Solutions to HW 4 2019.

Q1 $F = uxy + vwy + xy + uz + vz$

Kernels	Co-K.	
$vy + z$	u	$\rightarrow$ Level-0
$uy + wy + z$	v	
$uv + vw + x$	y	
$u + v$	z	$\rightarrow$ Level-0
$u + w$	vy	$\rightarrow$ Level-0.
F	(1)	

Cube-intersection matrix & the compute_Kernel() methods can be used to compute K/co-K pairs.

Let us consider the compute_kernel() method, given on the slides on the class website.

$F = uvy + vwy + xy + uz + vz.$

Vars: $\{u, v, w, x, y, z)$

Cubes $(F, u) = \{uvy, uz\}$

Largest products C containing u & contained in $\{uvy, uz\}$

$\boxed{\Rightarrow C = u}$ $\quad F/F_c = \underline{uvy} + vwy + xy + \underline{uz} + vz / u$

$= \underline{vy + z} = K$

$(vy+z)(u)$ — 1

Cubes$(F, v) = \{uvy, vwy, vz\}$

$C = v$ $\quad F/F_c = (uy + wy + z)$ — 2

↓ recursive call for kernel computation

$C = y \quad F = uy + wy + z$

$F/y = (u + w)$ — 3

Cubes$(F, w) = \{vwy\}$ < 2, so ignore

Cubes$(F, x) = \{xy\}$ < 2, so ignore

cubes$(F, y) = \{uvy, vwy, xy\}$

$C = \{y\}$

$F/y = \underline{\underline{uv + vw + x}}$ — 4

recursive call

Common var = v. $C = \{v\}$

$$F/v = \frac{uv + vw + x}{v} = \underline{\underline{u + w}}$$

= Kernel 3

recomputed

Cubes$(F, z) = \{uz, vz\}$

$C = z$. $F/C = \underline{\underline{u + v}}$ — 5

Using the cube-matrix intersection = straight-forward from the book.

Q2

$F = ab + ac + ad' + bc + bd'$

$G = a + b.$

Weak division:

Divide by a : $\{b, c, \bar{d}\} = V^a$

Divide by b : $\{a, c, \bar{d}\} = V^b$

$$V^a \cap V^b = \{c, \bar{d}\} = (c + \bar{d})$$

$$F = Q \cdot G + R$$

$$Q = (c + \bar{d})$$

$$R = F - (a+b)(c+\bar{d})$$

$$= \{ab, ac, a\bar{d}, bc, b\bar{d}\} - \{ac, a\bar{d}, bc, b\bar{d}\}$$

$$= ab$$

$$\boxed{F = \underbrace{(a+b)}_{G} \; \underbrace{(c+\bar{d})}_{Q} + \underbrace{ab}_{R}}$$

$G = c + \bar{d}$

Weak division: $\underset{V_0^c}{(a, b)} \cap \underset{V^{\bar{d}}}{(a, b)}$

$= (a, b) = (a+b)$

$Q = (a+b)$

$R = F - (c+\bar{d})(a+b) = ab$

$$\boxed{F = \underset{G}{(c+\bar{d})}\underset{Q}{(a+b)} + \underset{R}{ab}.}$$

Q3 $f = uvy + vwy + xy + uz + vz$

$g = uxy + xz + uvz + yz + vwz.$

If we take help of SIS

[See my experiment attached with this solution set].

Print kernels of f & g.

Cok / K. of f

(u) (vy + z)

(v) (uy + wy + z)

(vy) (u + w) — 1

(y) (uv + vw + x) — 2

(z) (u + v)

(1) (f)

cok/K of g.

(u) (vz + xy)

(vz) (u + w) — 3

(x) (uy + z)

(y) (ux + z)

(z) (uv + vw + x + y) — 4

(1) (g)

1 & 3 are common kernels. = 2-cube kernels

we could extract them.

'fx' command in SIS

2 & 4 have common sub-expressions:

$$K_f = \{uv + vw + x\}$$

$$K_g = \{uv + vw + x + y\}$$

Common subexpression extraction (CSE) = $K_f \cap K_g$

CSE = $\{uv + vw + x\}$

$$f/\text{CSE} = y(\underline{uv+vw+x}) + \overbrace{\underline{uz+vz}}^{z(u+v)}$$

$$g/\text{CSE} = \underbrace{\underline{uxy+yz}}_{y(ux+z)} + z(\underline{uv+vw+x})$$

$$h = uv+vw+x.$$

$$\left[\begin{aligned} f &= y \cdot h + z(u+v) \\ g &= zh + y(ux+z) \\ h &= uv+vw+x \end{aligned}\right]$$

$$= v(u+w)+x$$

15 factored form literals = 18 SOP literals.

resb-a, simplify → don't change the network.

You get a similar result in SIS when you run fx; resub-a; simplify.

See attached experiment.

Q4 $F = ab + ad + cd = 6$ SoP literals

$G = a + c.$

(i) Can we do $F = G \cdot H$?

$G = a + c \supseteq F$ ✓

$a \supset ab + ad$

$c \supset cd.$

$F = G \cdot H$ possible

$H = F + \overline{G} \,\big|_{D.C.}$

$\overline{G} = \overline{a}\,\overline{c}$

ab \ cd	00	01	11	10
00	$\overline{G}$	$\overline{G}$	F	
01	$\overline{G}$	$\overline{G}$	F	
11	F	F	F	F
10		F	F	

H = simplify F w/ $\overline{G}$ as D.C.

$H = ab + d$

$F = (a+c)(ab+d) = 5$ F.F. literals.

(ii) $F = G + H$? Not possible.

OR-decomp requires $G \subseteq F$

but we have $G \supset F$.

(iii) $F = G \overline{\oplus} H$ is <u>ALWAYS</u> possible.

$$\cancel{F = G \oplus (G \oplus F)}$$

$$F = \underbrace{G \oplus G}_{0} \oplus F$$

$$= G \overline{\oplus} G \overline{\oplus} F$$

$$= \underbrace{G}_{G} \overline{\oplus} \underbrace{(G \overline{\oplus} F)}_{H}$$

$$F = (a+c) \overline{\oplus} \left[(a+c) \overline{\oplus} (ab + ad + cd) \right]$$

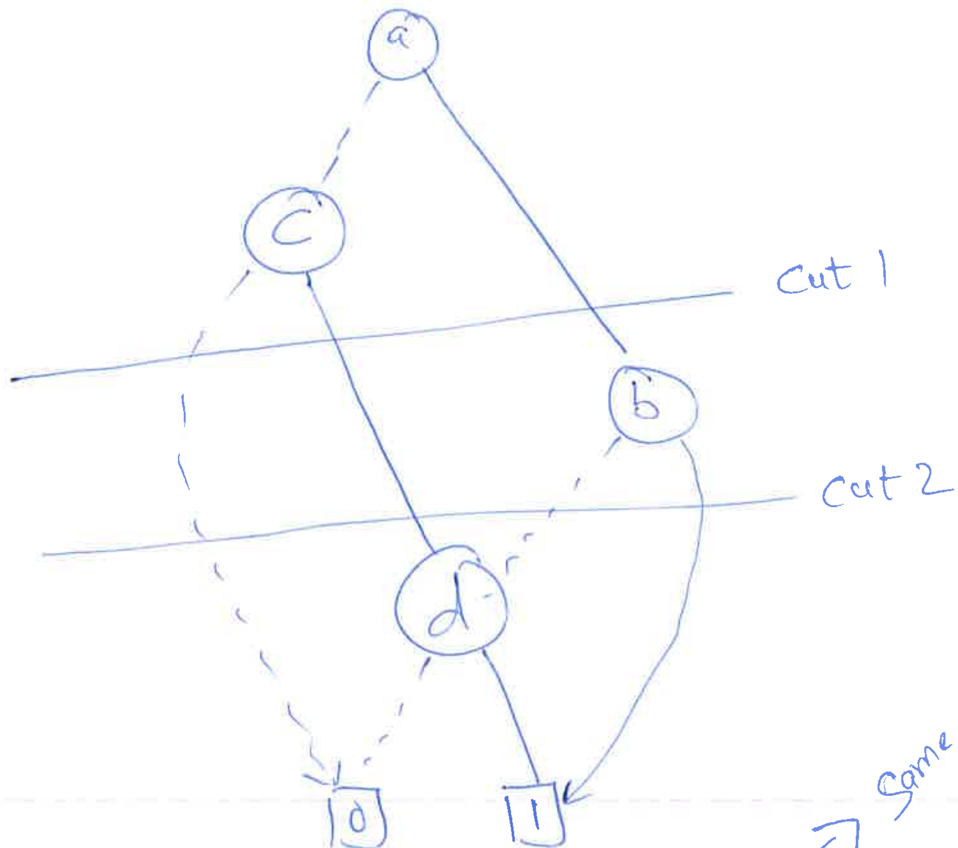
$\rightarrow$ may not be beneficial.

Q5

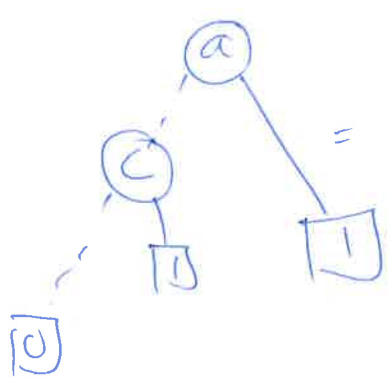
10

$F = ab + ad + cd.$

Var order $a > c > b > d$.

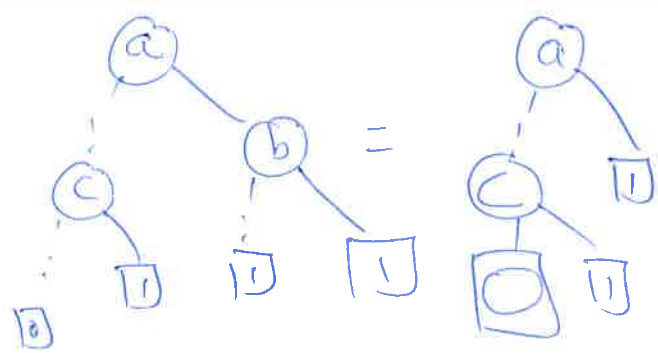


Cut 1 & Cut 2 both have same Σ_0-edges $\Rightarrow$ same divisor



$= a + c$

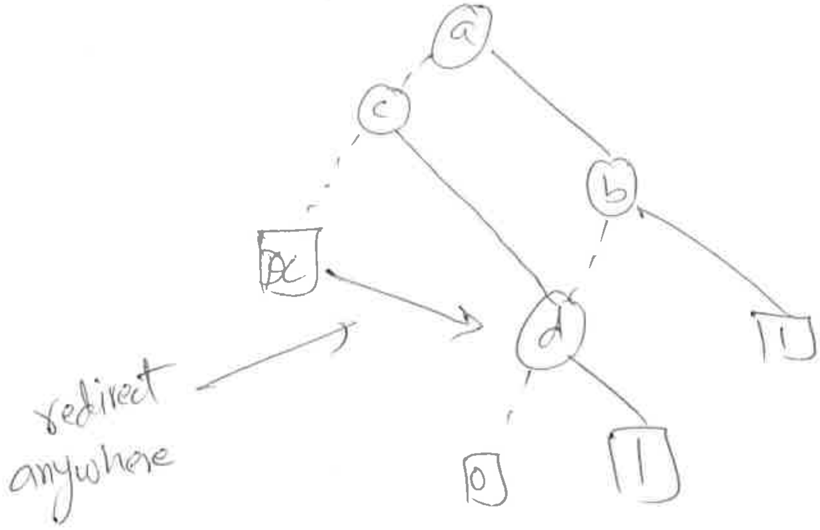
cut 1.

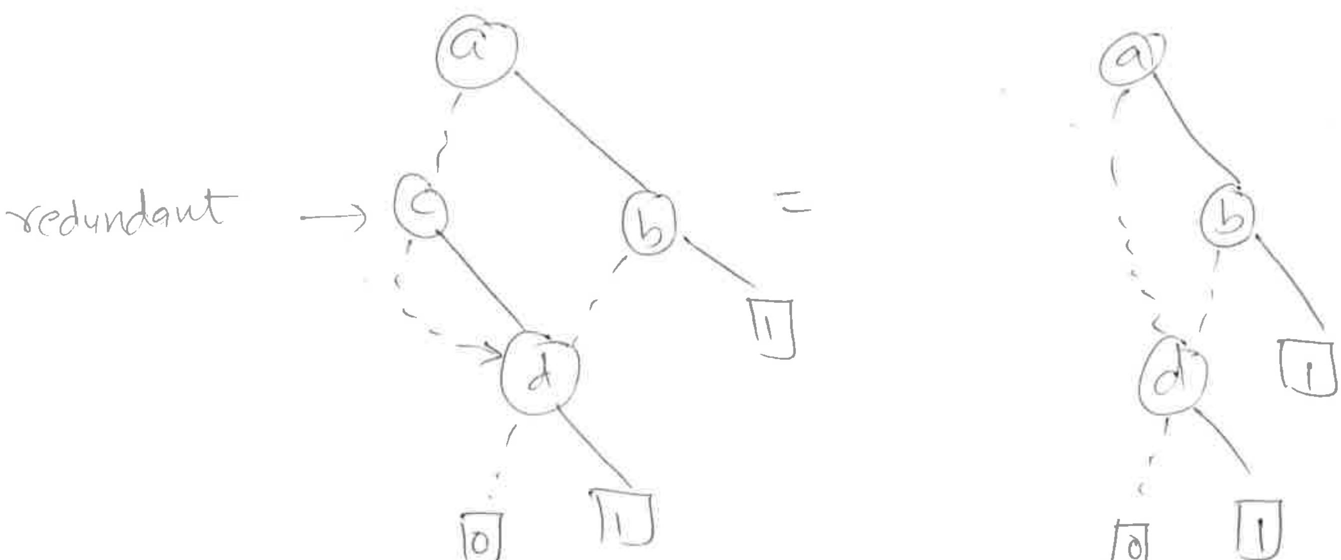


Cut 2.

$\boxed{G = a + c}$ can be identified.

G = F w/ $\bar{G}$ as D.C. on BDD





$= ab + a\bar{b}d + \bar{a}d$

$a[b + \bar{b}d] + \bar{a}d = ab + ad + \bar{a}d$

$= \underline{ab + d}$ same H

Q 6 $F = ab + acd + a\bar{c}\bar{d} + \bar{a}\bar{b}c + \bar{a}\bar{b}d$

$G = a + \bar{b}$

Weak Div.: $V^{a} = \{ab, acd, a\bar{c}\bar{d}\} \div a = \{b, cd, \bar{c}\bar{d}\}$

$V^{\bar{b}} = \{\bar{a}\bar{b}c, \bar{a}\bar{b}d\} = \{\bar{a}c, \bar{a}d\}$

$V^{a} \cap V^{\bar{b}} = \phi$

can't divide

$F = \underbrace{G \cdot H}_{} + R$ ✗

disjoint support } not possible

$G = a + \bar{b} \supset F$ so $F = G \cdot H$ is possible.

$\bar{G} = \bar{a}b$

ab \ cd	00	01	11	10
00		F	F	F
01	$\bar{G}$	$\bar{G}$	$\bar{G}$	$\bar{G}$
11	F	F	F	F
10	F		F	

$H = b + cd + \bar{a}c + \bar{a}d + a\bar{c}\bar{d}$

$F = G \cdot H$ possible with overlapping support in G & H.