Lecture 3 - Boolean Algebra contd.

- Shannon's Expansion: $f = xf_x + x'f_{x'}$
- If $f_x \supseteq f_{x'}$, f is +ve unate in x
- Reduction due to unateness: $f = xf_x + f_{x'}$
- Corollary of Shannon's Theorem:
  - $f \cdot g = x \cdot f_x \cdot g_x + x' \cdot f_{x'} \cdot g_{x'}$
- If f, g are +ve unate in x
  - $f \cdot g = x \cdot f_x \cdot g_x + f_{x'} \cdot g_{x'}$
- Significance?

Binate Functions

- $f = a'b'c + a'bc' + abc$ and $g = abc + a'b'c'$
- Are f and g unate in any variable?
- $f \cdot g = af_a g_a + a' f_{a'} g_{a'}$
- $f_a =$ $\qquad\qquad\qquad\qquad$ $f_{a'} =$
- $g_a =$ $\qquad\qquad\qquad\qquad$ $g_{a'} =$
- Is f_a +ve unate in b? ($f_{ab} \supseteq f_{ab'}$)?
- Is g_a +ve unate in b? ($g_{ab} \supseteq g_{ab'}$)

Shannon's expansion - Application

- $f = ab + ac + bc, g = ab' + ac'$
- $f_a = b + c; g_a = b' + c'$
- $f_{a'} = bc, g_{a'} = b'c'$
- $f \cdot g$ is compute intensive: Apply Shannon's theorem
- $f \cdot g = a \cdot f_a \cdot g_a + f_{a'} \cdot g_{a'}$
- Let us try to find solution in $f_{a'} \cdot g_{a'}$
- $f_{a'} \cdot g_{a'} = 0$. Does that mean $f \cdot g == 0$?
- $f_a \cdot g_a =$

What if NO Solution Exists?

- $f = ab + ac + a'bc, g = ab'c'$
- $f \cdot g = 0$ (obviously)
- Suppose $f \cdot g$ is compute intensive
- $f_a =$ $f'_a =$
- $g_a =$ $g'_a =$
- $f \cdot g = a \cdot f_a \cdot g_a + f_{a'} \cdot g_{a'}$
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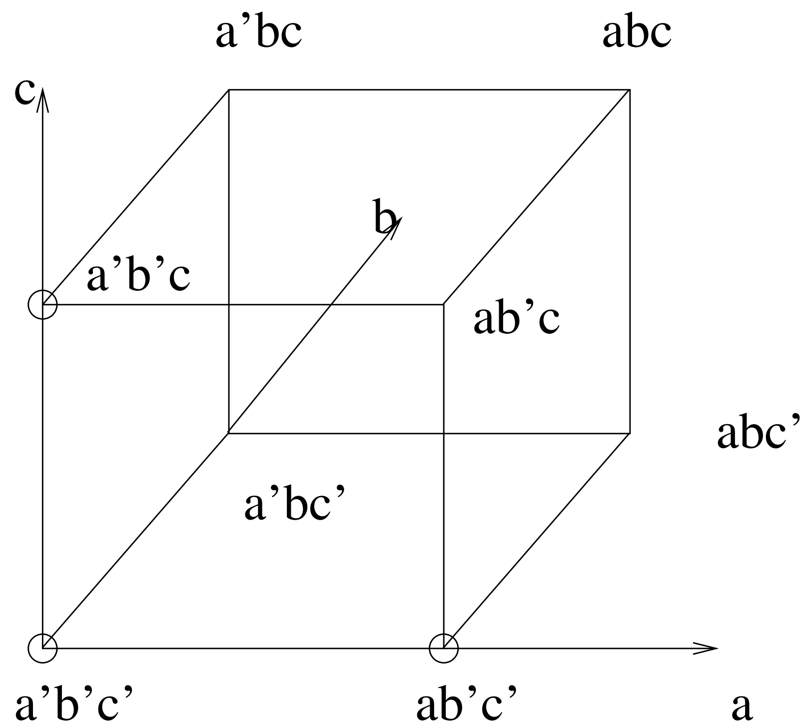
Boolean Difference

- Computed as $f_x \oplus f_{x'}$
- If $f_x \oplus f_{x'} = 0$, then $f_x = f_{x'}$
- When is $f_x = f_{x'}$?
- $f = ab + ac + bc$, $f_a \oplus f_{a'} = b'c + bc'$
- As a changes, f changes if $b'c + bc' = \text{TRUE}$


a'bc
abc
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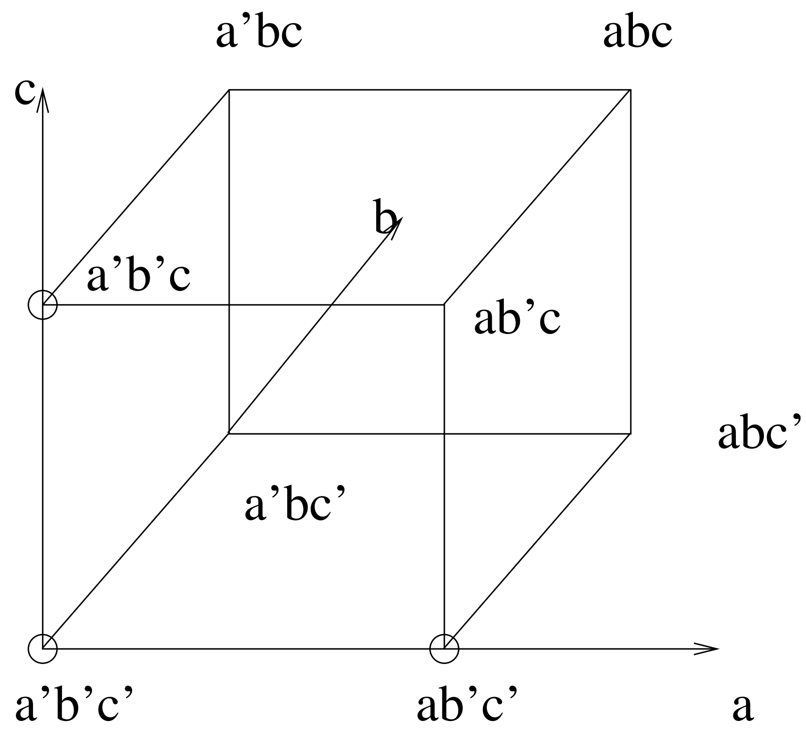
abc'
a'bc'
a'b'c'
ab'c'
a

Consensus and Smoothing

- Consensus: $f_x \cdot f_{x'}$
- Represents the component in f independent of x
- $f = ab + bc + ac$, consensus w.r.t. $a =$
- Smoothing: $f_x + f_{x'}$
- Makes the function independent of that variable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