HW6 Solutions

Q1 P.7, ch 11.

(a) — SDC are straightforward.

$$SDC_{y_2} = y_2 \oplus ac \quad \text{and so on....}$$

(b) $ODC_{y_4} = (y_5 = 0 \;\&\; y_2 = 1)$

$$= \bar{y}_5 + y_2$$

$$= ab + ac$$

$ODC_{y_5} = y_4 = 0, \; y_2 = 1$

$$= \bar{y}_4 + y_2$$

$$= \overline{a\bar{c}} + ac = \bar{a} + c + ac = \bar{a} + c$$

(c) $F_4 = a\bar{c}$

$$F_4 + ODC_{y_4} = a\bar{c} + \cancel{ab} + ac = \underline{\underline{a}}$$

Don't care

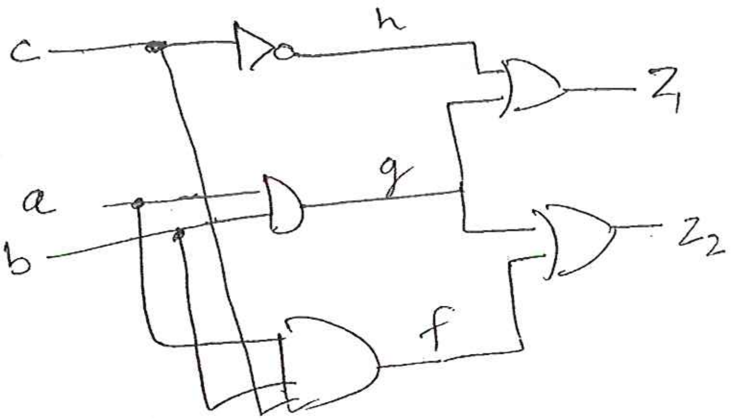
$$F_5 = \overline{ab} = \bar{a} + \bar{b}$$

$$F_5 + ODC_{y_5} = \bar{a} + \bar{b} + \bar{a} + c$$

D. C

$$= \bar{b}$$

Q2



ODC $h \equiv g=1 = ab$

ODC g @ Z_1 $\equiv h=1 = \bar{c}$

ODC g @ Z_2 $\equiv f=1 = abc$

ODC f @ Z_2 $\equiv g=1 = ab.$

Simplify h w/ ODC $\quad h + ODC_h$

$= \bar{c} + ab \quad = \bar{c}$

No simplification.

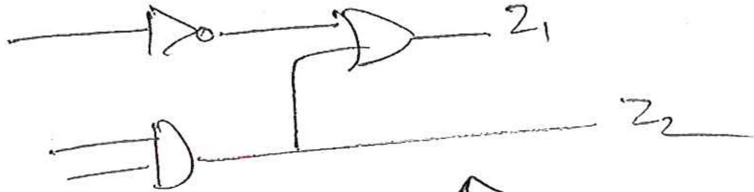
Simplify $g = ab$ w/ $\bar{c}$ or abc $\longrightarrow$ No simplificatio

Simplify f w/ $ODC_f = ab$

$= abc + ab = ab(c+1) = ab.$

But $ab = D.C$

So $f = 0$

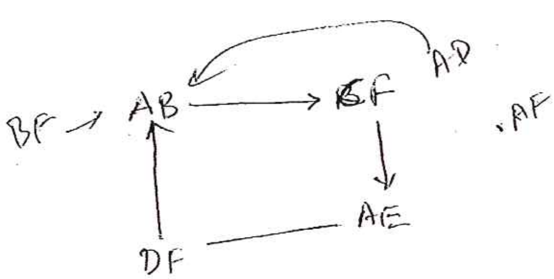


Simplified CKT

Q4

machine M1

	A	B	C	D	E
B	CF ✓				
C	×	×			
D	FD ✓	~~CD~~	×		
E	FD ✓	×	×	×	
F	✓	AB ✓	AE ✓	AB ✓	×



Compatibles:-

AB, AD, AE, AF

BF, CF, DF

$= \underline{(ADF)\ (AB)\ (AE)\ (BF)\ (CF)}$

(AF) × ×

$= \underbrace{(ADF)(ABF)(AE)(CF)}_{\text{Upper bound on \# of states}}$

Note that the set of maximal compatibles is itself the Solution ⟶ all of these are needed for covering

2. M1 is an incompletely specified machine. The first thing that you need to do is to identify compatible pairs. You can use either the merger table, or the merger graph. If you build the merger table, you can see that the only compatible (BD) gets cancelled because its implied pair (CD) is not compatible.
Compatibles are: (AF)(AE)(AD)(AB)(BF)(CF)(DF). Max compatibles: (ABF) (ADF) (CF) (AE).
Note that (AE) and (CF) are the only compatibles in the maximal set that contain C and E. Therefore, they have to be in the solution. That takes care of A, C, E, F. What about B and D? They are also in different compatible groups. Clearly, all 4 have to be in the solution. Now the question is whether their implied pairs are also in the maximal compatible sets? That they are.
Note: From the maximal compatible set, we can find a minimum number of compatibles that cover the entire machine. This guarantees covering, but not closure. On the other hand, the set containing all maximal compatibles is clearly a closed covering. Therefore, this is an upper bound on the # of states. However, in general, we may be able to get a better solution - a smaller set. And thats why we need to go into that compatibility graph and all that we studied in class.
In our current example, we don't need to solve the problem on the compatibility graph. This is because, from the maximal compatible set we can figure out that we need all maximal compatibles just to cover the machine. And since the set of all maximal compatibles also guarantees closure, we can just use the set of all maximal compatibles as our solution.

Q4 (M2)

Compatible pairs:- AB, AE, BC, BD, CD

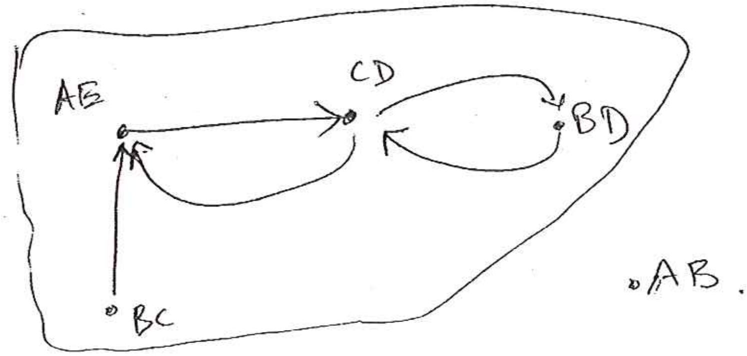
	A	B	C	D
B	✓			
C	✗	AE		
D	✗	CD	BD, AE	
E	CD	✗	✗	✗

AB, AE, BCD.

↓

cover Yes!

but closure?



Cover 1 :→ AE, CD, BD → 3 states.

Cover 2 :→ BC, AE, CD, BD

(BCD), (AE)

↑

2 states.

Q4 Now let us solve M2 using BCP.

Max. comp :- BCD , AB , AE

primes

P_1 ✓ BCD $\longrightarrow$ AE

P_2 ✓ AB $\longrightarrow$ ϕ

P_3 ✓ AE $\longrightarrow$ CD

BC $\longrightarrow$ AE ✗ not prime , $\subseteq P_1$

P_4 ✓ BD $\longrightarrow$ CD

CD $\longrightarrow$ BD, AE ✗ not prime, P_1 = cheaper.

A $\longrightarrow$ ϕ ✗ not prime , $\subseteq P_2$

B $\longrightarrow$ ϕ " "

P_5 ✓ C $\longrightarrow$ ϕ

P_6 ✓ D $\longrightarrow$ ϕ

P_7 ✓ E $\longrightarrow$ ϕ

State A :- $(P_2 + P_3)$

B :- $(P_1 + P_2 + P_4)$

C :- $(P_1 + P_5)$

D :- $(P_1 + P_4 + P_6)$

E :- $(P_3 + P_7)$

Cover

$P_1 \rightarrow P_3 \equiv \overline{P_1} + P_3$ C1

$P_3 \rightarrow$ CD $\subseteq$ BCD

$P_3 \rightarrow P_1 \equiv \overline{P_3} + P_1$ C2

BD $\rightarrow$ CD

$P_4 \rightarrow P_1$

$\overline{P_4} + P_1$ C3

Set-up BCP for M2.

Columns = prime compatibles $P_1 \ldots P_7$

rows = ~~clos~~ state covering + closure constraints

Select a minimum # of primes (columns) s.t. all constraints (rows) are SAT.

		P_1	P_2	P_3	P_4	P_5	P_6	P_7	
Covering	A	–	1	1	–	–	–	–	(P_2+P_3)
	B	1	1	–	1	–	–	–	$(P_1+P_2+P_4)$
	C	1	–	–	–	1	–	–	(P_1+P_5)
	D	1	–	–	1	–	1	–	$(P_1+P_4+P_6)$
	E	–	–	1	–	–	–	1	(P_3+P_7)
Closure	C1	0	–	1	–	–	–	–	$(\overline{P_1}+P_3)$
	C2	1	–	0	–	–	–	–	$(\overline{P_3}+P_1)$
	C3	1	–	–	0	–	–	–	$(\overline{P_4}+P_1)$

No essentials, no dominance

$\Rightarrow$ cyclic. M

Now compute MIS.

ignore rows with zeros (C_1, C_2, C_3)

$$L.B = MIS = \left\{ \overbrace{\begin{matrix} \text{row A} \\ \& \\ \text{row C} \end{matrix}}^{=2} \text{ or } \begin{matrix} \text{row B} \\ \& \\ \text{row E} \end{matrix} = 2 \right\} = \underline{\underline{2}}$$

maximally independent set of constraints.

Branch on P_1 (greedy, most entries)

$P_1 = 1$

	P2	P3	P7
A	1	1	—
E	—	1	1
C1	—	1	—

P_3 = essential

$P_1 + P_3 = \underline{L.B = 2}$

Stop.

P_1 & P_3 = solution.

(BCD) (AE)

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2 state M/C.

Q 6 Rectification   Bug $\Rightarrow$ $e = a \oplus b$, instead of $e = a\bar{b}$

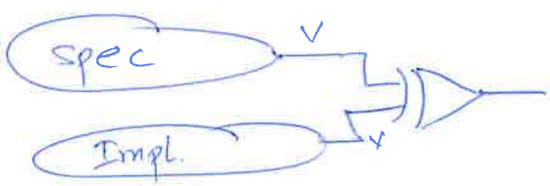
$\rightarrow$ (d) Rectification check at net e itself

Note that bug does not affect 'u'. It affects outputs v & w. Since $w = v + d$, a fix at e or v will also fix w. So, for simplicity, I will only consider output 'v' to construct miters.

Spec: $v = g + e = (a+b)c + a\bar{b}$

Imp: $v = (a+b)c + (a \oplus b)$ ↑ bug @ e

Let M = miter spec v/s. Imp. w.r.t. output 'v'



$M = \text{spec} \oplus \text{Impl.}$

$$M_0 = M(e=0) = [\underbrace{(a+b)c}_{e=0}] \oplus [(a+b)c + a\bar{b}]$$

Let $(a+b)c = x$, $M_0 = x \oplus [x + a\bar{b}]$

$$= \bar{x}(x + a\bar{b}) + x \cdot \bar{x} \cdot \overline{(a\bar{b})}$$

$$= \bar{x}a\bar{b} = \overline{(a+b)c} \cdot a\bar{b} = (\bar{a}\bar{b} + \bar{c})a\bar{b}$$

$$M_0 = a\bar{b}\bar{c} \quad a\bar{b}\bar{C}$$

$M_1 = M(e=1)$

$= [(a+b)c + \underset{e=1}{1}] \oplus [(a+b)c + a\bar{b}]$

$= \overline{((a+b)c + a\bar{b})}$ $\qquad x = (a+b)c$

$= \overline{x + a\bar{b}}$

$= \bar{x}(\bar{a}+b)$

$= \bar{x}\bar{a} + \bar{x}b$

$= (\bar{a}\bar{b}+\bar{c})\bar{a} \quad + \quad (\bar{a}\bar{b}+\bar{c})b$

$= \bar{a}\bar{b} + \bar{a}\bar{c} \quad + \quad b\bar{c}$

$M_0 \wedge M_1 = (a\bar{b}\bar{c})(\bar{a}\bar{b} + \bar{a}\bar{c} + b\bar{c}) = \phi$

$M_0 \wedge M_1 \Rightarrow$ rectification feasible @ 'e'

$\Rightarrow M_0 =$ rectification function

$\Rightarrow e = a\bar{b}\bar{c}$.

original

You can verify $\begin{cases} Y = (a+b)c + a\bar{b} \\ \quad = (a+b)c + a\bar{b}\bar{c} \end{cases}$

rectification patch

(f) Rectification check @ net 'g'.

$$M_0 = M_0(g=0) = [\underbrace{a\bar{b}+0}_{x}] \oplus [(a+b)c + \underbrace{a\bar{b}}_{x}]$$

$$x \oplus (x + (a+b)c)$$

$$= \bar{x}[x + (a+b)c] + x[\bar{x} \cdot \overline{(a+b)c}]$$

$$= \bar{x}(a+b)c + 0$$

$$= (\bar{a}+b)(a+b)c =$$

$$= (\bar{a}b + ab + b)c =$$

$$= \underline{\underline{bc}}$$

$$\boxed{M_0 = bc}$$

$$M_1 = M(g=1) = [a\bar{b}+1] \oplus [(a+b)c + a\bar{b}]$$

$$= 1 \oplus [\quad]$$

$$= \overline{((a+b)c + a\bar{b})} = \overline{(a+b)c} \cdot \overline{(a\bar{b})}$$

$$= (\overline{(a+b)} + \bar{c}) \cdot (\bar{a} + b)$$

$$M_1 = (\bar{a}\bar{b} + \bar{c})(\bar{a} + b) = \bar{a}\bar{b} + \bar{a}\bar{c} + b\bar{c}$$

$M_0 \wedge M_1 = \phi \Rightarrow$ bug @ 'e' rectified @ 'g'

with $g = M_0 = bc \leftarrow$ rectification function.

$$V = (a+b)c + a\bar{b} = bc + a\bar{b}$$

$$\underline{\underline{ac}} + bc + a\bar{b} = bc + a\bar{b}$$

↳ Consensus of $(bc, a\bar{b})$