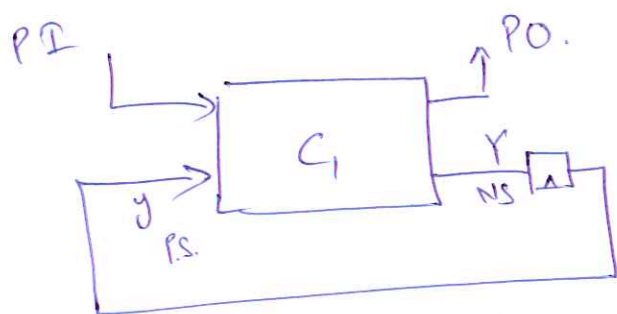


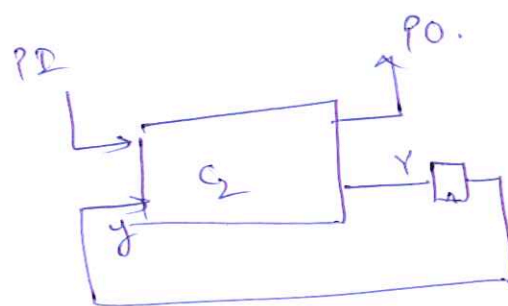
# Sequential Circuit Verification

①

Given two sequential circuits, starting from their respective initial states, do they have the same I-O-response? i.e., for all possible input sequences, do they produce the same output sequence?



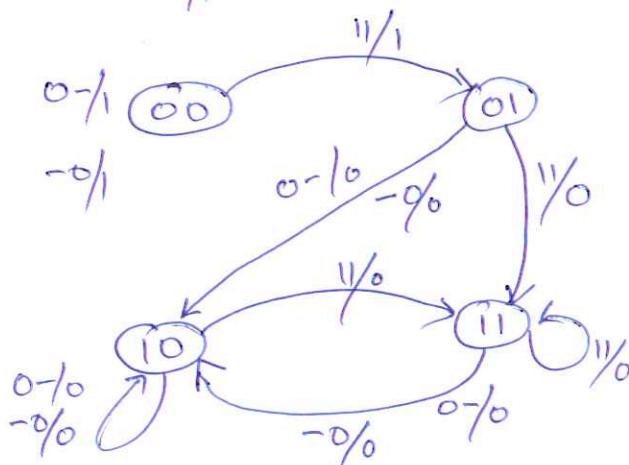
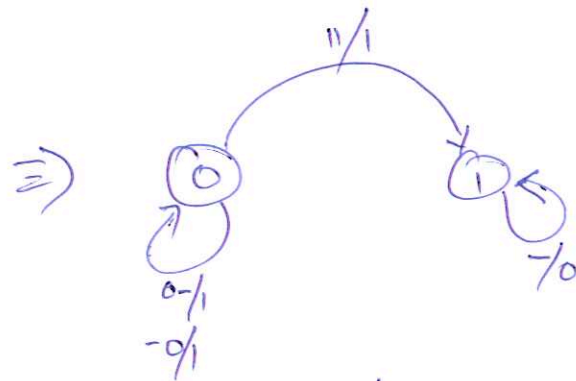
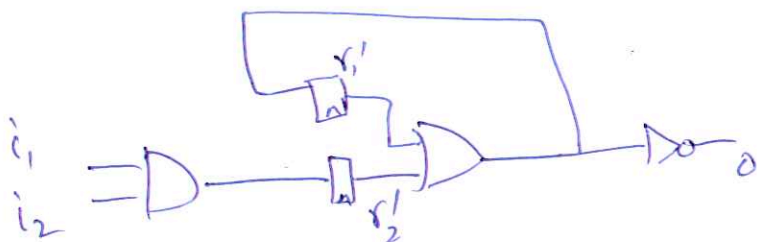
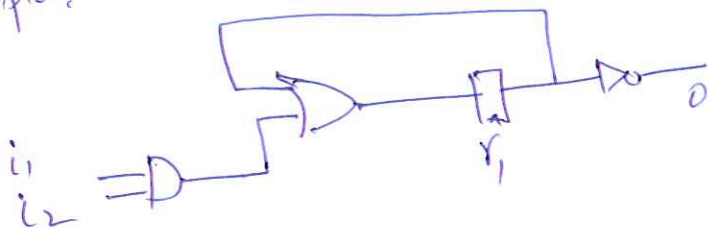
Seq. ckt. S1



seq. ckt. S2.

$C_1$  may not be equivalent to  $C_2$ ,  
but  $S_1 \equiv S_2$ .

Examples:



# Sequential Equivalence required! -

(2)

## ① Sequential optimizations

- retiming changes the # of D-FFs.

- code-reassignment

~~Code I~~ Code I

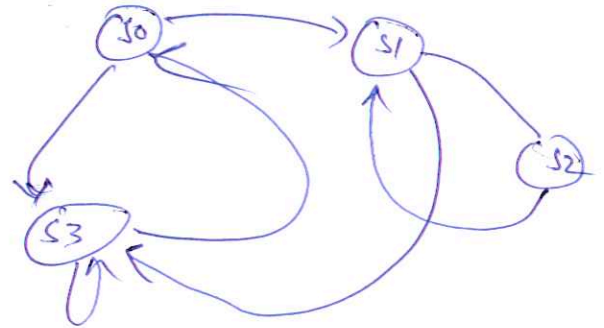
S0 = 00

S1 = 01

S2 = 10

S3 = 11

FSM



code II

S0 = 11

S1 = 10

S2 = 00

S3 = 01

but I-O behaviour  
is the same.

but combinational logic  $\neq$  same.

## ② New problems.

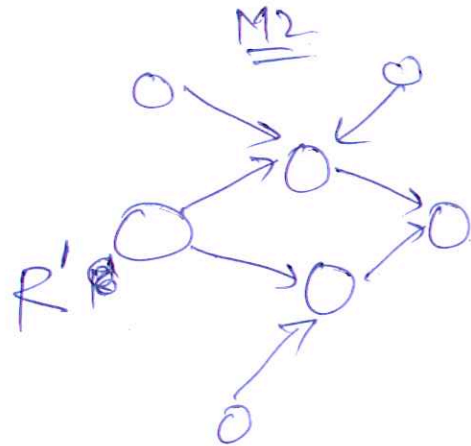
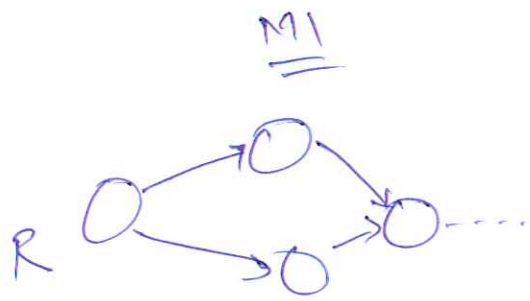
equivalence of symbolic C++ model

$\equiv$  RTL (clocked Verilog)

This problem slightly different from "testing" (sequential ATPG).

Seq. Verif  $\equiv$  state space analysis.

(3)



R & R'  $\equiv$  corresponding initial states.

Machine M1  $\equiv$  M2 if

(1) Both are identical

(2) Identical states, but different encodings

(3)  $M_1 \subseteq M_2$  or  ~~$M_1 \supseteq M_2$~~   $M_1 \supseteq M_2$

(4) Different reachable states, but same distinguishable states.

(5) Different unreachable states, but unreachable states = Don't care.

our problem: -

(4)

① given two FSMs (i.e. given their state tables or graphs)  
 $M_1$  &  $M_2$   
 $M_1 \equiv M_2?$

② Given two circuits, but not their state tables, are they equivalent?

① → "easier"

② → extract the underlying FSMs, & then prove equivalence.

Do this efficiently as a CAD solution.

Requires: - "Implicit state enumeration."

- Product FSM

- Implicit state enumeration, BFS-traversal

- BDDs.

- equivalence check.

FSM defined as:

(5)

$$M = (\Sigma, O, S, S^0, \Delta, \Lambda)$$

$\Sigma$  = input label

$O$  = output label

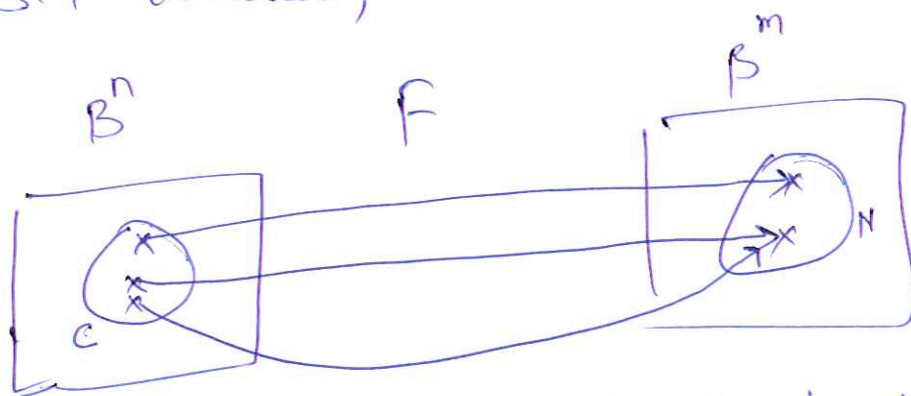
$S$  = set of states

$S^0 \subset S$  = initial (reset) state(s)

$\Delta: S \times \Sigma \rightarrow S$ , next state transition function.

$\Lambda: S \times \Sigma \rightarrow O$ , output function.

For FSM traversal, we will use image computation.



Domain.

$n$ -inputs

Co-domain

$m$ -outputs.

$$N = \text{Image of } F \text{ under } C = \text{Image}(F, C)$$

First we will understand how to ~~store~~ (6).

Solve  $M_1 \equiv M_2$  assuming STGs are given.

Then, we will see how to apply those concepts on a circuit.

### Product Machine

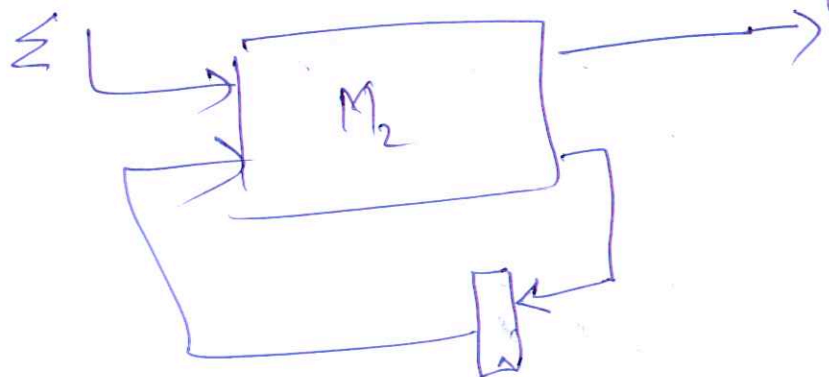
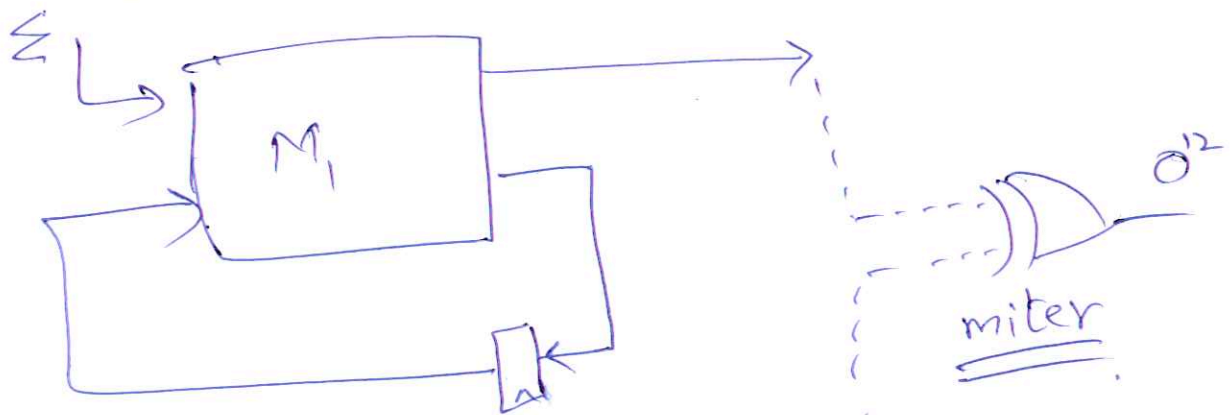
$$M_1 = \{ \Sigma, O, S^1, S^{01}, \Delta^1, \Lambda^1 \}$$

$$M_2 = \{ \Sigma, O, S^2, S^{02}, \Delta^2, \Lambda^2 \}$$

$$M_{12} = \{ \Sigma, O^{12}, S^{12}, S_0^{12}, \Delta^{12}, \Lambda^{12} \}$$

$$M_{12} = M_1 \times M_2$$

Product machine.



$$s^1 \in \Sigma^1, \quad s^2 \in \Sigma^2$$

Product of states  $s^{12} = s^1 \times s^2 = (s^1, s^2)$

(concatenation).

$$\Delta^1(s^1, x) \quad \& \quad \Delta^2(s^2, x)$$

( $x \in \Sigma$  = input)

$$\Delta^{12} = \Delta^{12}(s^{12}, x) : (s^1 \times s^2) \times \Sigma \rightarrow (s^1 \times s^2)$$

$$\Rightarrow [ (s^1 \times \Sigma) \rightarrow s^1 ] [ (s^2 \times \Sigma) \rightarrow s^2 ]$$

$$\Rightarrow [ \Delta^1(s^1, x), \Delta^2(s^2, x) ]$$

$\Rightarrow$  conjunction of transition relations.

Output  $\Lambda^{12} = \Lambda^{12}(s, x) : (s^1 \times s^2) \times \Sigma \rightarrow \{0, 1\}$ .

$$\Lambda^{12} = 1 \quad \text{if} \quad \lambda^1(s^1, x) = \lambda^2(s^2, x)$$

= 0 otherwise.