From "An Introduction to Gröbner Bases", by Adams & Loustaunau, pg. 27, Sec 1.5

Divide $y^2x + 4yx - 3x^2$ by $2y + x + 1$

Term order Deglex $y > x$ $\left[r = f - \frac{Lt(f)}{Lt(g)} \cdot g \right]$

$$\begin{array}{r|l} & \frac{1}{2}yx - \frac{1}{4}x^2 \\ \hline 2y+x+1 & y^2x + 4yx - 3x^2 \\ & y^2x + \frac{1}{2}yx \qquad +\frac{1}{2}yx^2 \\ & \hline & 0 + \frac{7}{2}yx - 3x^2 - \frac{1}{2}yx^2 \end{array}$$

cancel $Lt(f)$ → $0 + \frac{7}{2}yx - 3x^2 - \frac{1}{2}yx^2$

re-order → $-\frac{1}{2}yx^2 + \frac{7}{2}yx - 3x^2$ = 1-step remainder

$$\begin{array}{r|l} & -\frac{1}{2}yx^2 + \frac{7}{2}yx - 3x^2 \\ \hline 2y+x+1 & -\frac{1}{2}yx^2 \qquad +\frac{1}{4}x^2 \qquad +\frac{1}{4}x^3 \\ & \hline & \frac{7}{2}yx - \frac{11}{4}x^2 + \frac{1}{4}x^3 \end{array}$$

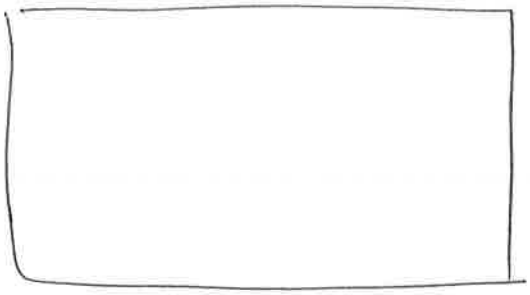
reorder → $\frac{1}{4}x^3 + \frac{7}{2}yx - \frac{11}{4}x^2$

↳ can't be cancelled, move it to the remainder

Try to cancel $\frac{7}{2}yx$

continued.....

$$\begin{array}{r|l} & \frac{1}{2}yx - \frac{1}{4}x^2 + \frac{7}{4}x \\ \hline 2y+x+1 & y^2x + 4yx - 3x^2 \end{array}$$



$$\frac{1}{4}x^3 + \frac{7}{2}yx - \frac{11}{4}x^2$$

$$+\frac{7}{2}yx \quad +\frac{7}{4}x^2 \quad +\frac{7}{4}x$$

$$-\quad\quad -\quad\quad -$$

$$\frac{1}{4}x^3 \quad - \frac{9}{2}x^2 - \frac{7}{4}x = r$$

final remainder, as no term in r can be cancelled by $LT(g) = 2y$

Note: $\deg(r) > \deg(g)$

$=3 \quad\quad =1$

lex $x > y$: $f = -3x^2 + xy^2 + 4xy$

$g = x + 2y + 1$

Now try division of same $f \div g$ but use this term order

$$-3x + y^2 + 10y + 3$$

$x+2y+1 \;\big|\; -3x^2 + xy^2 + 4xy$

$-3x^2 \quad -6xy \quad -3x$

$+ \qquad + \qquad +$

$xy^2 + 10xy + 3x$

$-xy^2 \qquad \pm 2y^3 \pm y^2$

$10xy + 3x - 2y^3 - y^2$

$-10xy \qquad \pm 20y^2 \pm 10y$

$3x - 2y^3 - 21y^2 - 10y$

$-3x \qquad \pm 6y \pm 3$

$-2y^3 - 21y^2 - 16y - 3$

final remainder.

Change term order $\Rightarrow$ change $Lt(f)$ & $Lt(g)$ $\Rightarrow$ Change quotient and remainders.