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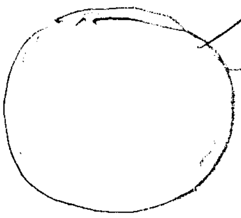
ME/CH EN 2300 FINAL EXAM

For any problem that requires use of specific heats, you may assume a constant value unless the problem statement indicates otherwise. To receive full credit for a problem, you must show all equations and all work. If a problem requires a system or control volume analysis, full credit will only be given if you **clearly indicate** the system or control volume that you are using. **Do not plug any numbers into your equations until the last step or until necessary.**

1. (18 pts) A flexible container has an initial volume of $0.01 m^3$ at a pressure of 100 kPa and an initial temperature of 27 C. The gas in the cylinder is air and can be treated as an ideal gas. The container expands to $0.02 m^3$ while the temperature is maintained at a constant value. Neglect kinetic and potential energy. (Most of the points for this problem can be gotten without plugging in any numbers)
   a) What is the net heat added/removed to/from the cylinder?
   b) What is the entropy change of the air in the cylinder?



flexible container – system

Initial $V_1 = .01 m^3$, $P_1 = 100 kPa$, $T_1 = 27 C$

Final $V_2 = .02 m^3$, $T_2 = 27 C$

$PV = mRT \Rightarrow m = \frac{PV}{RT} = \frac{100(.01)}{.287(300)} = .0116 kg$

a) Heat Added? Use 1st Law for C.V.

$\Delta E = Q - W \qquad \Delta E = \Delta U = 0$ since $T_2 = T_1$

so $Q = W$

$W = \int_{V_1}^{V_2} P\,dV$; const T, $P = \frac{mRT}{V}$ so $W = mRT\int_{V_1}^{V_2}\frac{dV}{V}$

$\boxed{\text{so } Q = mRT \ln\frac{V_2}{V_1} = .598 \text{ KJ or } 59.6 \text{ KJ/kg}}$ 10

entropy

$\Delta S = m\left[\int_{T_1}^{T_2} c_v \frac{dT}{T} + R\ln\frac{V_2}{V_1}\right]$ in KJ/K

$= mR\ln\frac{V_2}{V_1}$

or, for constant T

$\boxed{\Delta S = \frac{Q}{T}}\left(= mR\ln\frac{V_2}{V_1}\right)$ 8

.00233 KJ/K — same as — or .199 KJ/kg K

Grading:

a) $\Delta E = Q - W$ 2

$\Delta E = 0$ 2

$W = \int P\,dV = mRT\int\frac{dV}{V}$ 2

$Q = mRT\ln\frac{V_2}{V_1}$ 4

b) $\Delta S =$ 3

simplify $= R\ln\frac{V_2}{V_1}$ 4

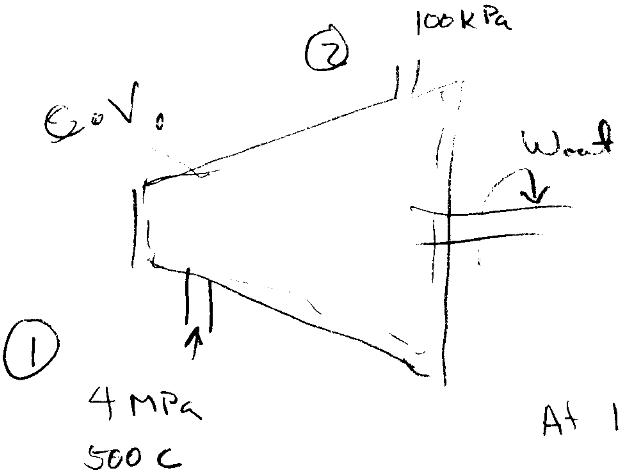
numbers correct 1

units wrong (−2)

not T_{abs} − 2

2. (18 pts) Steam (superheated vapor) enters a turbine at a pressure of 4 MPa and temperature of 500 C. The turbine can be treated as adiabatic with negligible differences in kinetic energy and potential energy between the incoming and outgoing flow. Water leaves the turbine as a saturated MIXTURE at 100 kPa. The turbine runs at its maximum theoretical efficiency. (You will have to correctly obtain quantitative numbers to get many of the points on this problem)

a) What is the quality of the steam at the exit?
b) Determine the work output of the turbine in kJ/kg.



max efficiency = isentropic

so $S_1 = S_2$

From superheated tables

$S_1 = 7.0922 = S_2$

At 100 kPa, $S_f = 1.3028$ $S_g = 7.3589$

$S_2 = (1-x)S_f + xS_g$

$= S_f - xS_f + xS_g = S_f + x(S_g - S_f)$

$$\Rightarrow \boxed{x = \frac{S_2 - S_f}{S_g - S_f} = \frac{7.0922 - 1.3028}{7.3589 - 1.3028} = \frac{5.7894}{6.0561} = .95}$$

b) Work Use energy for C.V.

$$\frac{dE}{dt} = \dot{Q} - \dot{W} + \dot{m}_{in}\left(h_{in} + \frac{V_{in}^2}{2} + gz_{in}\right) - \dot{m}_{out}\left(h_{out} + \frac{V_{out}^2}{2} + gz_{out}\right)$$

$\dot{W} = \dot{m}_{in}(h_{in} - h_{out})$ or $w = (h_{in} - h_{out})$

from superheated table, $h_{in} = 3446$

$h_{out} = x h_g + (1-x) h_f$

$= .95(2675) + .05(417.5)$

$= \underline{2562 \text{ kJ/kg}}$

$w = 3446 - 2562 = 883$ kJ/kg

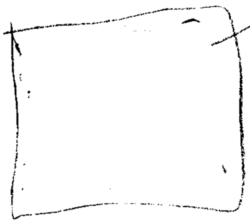
Grading,	
max eff = isentrop.	2
S_1 from table	2
$S_1 = S_2$	2
Quality for (2)	3
Energy Equation	2
Reduce correct	3
h out correct	4

3. (18 pts) 5 liters of water, initially at 27C are cooled to 5 C in a perfectly insulated refrigerator. The refrigerator has a power input of 500 W and has a COP of 2.0. Assuming infinitely fast heat transfer between the water and the air in the refrigerator, how long will it take the water to cool to 5 C?

$$COP = \frac{Q_{out}}{W_{in}}$$

Q: From energy $\Delta E = Q - W$ $\Delta E = \Delta U$

no W on water



5 liters of water is C.V.

So $Q = \Delta U = m \Delta u$

$= mc(T_1 - T_2)$

So $W_{in} = (2)(mc(T_1 - T_2))$ $c = 4.22$ kJ/kg K

$= 928$ kJ $m = 5$ kg

$W_{in} = \dot{W}(t)$

So $t = \frac{W_{in}}{\dot{W}} = \frac{(928 \text{ kJ})}{.5 \text{ kW}} = 464$ sec $= 7.7$ min

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Grading: $COP = \frac{Q_{out}}{W_{in}}$ 4

units correct
#s correct } 3

from energy $Q = \Delta U$ 2

$= mc(T_1 - T_2)$ 4

$W_{in} = 2(mc(T_1 - T_2))$ 2

$t = \frac{W_{in}}{\dot{W}}$ 3

4. (18 pts) Superheated water vapor is at a temperature of 374 C and 11.3 MPa.

a) Can the steam be treated as an ideal gas? Explain.
b) What is the error in computing the specific volume of the steam if it is treated as an ideal gas?

For ideal gas need to have

$P_R = \frac{P}{P_{cr}} << 1$

or $T_R = \frac{T}{T_{CR}} > 2$

For water: $P_{cr} =$ ~~647.00~~ 22.06 MPa

$T_{cr} = 647.1$ K

So $P_R = .5$

$T_R = 1$

Doesn't satisfy criteria

Error:
Look at Compressibility Chart with
$P_R = .5$ & $T_R = 1$

gives $Z = \frac{Pv}{RT} = .81$ from A-15

so $v_{real} \approx .81\, v_{ideal}$

% error $= 1 - .81 = \underline{19\%}$

or actually $\frac{\text{true} - \text{calcul}}{\text{true}} = 25.8\%$

grading

P_{cr} 1

T_{cr} 1

P_R 2

T_R 2

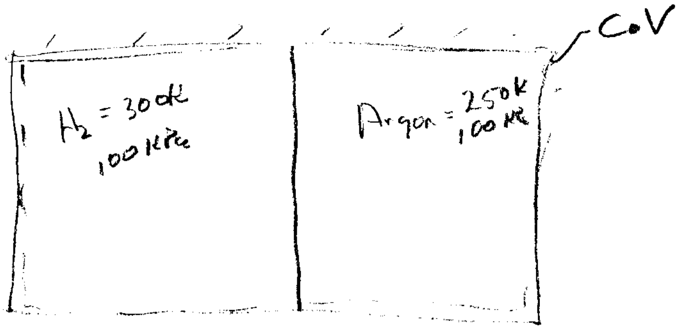
criteria 4

Use of Z chart (or other harder way) 4

percentage 4

10

5. (18 pts) A 0.1 m^3 insulated container is divided in two equal volumes. One half contains hydrogen at a temperature of 300 K and a pressure of 100 kPa. The other half contains argon at a temperature of 250 K and a pressure of 100 kPa. If the partition between the two halves is removed and the gasses are allowed to come into thermal equilibrium, what is the final temperature of the mixture?



Grading

Energy Equation

$\Delta E = Q - W$ — no work, heat transfer so $\Delta E = \Delta U = 0$ — 2

$\Delta E = m_{H_2} \Delta u_{H_2} + m_{Ar} \Delta u_{AR} = 0$ — 2

$0 = m_{H} C_{v_{H_2}} (T_2 - T_{1_{H_2}}) + m_{AR} C_{v_{AR}} (T_2 - T_1)_{AR}$ — 4

$m_{H_2} = \dfrac{P_1 V_1}{R_{H_2} T_{1_{H_2}}} = \dfrac{(100\ kPa)(.05\ m^3)}{(4.12)(300\ K)} = .004\ kg$ — 2

$m_{AR} = \dfrac{P_1 V_1}{R_{AR} T_{1AR}} = \dfrac{(100\ kPa)(.05\ m^3)}{(.208)(250\ K)} = .09\ kg$ — 2

$C_{v_{H_2}} = 10.18 \quad C_{v_{AR}} = .312\ kJ/kg\ K$ — 2

Solve for T_2 — 3 (1 + 2 correct)

$T_2 = \dfrac{m_{H_2} C_{v_{H_2}} T_{1_{H_2}} + m_{AR} C_{v_{AR}} T_{1AR}}{m_{H_2} C_{v_{H_2}} + m_{AR} C_{v_{AR}}} = \dfrac{12.21 + 7.02}{.0407 + .028}$

$= 280\ K$

6. (10 pts) Water is at a pressure of 200 kPa and has an internal energy of 1000 kJ/kg.

a) What is the temperature of the water? Explain.
b) What is the entropy of the water (in kJ/kg K)?
c) If the internal energy of the water is lowered by 500 kJ/kg, what is the state of the water? Explain.

a) @ 200 kPa
u of 1000 is between
u_f & u_g so
Saturated

$T = 120.21$

a Correct table lookup & explanation
3

b) compute quality and find s 4

b) Need to get quality

$1000 = x(2529.1) + (1-x)\,504.5$

$\boxed{x = .2447}$

$s = x s_g + (1-x) s_f$

≈ 2.9 kJ/kg K

c) correct explanation 3

c) @ 200 kP, if u lowered to 500 k, will be a liquid.