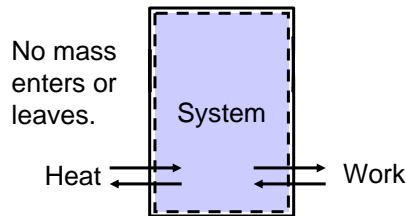


II. Energy and the First Law of Thermodynamics

A. Generic Statement of the First Law for a Closed System

$$\left(\begin{array}{l} \text{Time rate of change} \\ \text{of energy within} \\ \text{system} \end{array} \right) = \left(\begin{array}{l} \text{rate energy} \\ \text{enters system} \end{array} \right) - \left(\begin{array}{l} \text{rate energy} \\ \text{leaves system} \end{array} \right)$$



Rate form of energy balance:

$$\frac{dE_{\text{system}}}{dt} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} \quad (4-12)$$

Integrated form of energy balance:

$$\Delta E_{\text{system}} = E_{\text{in}} - E_{\text{out}} \quad (4-11)$$

Note: $\Delta E_{\text{system}} + \Delta E_{\text{surroundings}} = 0$

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II. Energy and the First Law of Thermodynamics

B. Specific Statement of First Law for a Closed System

1. Energy is conserved.
2. Energy can cross the boundary of a closed system by only two mechanisms: heat transfer and work transfer.
3. The change in energy of a closed system is equal to the net heat transferred to the system minus the net work performed by the system (4-17).

Total energy:

$$\Delta E = Q_{\text{in,net}} - W_{\text{out,net}} \quad (\text{kJ})$$

$$E = mu + \frac{1}{2}mV^2 + mgz \quad (\text{kJ})$$

The differential form of (4-17):

$$dE = \delta Q_{\text{in,net}} - \delta W_{\text{out,net}} \quad (\text{kJ})$$

Energy per unit mass:

$$\Delta e = q_{\text{in,net}} - w_{\text{out,net}} \quad (\text{kJ/kg})$$

$$e = u + \frac{1}{2}V^2 + gz \quad \left(\frac{\text{kJ}}{\text{kg}} \right)$$

$$w = W / m \quad \text{and} \quad q = Q / m$$

$$de = \delta q_{\text{in,net}} - \delta w_{\text{out,net}} \quad (\text{kJ/kg})$$

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II. Energy and the First Law of Thermodynamics

4. (Rate form) The rate of change in energy of a closed system is equal to the rate of heat transfer to the system, minus the rate of work performed by the system plus.

$$\frac{dE}{dt} = \dot{Q}_{in,net} - \dot{W}_{out,net} \quad (kW)$$

Integration gives the previous form, (4-17):

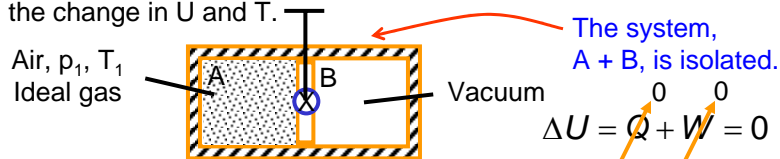
$$\Delta E = \int_{t_1}^{t_2} \frac{dE}{dt} dt \quad Q = \int_{t_1}^{t_2} \dot{Q} dt \quad W = \int_{t_1}^{t_2} \dot{W} dt$$

$$\Delta E = Q_{in,net} - W_{out,net} \quad (kJ)$$

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II. Energy and the First Law of Thermodynamics

5. Example 1 (ideal gas). We have two rigid, insulated chambers connected by a valve. Chamber A is filled with air at 10 bar (gage) and 300 K and B is empty. At these conditions, the air behaves as an ideal gas. Now open the valve and allow the entire system (chambers A and B) to reach equilibrium. Find the change in U and T.



$$\therefore U = \frac{5}{2} mRT = \text{constant} \quad PV = mRT = \text{constant}$$

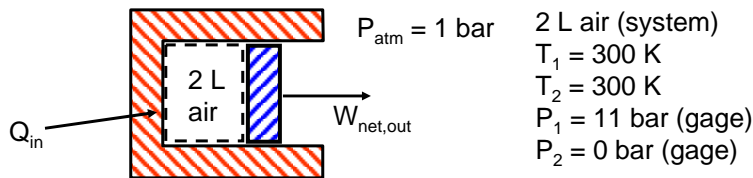
Because U is constant and a function of only T (because the air behaves as an ideal gas) **this is a constant temperature process.** Joule verified this experimentally.

\therefore At equilibrium, $T_1 = T_2$ and $P_1V_1 = P_2V_2$. This is an **irreversible** process.

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II. Energy and the First Law of Thermodynamics

6. Example 2 (ideal gas). A cylinder, containing $V_1 = 2.0$ L of air at 11 bar (gage) and 300 K, is fitted with a frictionless piston. The constant atmospheric pressure outside the cylinder is 1 bar. The air expands **reversibly** and **isothermally** to a final pressure of 0 bar (gage). Treating the air as an ideal gas, calculate Q_{in} , W_{out} , and V_2 .



First Law: $\Delta U = Q_{in} - W_{out} = 0 \Rightarrow Q_{in} = W_{out}$

Reversible expansion: $W_{out} = \int_1^2 P dV$

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II. Energy and the First Law of Thermodynamics

6. Example 2. Isothermal expansion of ideal gas.

Ideal gas: $PV = mRT$ or $P = \frac{mRT}{V}$

Calculate work:

$$W_{out} = \int_1^2 P dV = mRT \int_1^2 \frac{dV}{V} = mRT \ln \frac{V_2}{V_1} = P_1 V_1 \ln \frac{P_1}{P_2}$$

$$W_{out} = Q_{in} = P_1 V_1 \ln \frac{P_1}{P_2} = (1200 \text{ kPa})(0.002 \text{ m}^3) \ln \frac{12}{1} = 5.964 \text{ kJ}$$

Calculate final volume:

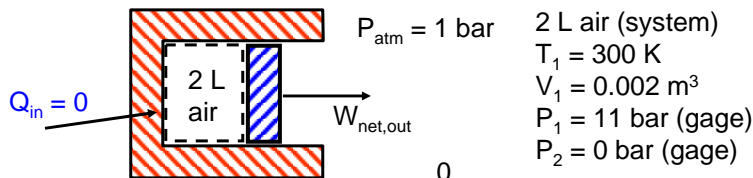
$$V_2 = V_1 \frac{P_1}{P_2} = 0.002 \frac{12}{1} = 0.024 \text{ m}^3$$

Note: in using the ideal gas law, always use absolute pressure rather than gage.

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II. Energy and the First Law of Thermodynamics

7. Example 3 (ideal gas). A cylinder containing 2.0 L of air at 11 bar (gage) and 300 K is fitted with a frictionless piston. The constant atmospheric pressure outside the cylinder is 1 bar. The air is expanded **reversibly** and **adiabatically** to a final pressure of 0 bar (gage). Treating the air as an ideal gas, calculate Q_{in} , W_{out} , V_2 and T_2 .



First Law: $\Delta U = Q_{in} - W_{out}$

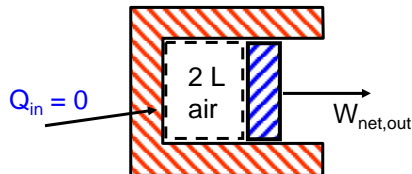
Reversible expansion:

$$W_{out} = \int_1^2 P dV \text{ or } \delta W = P dV (\text{kJ}) \text{ or } \delta w = P dv (\text{kJ/kg})$$

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II. Energy and the First Law of Thermodynamics

7. Example 3. **Reversible, adiabatic expansion of ideal gas.**



Write first law in differential form:

$$\Delta(mu) = Q - W \text{ (kJ)}$$

$$du = \delta q - \delta w \text{ (kJ/kg)}$$

$$du = -P dv$$

Recall for air that $u = \frac{5}{2}RT$ or $du = \frac{5}{2}RdT$ or $du = c_v dT$

Ideal gas: $Pv = RT$ or $P = \frac{RT}{v}$

$$c_v = \frac{5}{2}R$$

Integrate 1st law: $c_v \int_1^2 \frac{dT}{T} = -R \int_1^2 \frac{dv}{v}$ or $c_v \ln \frac{T_2}{T_1} = -R \ln \frac{v_2}{v_1}$

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II. Energy and the First Law of Thermodynamics

7. Example 3. Reversible, adiabatic expansion of ideal gas.

$$\text{Rearrange: } c_v \ln \frac{T_2}{T_1} = -R \ln \frac{v_2}{v_1} \text{ to give } \frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\frac{R}{c_v}}$$

By convention, this is written

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{k-1} \text{ or } T v^{k-1} = \text{constant}$$

$$\text{where } k = \frac{R}{c_v} + 1$$

For air at room temperature, $k = 7/5 = 1.4$.

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II. Energy and the First Law of Thermodynamics

7. Example 3. Reversible, adiabatic expansion of ideal gas.

$$\text{Use the ideal gas law to rearrange previous equation: } \frac{T_2}{T_1} = \frac{v_2 P_2}{v_1 P_1}$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2} \right)^k \text{ or } P v^k = \text{constant}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \text{ or } T P^{\frac{1-k}{k}} = \text{constant}$$

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II. Energy and the First Law of Thermodynamics

7. Example 3. Reversible, adiabatic expansion of ideal gas.

We are finally ready to calculate Q_{in} , W_{out} , V_2 and T_2 .

$Q_{in} = 0$ because the process is adiabatic.

$$\begin{aligned}W_{out} &= -\Delta U = mc_v(T_1 - T_2) \\&= mc_v T_1 \left(1 - \frac{T_2}{T_1}\right) = mc_v T_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right] \\&= \frac{5}{2} P_1 V_1 \left[1 - \left(\frac{P_2}{P_1}\right)^{\frac{2}{7}}\right] = \frac{5}{2} (1200 \text{ kPa})(0.002 \text{ m}^3) \left[1 - \left(\frac{1}{12}\right)^{\frac{2}{7}}\right] = 3.050 \text{ kJ}\end{aligned}$$

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II. Energy and the First Law of Thermodynamics

7. Example 3. Reversible, adiabatic expansion of ideal gas.

Calculate T_2 and V_2 .

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 300 \text{ K} \left(\frac{1}{12}\right)^{\frac{2}{7}} = 147.5 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\frac{R}{c_v}} \text{ or } \frac{V_1}{V_2} = \left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}}$$

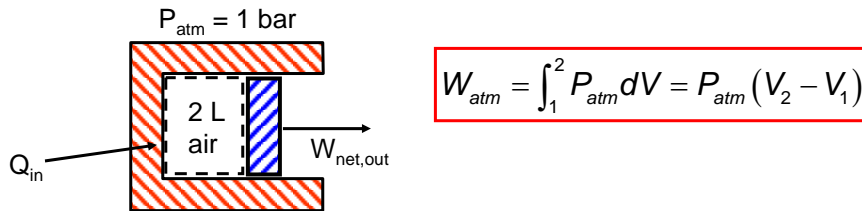
$$V_2 = \frac{V_1}{\left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}}} = \frac{0.002 \text{ m}^3}{\left(\frac{147.5}{300}\right)^{5/2}} = 0.01180 \text{ m}^3$$

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II. Energy and the First Law of Thermodynamics

8. Example 4. Work done against the atmosphere and net work available in Examples 2, 3.

In Examples 2 and 3, we have not considered the work done against the atmosphere. In both cases that will be



In Example 2, $W_{atm} = 100 \text{ kPa}(0.0240 - 0.002) \text{ m}^3 = 2.20 \text{ kJ}$

The net work available in Ex. 2 is $W_{net,out} = 5.96 - 2.20 = 3.76 \text{ kJ}$.

In Example 3, $W_{atm} = 100 \text{ kPa}(0.0118 - 0.002) \text{ m}^3 = 0.980 \text{ kJ}$

The net work available in Ex. 3 is $W_{net,out} = 3.05 - 0.98 = 2.07 \text{ kJ}$.

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II. Energy and the First Law of Thermodynamics

9. Example 5. Estimating the energy released in an **explosion**.

A 2-L, plastic soda bottle is filled with air at 11 bar (gage) and 300 K. The atmospheric pressure is 1 atm. The bottle suddenly explodes. Estimate the energy released.

Key assumptions: (1) ideal gas, (2) adiabatic. This process is highly **irreversible**.

First Law: $\Delta U = \overset{0}{Q}_{in} - W_{out}$

Work performed: $W_{out} = W_{atm} = P_{atm} (V_2 - V_1) = P_2 (V_2 - V_1)$

Ideal gas: $PV = mRT$ or $V_2 = \frac{mRT_2}{P_2}$

Change in U: $u = \frac{5}{2}RT$ or $\Delta U = mc_v(T_2 - T_1)$ where $c_v = \frac{5}{2}R$

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II. Energy and the First Law of Thermodynamics

9. Example 5. Estimating the energy released in an **explosion of a 2-L soda bottle**.

From first law: $\Delta U = -W_{out}$ or $mc_v(T_2 - T_1) = P_{atm}(V_1 - V_2)$

$$\text{Solving for } T_2: T_2 = T_1 \frac{\frac{P_2 + \frac{c_v}{R}}{P_1}}{1 + \frac{c_v}{R}} = \frac{12 + \frac{5}{2}}{1 + \frac{5}{2}} = 221.4 \text{ K}$$

$$\text{Energy released: } W_{out} = -\Delta U = mc_v(T_1 - T_2) = P_1 V_1 \frac{c_v}{R} \left(1 - \frac{\frac{P_2 + \frac{c_v}{R}}{P_1}}{1 + \frac{c_v}{R}} \right)$$

$$W_{out} = (1200 \text{ kPa})(0.002 \text{ m}^3) \frac{5}{2} \left(1 - \frac{12 + \frac{5}{2}}{1 + \frac{5}{2}} \right) = 1.57 \text{ kJ}$$

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II. Energy and the First Law of Thermodynamics

10. Summary of relations used to calculate work in ideal gas, **closed system (batch)**, compression and expansion processes.

Isobaric (constant pressure): $P = \text{constant}$

Isothermal: $PV = \text{constant}$

Adiabatic and reversible: $PV^k = \text{constant}$ where $k = \frac{R}{c_v} + 1$

Polytropic: $PV^n = \text{constant}$, where $1 < n < k$ (most of the time)

$$W_{out,poly} = \frac{P_2 V_2 - P_1 V_1}{1 - n} = \frac{mR(T_2 - T_1)}{1 - n}, n \neq 1$$

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II. Energy and the First Law of Thermodynamics

11. Final remarks on reversible and actual processes

- a. Examples 2 and 3 involved batch reversible **expansion** processes. Actual processes deliver less work than reversible processes. Define an **efficiency** to account for this:

$$W_{actual} = \eta W_{reversible}, 0 < \eta < 1, \text{ expansion}$$

- b. Actual compression processes require more work than reversible **compression** processes. Define an **efficiency** to account for this:

$$W_{actual} = \frac{W_{reversible}}{\eta}, 0 < \eta < 1, \text{ compression}$$

For batch compression and expansion processes, $\eta \cong 0.75$.
The efficiency must be determined experimentally.

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