Q1 (a) True!

$$f = x f_x + \bar{x} f_{\bar{x}}$$

$$= x f_x \oplus \bar{x} f_{\bar{x}}$$

[$x = 0$ or 1, not both, so OR in Shannon's expansion = XOR]

$$= x f_x \oplus (1 \oplus x) f_{\bar{x}}$$

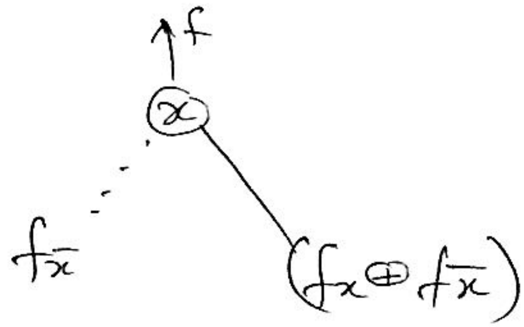
$$= x f_x \oplus f_{\bar{x}} \oplus x f_{\bar{x}}$$

$$= f_{\bar{x}} \oplus x (f_x \oplus f_{\bar{x}})$$

This is called Davio's expansion., or Kroenecker's expansion or Reed-Müller's expansion. One can also build a decision diagram based on this decomposition.

Shannon

Davio



(b) $f = (x + f_{\bar{x}})(\bar{x} + f_x) = x \cdot \bar{x} + x f_x + \bar{x} f_{\bar{x}} + f_x f_{\bar{x}}$

$$= 0 + x f_x + \bar{x} f_{\bar{x}} + (1) f_x f_{\bar{x}}$$

$$= x f_x + \bar{x} f_{\bar{x}} + (x + \bar{x}) f_x \cdot f_{\bar{x}} =$$

$$x f_x + x f_x f_{\bar{x}} + \bar{x} f_{\bar{x}} + \bar{x} f_x f_{\bar{x}}$$

$$= x f_x (1 + f_{\bar{x}}) + \bar{x} f_{\bar{x}} (1 + f_x)$$

$$= x f_x \cdot 1 + \bar{x} f_{\bar{x}} \cdot 1 = x f_x + \bar{x} f_{\bar{x}}.$$

Also,

$$(x + f_{\bar{x}})(\bar{x} + f_{\bar{x}}) = 0 + \underbrace{x f_x + \bar{x} f_{\bar{x}}}_{f} + \underbrace{f_x \cdot f_{\bar{x}}}$$

<u>Consensus</u> of f w.r.t. x

↳ largest function, smaller than f, contained in f and does not have x in its support.

$f \supset f_x \cdot f_{\bar{x}}$ So $f \cup f_x \cdot f_x = \underline{f}$

So $x f_x + \bar{x} f_{\bar{x}} + \underbrace{f_x f_{\bar{x}}} = x f_x + \bar{x} f_{\bar{x}}$

Contained

6. f = +ve unate in x.

$f_x \supset f_{\bar{x}}$ $\quad\quad f = x f_x + f_{\bar{x}}$

$$\bar{f} = \overline{x f_x + f_{\bar{x}}}$$

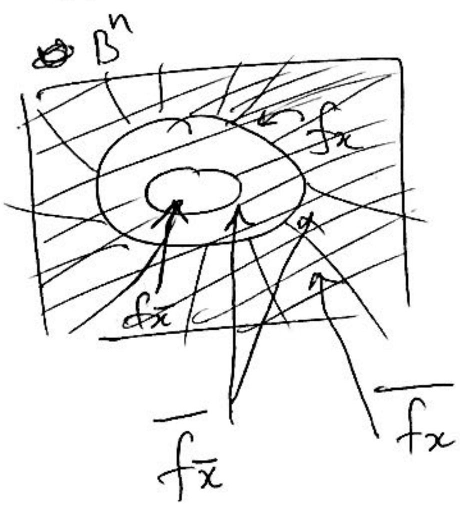
$$= \overline{(x f_x)} \cdot \overline{f_{\bar{x}}}$$

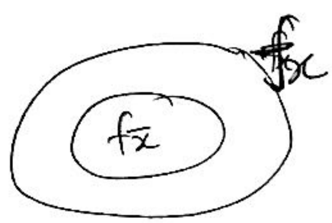
$$= (\bar{x} + \overline{f_x})(\overline{f_{\bar{x}}})$$

$$= \bar{x}\,\overline{f_{\bar{x}}} + \overline{f_x} \cdot \overline{f_{\bar{x}}}$$

$$\boxed{f_x \supset f_{\bar{x}} \Rightarrow \overline{f_x} \subset \overline{f_{\bar{x}}}}$$

See venn diagram

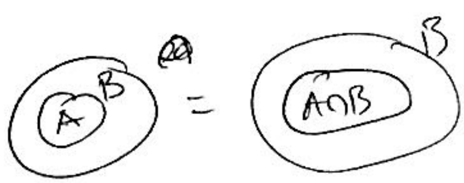




$$\boxed{\bar{x}\,\overline{f_{\bar{x}}} + \overline{f_x}}$$

True.

$A \subset B \Rightarrow A \cap B = A$



(4)

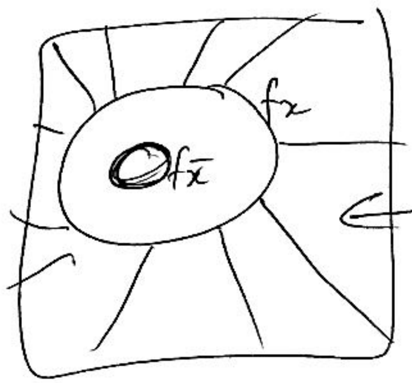
d. $f =$ +ve unate. $\Rightarrow f_x \supset f_{\bar{x}}$

Boolean difference.

$$\frac{\partial f}{\partial x} = f_x \oplus f_{\bar{x}}$$

$$= f_x \cdot \overline{f_{\bar{x}}} + \underbrace{\overline{f_x} \cdot f_{\bar{x}}}_{=0.} = f_x \cdot \overline{f_{\bar{x}}}$$

True



$\overline{f_x}$

$\overline{f_x} \wedge f_{\bar{x}} = \phi$

(e) True. $f = 1 \Leftrightarrow f_x = 1 \wedge f_{\bar{x}} = 1$

~~Assume~~ if $f_x = 1$ & $f_{\bar{x}} = 1$

If case. $\Rightarrow x f_x + \bar{x} f_{\bar{x}} = x \cdot 1 + \bar{x} \cdot 1 = 1.$

only if case. Assume $f_x \neq 1$ = set of cubes $\{c_1 \cdots c_n\}$. not containing x.

$f_{\bar{x}} = 1$

$(\bar{a} + ab = \bar{a} + b)$

$$f = x f_x + \bar{x} f_{\bar{x}}$$

$$= x[c_1 + c_2 + \cdots c_n] + \bar{x} \cdot 1$$

$$= c_1 + c_2 + \cdots + c_n + \bar{x} \quad \text{cannot be } 1$$

Q2. $f = f + c$ because $c =$ implicant of f.

$$f_c = (f+c)\Big|_{c=1}$$

$$= f(c=1) + c(c=1)$$

$$= f(c=1) + 1$$

$$f_c = 1$$

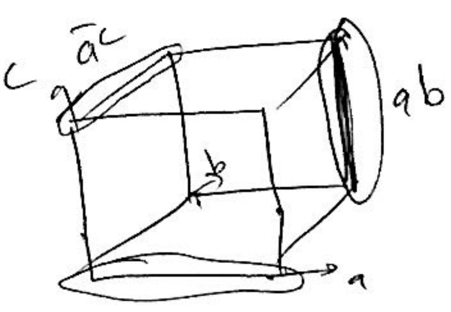
Q3 (a) $f = ab + \bar{a}c + \bar{b}\bar{c}$

Find g: smallest function, contains f, without b in it's support

$\Rightarrow$ smoothing of f w.r.t. b.

$$\exists_b f = f_b + f_{\bar{b}}$$

$$\left.\begin{array}{l} f_b = a + \bar{a}c + 0 = a + c \\ f_{\bar{b}} = \bar{a}c + \bar{c} = \bar{a} + \bar{c} \end{array}\right\} f_b + f_{\bar{b}} = 1$$



Clearly $g =$ whole cube

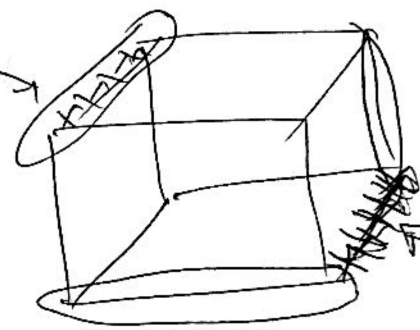
if $g \supset f$ and $b \notin$ support(g)

(b) h = Consensus of f w.r.t b.

$$= \forall_b f = f_b \cdot f_{\bar b}$$

$$= (a+c)(\bar a + \bar c)$$

$$= a\bar c + \bar a c$$



$\underbrace{\bar a c + a\bar c}$ cf

Q4. Is Z sensitive to changes in a?

$$\Rightarrow \frac{\partial Z}{\partial a} = \text{non-zero?}$$

$$Z = k + m = a \cdot j + jc = (a+c)j$$

$$= (a+c)(abc+d)$$

$$\frac{\partial z}{\partial a} = Z_a \oplus Z_{\bar a}$$

$$Z_a = Z(a=1) = bc+d$$

$$Z_{a=0}\ (c)(d) = cd.$$

$= \cancel{\text{c}\ldots}$

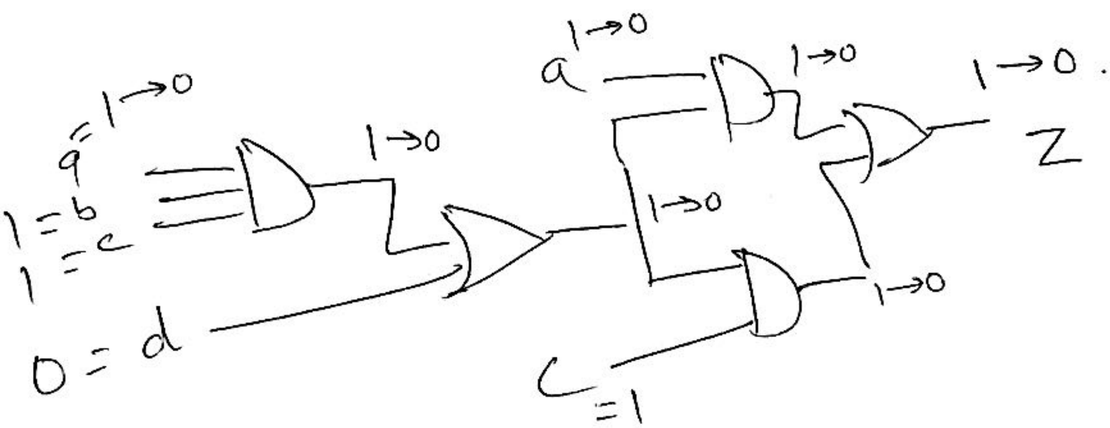
$$Z_a \oplus Z_{\bar a} = (bc+d)\overline{(cd)} + \overline{(bc+d)} \cdot cd$$

$$= (bc+d)(\bar c + \bar d) + \underbrace{(\overline{bc}) \cdot (\bar d)(cd)}_{0}$$

$$= \underbrace{bc\bar{d} + \bar{c}d}_{\frac{\partial Z}{\partial a}}$$

Z = sensitive to changes in 'a' when $bc\bar{d}$ ✓ or $\bar{c}d$ ✓

bcd: {110, x01}.



Apply bcd = 110.

change a from 1 to 0 & see if Z changes w/ a.

But put b=0 c=0, d=0.

$$\frac{\partial Z}{\partial a} = bc\bar{d} + \bar{c}d$$
$$= 0 + 0 = 0.$$

For $\underline{\underline{bcd = 000}} \Rightarrow j = 0 \Rightarrow Z = 0.$

Change 'a'. 1→0 or 0→1 } $\underline{\underline{Z=0}}$ no change.

Q5 Prove that $T_{x/0} = x \frac{\partial f}{\partial x}$.

From Fig 2 in the HW.

$T_{x/0}$ = set of all input vectors that differentiate between C_1 & C_2.

XOR.

$$T_{x/0} = C_1 \oplus C_2$$

@ $C_1 = f(x)$ = fault-free ckt.

$C_2 = f(x=0) = f_{\bar{x}}$

$$T_{x/0} = f(x) \oplus f_{\bar{x}}$$

$$= x f_x + \bar{x} f_{\bar{x}} \oplus f_{\bar{x}}$$

$$= x f_x \oplus \bar{x} f_{\bar{x}} \oplus f_{\bar{x}} \quad [\text{Q1 (a)}]$$

$$= x f_x \oplus f_{\bar{x}} [\bar{x} \oplus 1]$$

$$= x f_x \oplus f_{\bar{x}} \cdot x$$

$$= x [f_x \oplus f_{\bar{x}}] = x \frac{\partial f}{\partial x}.$$