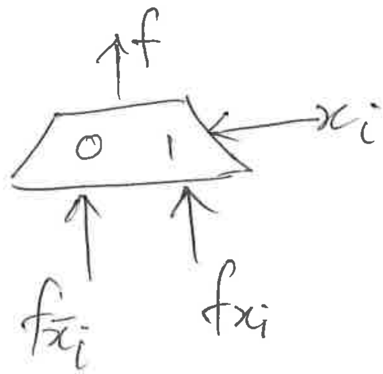
Notes on Generalized Cofactors

$$f(x_1 \ldots x_n) = x_i f_{x_i} + \overline{x_i} f_{\overline{x_i}}$$

x_i = literal

Synthesize w/ a mux $\Rightarrow$



Generalize Shannon's Expansion.

Let f = Boolean function (set of cubes)
g = Boolean function (" " ").

$$f = g \cdot f_g + \overline{g} \cdot f_{\overline{g}}$$

$f_g, f_{\overline{g}}$ = generalized cofactors.

Similar to <u>orthonormal</u> expansion:

Let ϕ_i's $i = 1, 2, \ldots, K$ be a set of Boolean functions such that: $\sum_{i=1}^{K} \phi_i = 1$ and

$\phi_i \cdot \phi_j = 0 \quad \forall\, i \neq j$

Then

$$f = \phi_1 f_{\phi_1} + \phi_2 f_{\phi_2} + \cdots + \phi_K f_{\phi_K}$$

f_{ϕ_i} = generalized cofactors.

<u>Example</u>:

$$f = ab + ac + bc$$

$$\phi_1 = ab \qquad \phi_2 = \bar{a} + \bar{b}$$

$$\boxed{\phi_1 + \phi_2 = 1 \qquad \phi_1 \wedge \phi_2 = 0}$$

So ϕ_1, ϕ_2 = a <u>basis</u>

So $f = \phi_1 f_{\phi_1} + \phi_2 f_{\phi_2}$

$$f_{\phi_1} = ab \qquad f_{\phi_2} = (ac + bc)$$

or

$$f_{\phi_1} = a \qquad f_{\phi_2} = (ac + bc)$$

$$(ab)\cdot(ab) + (\bar{a}+\bar{b})\cdot(ac+bc)$$

$$= ab + \bar{a}bc + a\bar{b}c$$

$$= ab + bc + ac = f$$

$$a(ab) + (\bar{a}+\bar{b})(ac+bc)$$

$$= \underline{f}$$

Generalized Cofactors are not unique.
But they satisfy these bounds.

$$\boxed{f \cdot \phi_i \subseteq f_{\phi_i} \subseteq f + \overline{\phi_i}}$$

↳ Can you prove this?

Demonstrate: $f = ab + ac + bc$, $\phi_1 = ab$, $\phi_2 = \bar{a} + \bar{b}$

$f \cdot \phi_1 = (\ )(\ ) = \underline{\underline{ab}}$

$f + \overline{\phi_1} = ab + ac + bc + \bar{a} + \bar{b} = 1 =$ everything

$a \backslash bc$	00	01	11	10
0	X	X	X	X
1	X	X	X f_{ϕ_1}	X f_{ϕ_1}

Think of $f \cdot \phi_1 =$ <u>Care-set</u> of the function

$f + \overline{\phi_1} =$ Care-set $\cup$ Don't care set $=$ X's in K-map.

$f_{\phi_1} =$ <u>anything</u> between f_{ϕ_1} and $f_{\phi_1} \cup X_s$ (D.C).

$f_{\phi_1} = ab$, or $f_{\phi_1} = a$ or $f_{\phi_1} = 1$ [they all work]

Likewise. $\phi_2 = \bar{a} + \bar{b}$

$$f \cdot \phi_2 = a\bar{b}c + \bar{a}bc$$

$$f + \bar{\phi}_2 = ab + ac + bc = X's \text{ on K-map.}$$

a \ bc	00	01	11	10
0			$\bar{a}bc$ X	
1		$a\bar{b}c$ X	X	X

$$f_{\phi_2} = ac + bc$$

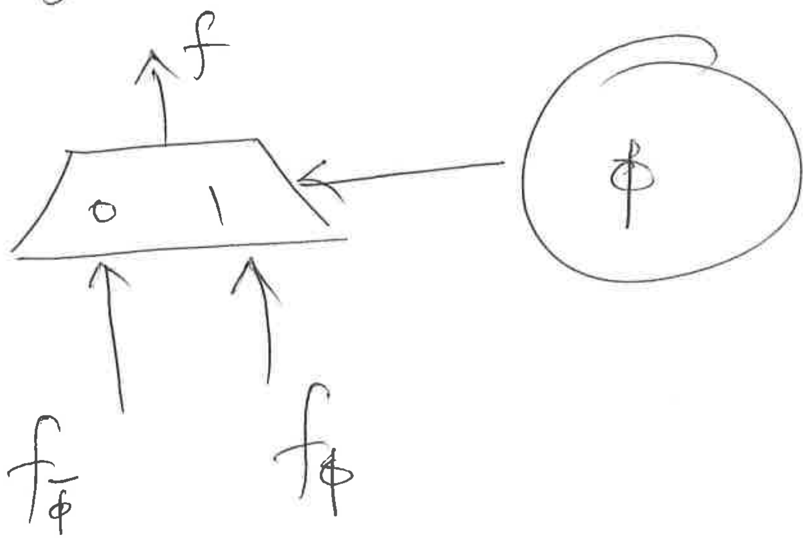
Implications on synthesis:

HW Given f ✓

find a ϕ ~~set~~.

and corresponding

f_ϕ, $f_{\bar{\phi}}$



s.t.

$$f = \phi f_\phi + \bar{\phi} f_{\bar{\phi}}$$

[Boolean function decomposition]