{m}_2 \quad or \quad \dot{m}_1 = \dot{m}_2 = \dot{m}$$

From the inlet volumetric flow rate,

$$\dot{m} = \rho V_1 A_1 = \rho \dot{V} = 10\frac{L}{min}\frac{1\ min}{60\ s}\frac{1\ kg}{1\ L} = 0.1667\frac{kg}{s}$$

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c. Example

Analysis

The inlet velocity is

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{0.1667\,\frac{kg}{s}}{1000\,\frac{kg}{m^3}\,\pi(0.0075\ m)^2} = 0.943\,\frac{m}{s}$$

and the exit velocity is

$$V_2 = V_1\frac{A_1}{A_2} = 0.943\,\frac{m}{s}\left(\frac{1.5}{0.75}\right)^2 = 3.77\,\frac{m}{s}$$

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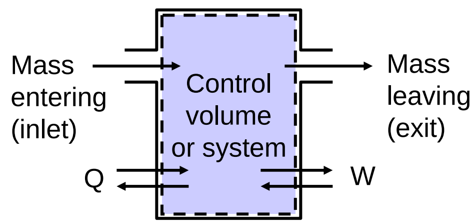
C. Conservation of Energy for an Open System

1. The law of conservation of energy can be applied to a control volume



$$\begin{pmatrix}\text{Time rate of change}\\ \text{of energy within}\\ \text{control volume}\end{pmatrix} = \begin{pmatrix}\text{net rate of energy}\\ \text{crossing boundary}\\ \text{as work and heat}\end{pmatrix} + \begin{pmatrix}\text{total rate of}\\ \text{energy entering}\\ \text{CV with mass}\end{pmatrix} - \begin{pmatrix}\text{total rate of}\\ \text{energy exiting}\\ \text{CV with mass}\end{pmatrix}$$

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2. Mathematical statement of conservation of energy for a control volume (single inlet, single outlet).

Recall that $e = u + \frac{1}{2}V^2 + gz \quad \left(\frac{kJ}{kg}\right)$

$$\frac{dE_{CV}}{dt} = \dot{Q}_{in,net} - \dot{W}_{out,net} + \dot{m}_i\left(u + \frac{V^2}{2} + gz\right)_i - \dot{m}_e\left(u + \frac{V^2}{2} + gz\right)_e \quad \text{(A)}$$

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3. The work term

a. Flow work

$$\dot{W} = F \bullet \text{Vel} \quad (kW)$$

$$\dot{W}_{flow} = P \bullet A \bullet \text{Vel}$$

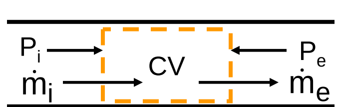
$$\dot{W}_{flow} = \dot{m}(Pv)$$

$$\dot{m} = \frac{1}{v} A \bullet \text{Vel} \quad (kg/s)$$

b. Rewrite work term in (A)

$$\dot{W}_{out,net} = \dot{W}_{nonflow} + \dot{W}_{flow}$$

$$= \dot{W}_{nonflow} - \dot{m}_i(Pv)_i + \dot{m}_e(Pv)_e = \dot{W}_{CV} - \dot{m}_i(Pv)_i + \dot{m}_e(Pv)_e$$



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c. The control volume energy equation (A) becomes

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV,in} - \dot{W}_{CV,out} + \dot{m}_i\left(u + Pv + \frac{V^2}{2} + gz\right)_i - \dot{m}_e\left(u + Pv + \frac{V^2}{2} + gz\right)_e$$

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV,in} - \dot{W}_{CV,out} + \dot{m}_i\left(h + \frac{V^2}{2} + gz\right)_i - \dot{m}_e\left(h + \frac{V^2}{2} + gz\right)_e$$

Single inlet - single outlet

$$\frac{dE_{CV}}{dt} = \dot{Q}_{CV,in} - \dot{W}_{CV,out} + \sum_{in}\dot{m}_i\left(h + \frac{V^2}{2} + gz\right)_i - \sum_{out}\dot{m}_e\left(h + \frac{V^2}{2} + gz\right)_e$$

Multiple inlet - multiple outlet (see 5-59, p. 255)

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4. Steady state control volume equations
   a. Conservation of energy

$$0 = \dot{Q}_{CV} - \dot{W}_{CV} + \sum_{in}\dot{m}_i\left(h + \frac{V^2}{2} + gz\right)_i - \sum_{out}\dot{m}_e\left(h + \frac{V^2}{2} + gz\right)_e \qquad (5\text{-}37)$$

   b. Conservation of mass

$$0 = \sum_{in}\dot{m}_i - \sum_{out}\dot{m}_e \qquad (5\text{-}18)$$

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5. Steady state control volume equations

c. single inlet - single outlet

$$0 = \dot{Q}_{in,net} - \dot{W}_{out,net} + \dot{m}_1\left(h + \frac{V^2}{2} + gz\right)_1 - \dot{m}_2\left(h + \frac{V^2}{2} + gz\right)_2$$

Recall $0 = \dot{m}_1 - \dot{m}_2$

Then

$$\dot{Q}_{in,net} - \dot{W}_{out,net} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] \quad (5\text{-}38)$$

or

$$q - w = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \quad (5\text{-}39)$$