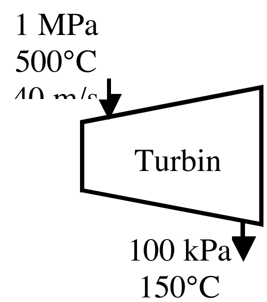
Homework # 9 Solutions

#1 5-60 Air is expanded in an adiabatic turbine. The mass flow rate of the air and the power produced are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** The turbine is well-insulated, and thus there is no heat transfer. **3** Air is an ideal gas with constant specific heats.

Properties The constant pressure specific heat of air at the average temperature of (500+150)/2=325°C=598 K is c_p = 1.051 kJ/kg·K (Table A-2*b*). The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1).

Analysis (*a*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{system}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) + \dot{W}_{out}$$

$$\dot{W}_{out} = \dot{m}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right) = \dot{m}\left(c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right)$$

The specific volume of air at the inlet and the mass flow rate are

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(500+273\ \text{K})}{1000\ \text{kPa}} = 0.2219\ \text{m}^3/\text{kg}$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(0.2\ \text{m}^2)(40\ \text{m/s})}{0.2219\ \text{m}^3/\text{kg}} = \mathbf{36.06\ kg/s}$$

Similarly at the outlet,

$$v_2 = \frac{RT_2}{P_2} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(150+273\ \text{K})}{100\ \text{kPa}} = 1.214\ \text{m}^3/\text{kg}$$

$$V_2 = \frac{\dot{m}v_2}{A_2} = \frac{(36.06\ \text{kg/s})(1.214\ \text{m}^3/\text{kg})}{1\ \text{m}^2} = 43.78\ \text{m/s}$$

(*b*) Substituting into the energy balance equation gives

$$\begin{aligned}\dot{W}_{out} &= \dot{m}\left(c_p(T_1 - T_2) + \frac{V_1^2 - V_2^2}{2}\right)\\ &= (36.06\ \text{kg/s})\left[(1.051\ \text{kJ/kg}\cdot\text{K})(500-150)\text{K} + \frac{(40\ \text{m/s})^2 - (43.78\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right)\right]\\ &= \mathbf{13{,}260\ kW}\end{aligned}$$

#3 5-122 An evacuated bottle is surrounded by atmospheric air. A valve is opened, and air is allowed to fill the bottle. The amount of heat transfer through the wall of the bottle when thermal and mechanical equilibrium is established is to be determined.

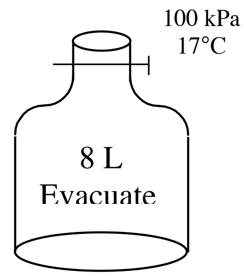
Assumptions **1** This is an unsteady process since the conditions within the device are changing during the process, but it can be analyzed as a uniform-flow process since the state of fluid at the inlet remains constant. **2** Air is an ideal gas with variable specific heats. **3** Kinetic and potential energies are negligible. **4** There are no work interactions involved. **5** The direction of heat transfer is to the air in the bottle (will be verified).

Properties The gas constant of air is 0.287 kPa.m^3/kg.K (Table A-1).

Analysis We take the bottle as the system, which is a control volume since mass crosses the boundary. Noting that the microscopic energies of flowing and nonflowing fluids are represented by enthalpy h and internal energy u, respectively, the mass and energy balances for this uniform-flow system can be expressed as

Mass balance:

$$m_{in} - m_{out} = \Delta m_{system} \rightarrow m_i = m_2 \quad (\text{since } m_{out} = m_{initial} = 0)$$



Energy balance:

$$\underbrace{E_{in} - E_{out}}_{\substack{\text{Net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta E_{system}}_{\substack{\text{Change in internal, kinetic,}\\ \text{potential, etc. energies}}}$$

$$Q_{in} + m_i h_i = m_2 u_2 \quad (\text{since } W \cong E_{out} = E_{initial} = ke \cong pe \cong 0)$$

Combining the two balances:

$$Q_{in} = m_2(u_2 - h_i)$$

where

$$m_2 = \frac{P_2 V}{RT_2} = \frac{(100\text{ kPa})(0.008\text{ m}^3)}{(0.287\text{ kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\text{ K})} = 0.0096\text{ kg}$$

$$T_i = T_2 = 290\text{ K} \xrightarrow{\text{Table A-17}} \begin{array}{l} h_i = 290.16\text{ kJ/kg} \\ u_2 = 206.91\text{ kJ/kg} \end{array}$$

Substituting,

$$Q_{in} = (0.0096\text{ kg})(206.91 - 290.16)\text{ kJ/kg} = -0.8\text{ kJ}$$

or

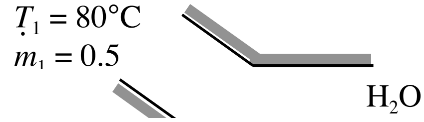
$$Q_{out} = \mathbf{0.8\ kJ}$$

Discussion The negative sign for heat transfer indicates that the assumed direction is wrong. Therefore, we reverse the direction.

#4 5-78 A hot water stream is mixed with a cold water stream. For a specified mixture temperature, the mass flow rate of cold water is to be determined.

Assumptions **1** Steady operating conditions exist. **2** The mixing chamber is well-insulated so that heat loss to the surroundings is negligible. **3** Changes in the kinetic and potential energies of fluid streams are negligible. **4** Fluid properties are constant. **5** There are no work interactions.

Properties Noting that T < $T_{sat\ @\ 250\ kPa}$ = 127.41°C, the water in all three streams exists as a



compressed liquid, which can be approximated as a saturated liquid at the given temperature. Thus,

$$h_1 \cong h_{f\,@\,80°\text{C}} = 335.02\ \text{kJ/kg}$$
$$h_2 \cong h_{f\,@\,20°\text{C}} = 83.915\ \text{kJ/kg}$$
$$h_3 \cong h_{f\,@\,42°\text{C}} = 175.90\ \text{kJ/kg}$$

Analysis We take the mixing chamber as the system, which is a control volume. The mass and energy balances for this steady-flow system can be expressed in the rate form as

Mass balance:

$$\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \Delta\dot{m}_{\text{system}}^{\nearrow 0\ \text{(steady)}} = 0 \quad\longrightarrow\quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

Energy balance:

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta\dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}_1h_1 + \dot{m}_2h_2 = \dot{m}_3h_3 \quad (\text{since } \dot{Q} = \dot{W} = \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

Combining the two relations and solving for $\dot{m}_2$ gives

$$\dot{m}_1h_1 + \dot{m}_2h_2 = (\dot{m}_1 + \dot{m}_2)h_3$$

$$\dot{m}_2 = \frac{h_1 - h_3}{h_3 - h_2}\dot{m}_1$$

Substituting, the mass flow rate of cold water stream is determined to be

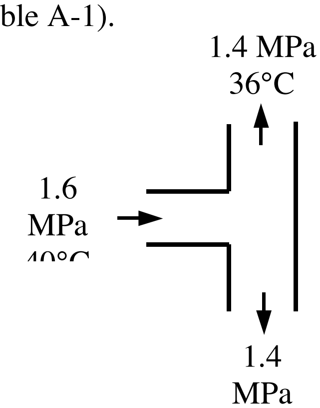
$$\dot{m}_2 = \frac{(335.02 - 175.90)\ \text{kJ/kg}}{(175.90 - 83.915)\ \text{kJ/kg}}(0.5\ \text{kg/s}) = \mathbf{0.865\ \ kg/s}$$

5-157 The mass flow rate of a compressed air line is divided into two equal streams by a T-fitting in the line. The velocity of the air at the outlets and the rate of change of flow energy (flow power) across the T-fitting are to be determined.

Assumptions **1** Air is an ideal gas with constant specific heats. **2** The flow is steady. **3** Since the outlets are identical, it is presumed that the flow divides evenly between the two.

Properties The gas constant of air is R = 0.287 kPa·m^3/kg·K (Table A-1).

Analysis The specific volumes of air at the inlet and outlets are



$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(40 + 273\ \text{K})}{1600\ \text{kPa}} = 0.05614\ \text{m}^3/\text{kg}$$

$$v_2 = v_3 = \frac{RT_2}{P_2} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(36 + 273\ \text{K})}{1400\ \text{kPa}} = 0.06335\ \text{m}^3/\text{kg}$$

Assuming an even division of the inlet flow rate,
the energy balance can be written as

$$\frac{A_1 V_1}{v_1} = 2\frac{A_2 V_2}{v_2} \longrightarrow V_2 = V_3 = \frac{A_1}{A_2}\frac{v_2}{v_1}\frac{V_1}{2} = \frac{0.06335}{0.05614}\frac{50}{2} = \mathbf{28.21\ m/s}$$

The mass flow rate at the inlet is

$$\dot{m}_1 = \frac{A_1 V_1}{v_1} = \frac{\pi D_1^2}{4}\frac{V_1}{v_1} = \frac{\pi(0.025\text{ m})^2}{4}\frac{50\text{ m/s}}{0.05614\text{ m}^3\text{/kg}} = 0.4372\text{ kg/s}$$

while that at the outlets is

$$\dot{m}_2 = \dot{m}_3 = \frac{\dot{m}_1}{2} = \frac{0.4372\text{ kg/s}}{2} = 0.2186\text{ kg/s}$$

Substituting the above results into the flow power expression produces

$$\begin{aligned}\dot{W}_{\text{flow}} &= 2\dot{m}_2 P_2 v_2 - \dot{m}_1 P_1 v_1 \\ &= 2(0.2186\text{ kg/s})(1400\text{ kPa})(0.06335\text{ m}^3\text{/kg}) - (0.4372\text{ kg/s})(1600\text{ kPa})(0.05614\text{ m}^3\text{/kg}) \\ &= \mathbf{-0.496\ kW}\end{aligned}$$

5. 6-9C No. Such an engine violates the Kelvin-Planck statement of the second law of thermodynamics.

6. 6-15C No. Such an engine violates the Kelvin-Planck statement of the second law of thermodynamics.