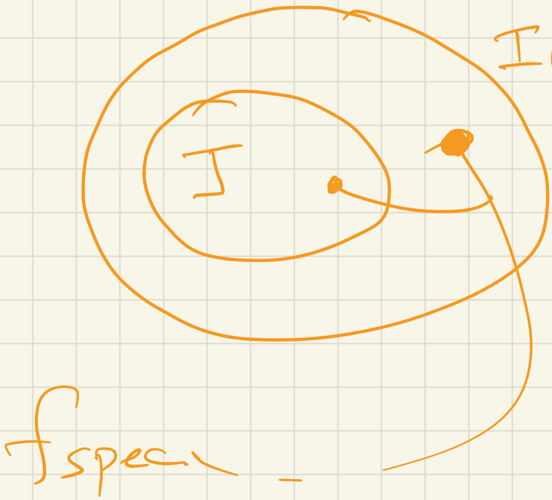
Nov 22.

Recap: over $\mathbb{F}_q[x_1, \dots, x_n]$



$$I(V_{\mathbb{F}_q}(J)) = J + J_0$$

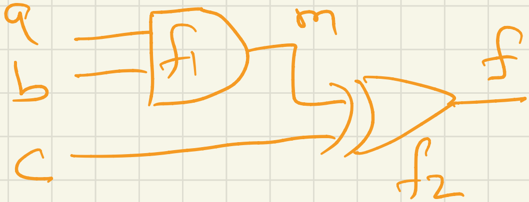
$J = \langle$ polynomials from the ckt $\rangle$

J_0 = ideal of vanishing polynomials.

$$I[V_{\mathbb{F}_q}(J)] = I[V_{\overline{\mathbb{F}}_q}(J + J_0)]$$

$$= \sqrt{J + J_0} \quad [\because \text{SNS}]$$

$$= J + J_0 \quad \left[\text{because } J + J_0 = \text{radical}\right]$$



$\underline{\mathbb{F}_2[a, b, c, m, f]}$

$f_1: m = a \wedge b \longrightarrow m = a \cdot b \rightarrow m + a \cdot b$

$f_2: f = m \oplus c \longrightarrow f = m + c \rightarrow f - (m + c)$

$J = \langle f_1, f_2 \rangle$, $J_0 = \langle a^2 - a, \dots, f^2 - f \rangle$

$V_{\mathbb{F}_2}(J) = ?$

a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

$$= \left\{ \begin{array}{ccccc} (a & b & c & m & f) \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & \vdots & 0 \\ & & \vdots & \vdots & \\ 1 & 1 & 1 & 1 & 0 \end{array} \right\}$$

$= V_{\mathbb{F}_2}(J) = V_{\mathbb{F}_2}(J + J_0)$

$f_{spec}: f = a \cdot b \oplus c$

$f_{spec}: f - (ab + c)$

Does f_{spec} vanish on $V_{F_2}(J)$?

$f_{spec}: f + ab + c.$

$$V_{F_2}(J) : \left\{ \begin{array}{ccccc} (a & b & c & m & f) \\ 0 & 0 & 0 & 0 & 0 \\ - & - & - & - & - \\ 1 & 1 & 1 & 1 & 0 \end{array} \right\} \begin{array}{l} \to p_1 \\ \\ \to p_8 \end{array}$$

$f_{spec}(p_1) = 0 + 0 \cdot 0 + 0 = 0.$

$- - - - -$

$f_{spec}(p_2) = 0$

$- - - - - -$

$f_{spec}(p_8) = 0 + 1 \cdot 1 + 1 = 0$

So, $f_{spec} \equiv c \iff f_{spec}$ vanishes everywhere on $V(J)$

$f \in I(V(J)) \longleftrightarrow$

$\in J + J_0.$

Product criteria

$f_1 \quad x_2 + x_0 x_1 \qquad\qquad f_2 : x_3 + x_1 x_2$

$$S(f_1, f_2) = \frac{L}{LT(f_1)} \cdot f_1 - \frac{L}{LT(f_2)} \cdot f_2$$

$$L = LCM(LM(f_1), LM(f_2))$$

$$= LCM(x_2, x_3) = x_2 \cdot x_3$$

$$\frac{\cancel{x_2} x_3}{\cancel{x_2}} (x_2 + x_0 x_1) - \frac{x_2 \cancel{x_3}}{\cancel{x_3}} (x_3 + x_1 x_2)$$

$$= \cancel{x_3 x_2} + x_0 x_1 x_3 - \cancel{x_2 x_3} - x_1 x_2^2$$

$$= f_1 f_2$$