III. Properties of a Pure Substance

G.The Ideal-Gas Equation of State

1. The ideal-gas equation of state models the PvT behavior of gases at low pressure.

$$PV = NR_u T \;\; or \;\; P\bar{v} = R_u T$$

molar basis

$$PV = mRT \;\; or \;\; Pv = RT$$

mass basis (3-10)

N = number of moles of gas, kmol
T = absolute temperature, K
R_u = universal gas constant = 8.314 kJ/(kmol K)
V = volume, m^3
v = specific volume, m^3/kg; $\bar{v}$, $m^3/kmol$
R = R_u/M = specific gas constant, kJ/(kg K)
M = molecular weight of gas, kg/kmol

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2. Notes on use of ideal-gas equation of state
   a. The temperature, T, must be absolute (Kelvin or Rankine)
   b. The pressure, p, must be absolute
   c. The universal gas constant, R_u, is different from the specific gas constant, R. Use the most convenient units.

Substance	R, kPa•m³/kg•K or kJ/kg•K	R_u, kPa•m³/kmol•K or kJ/kmol•K
Air	0.287	8.314
Helium	2.077	8.314
Hydrogen	4.124	8.314
Carbon dioxide	0.189	8.314

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3.0 Example - Does superheated r-134a obey the ideal-gas law at P = 1.0 MPa and T = 100°C? Compare the specific volume calculated from the ideal-gas law with the property data in Table A-13.

From Table A-13, v_{actual} = 0.02755 m^3/kg
From Table A-1, M = 102.03 and R = 0.08149 kPa•m^3/kg•K
From the ideal-gas law

$$v_{ideal} = \frac{RT}{p} = \frac{0.08149 \frac{kPa \bullet m^3}{kg \bullet K}(100 + 273.15)K}{1000\ kPa} = 0.03041 \frac{m^3}{kg}$$

$$\%error = \frac{v_{ideal} - v_{actual}}{v_{actual}} x100\% = 10.4\%$$

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H. The Compressibility Factor and the Principle of Corresponding States

1. Compressibility factor

$$Z = \frac{v_{actual}}{v_{ideal}} = \frac{Pv_{actual}}{RT} \qquad (3\text{-}17, 18, 19)$$

From the proceeding example,

$$Z = \frac{v_{actual}}{v_{ideal}} = \frac{0.02755}{0.03041} = 0.906$$

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2. Principle of Corresponding States

a. Definition - the compressibility factor, Z, is about the same for most gases when they have the same reduced temperature and pressure.

$$P_R = \frac{P}{P_c} \quad \text{and} \quad T_R = \frac{T}{T_c} \qquad (3.27)$$

where absolute temperature and pressure are used.

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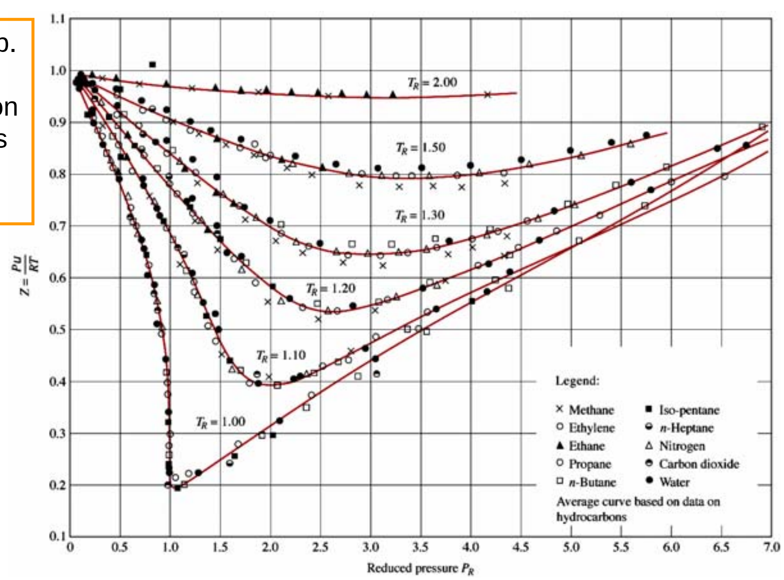
Fig. 3-51, p. 141: Comparison of Z factors for several gases.



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b. Example - for the conditions of the previous example for r-134a, use Fig. A-15, p. 908, to calculate Z

$$Z = f(P_R, T_R)$$

$$P_R = \frac{P}{P_c} = \frac{1.0\ MPa}{4.067\ MPa} = 0.2459$$

$$T_R = \frac{T}{T_c} = \frac{373.15\ K}{374.3\ K} = 0.9969$$

$$Z = 0.913$$

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3. How do we know if the ideal-gas law applies?
   a. $P_R << 1$ at all temperatures (see Fig. A-15)
   b. $T_R > 2$ except where $P_R >> 1$ (see Fig. A-15)
4. When to use the ideal-gas law?

The text has ideal-gas property tables for air, N_2, O_2, CO_2, water vapor, and CO (Tables A-17 to A-25). The use of the air table (A-17) is not fully explained until Chapter 7 (p. 359) in the text. Note that $P_r \neq P_R$ and $v_r \neq v_R$.

Use the ideal-gas law to calculate the density or specific volume of air and all other gases for which the ideal-gas law applies.

As a general rule, do not use the ideal-gas law or the ideal-gas tables to calculate the properties of water or refrigerants in any state - use the tables.

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5. Example from first examination, Fall Semester 2000

Find the mass (kg) and weight (N) of the air contained in a 4.00-m-by-7.00-m-by-8.00-m room if the temperature and pressure of the air are 22.0 C and 0.825 atm.

Assumption: the ideal-gas law applies

Data: R = 0.2870 (kPa m^3)/(kg K) (Table A-1)

g = 9.81 m/s^2 (inside front pages of book)

T = 22 + 273.15 = 295.15 K

Calculations: $PV = mRT$

$$m = \frac{PV}{RT} = \rho V = \frac{0.825\ atm \dfrac{101.325\ kPa}{1\ atm}(4)7(8)m^3}{0.2870 \dfrac{kPa \bullet m^3}{kg \bullet K}(295.15\ K)} = 0.9868 \frac{kg}{m^3} 224\ m^3 = 221\ kg$$

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$$weight = mg = 221(kg)9.81\frac{m}{s^2} = 2170\ N$$