

Acceleration Structures

CS 6965 Fall 2011

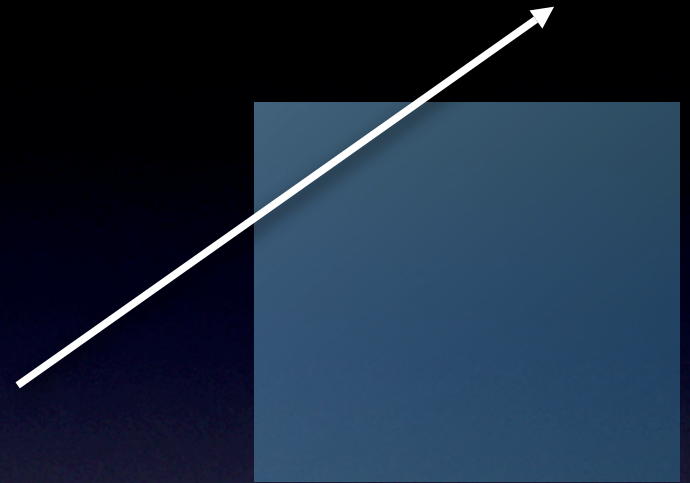
Program 2

- Run Program 1 in simhwrt
 - Also run Program 2 and include that output
- Lab time?

- Inheritance probably doesn't work

Boxes

- Axis aligned boxes
- Parallelepiped
- 12 triangles?
- 6 planes with squares?



Ray-box intersection

$$\vec{N} \cdot \vec{P} - d = 0$$

$$t = \frac{d - \vec{N} \cdot \vec{O}}{\vec{N} \cdot \vec{V}}$$

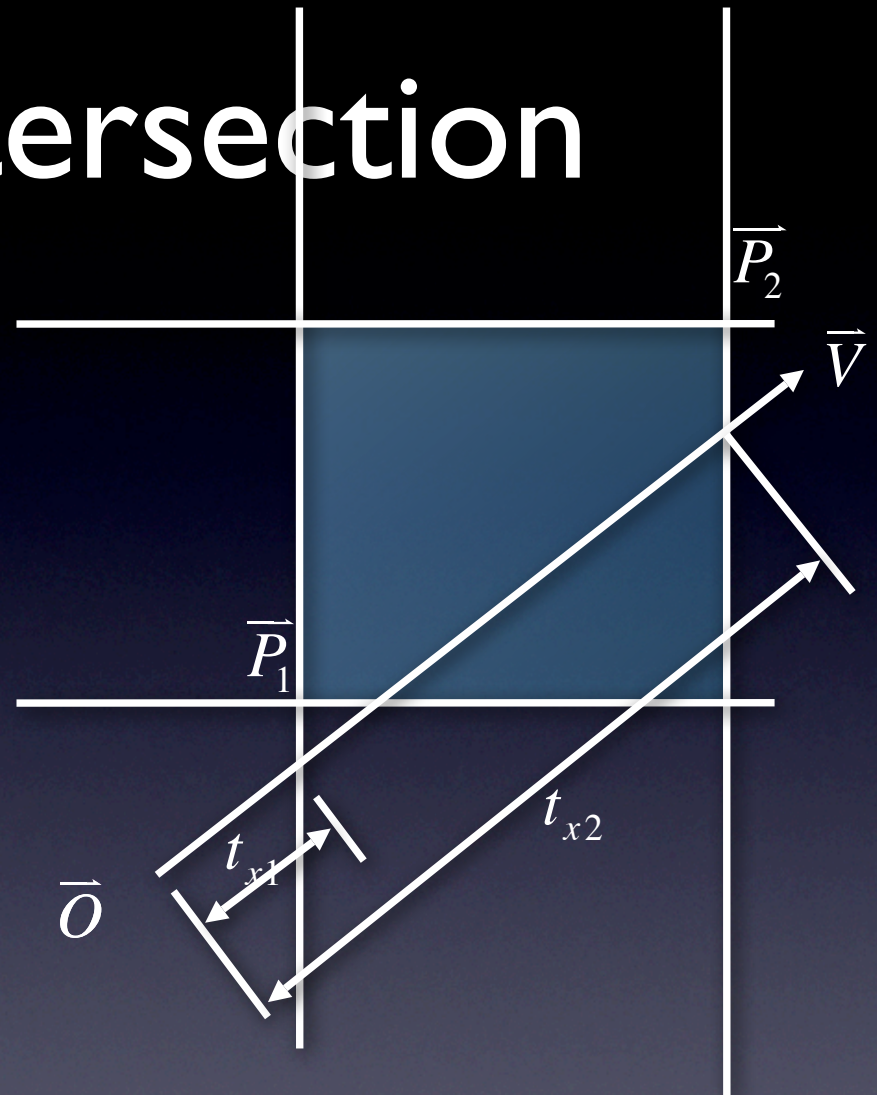
$$\text{x plane: } \vec{N} = [1 \ 0 \ 0]$$

$$t = \frac{d - O_x}{V_x}$$

$$d_1 = P_{1x}, d_2 = P_{2x}$$

$$t_{x1} = \frac{P_{1x} - O_x}{V_x}, t_{x2} = \frac{P_{2x} - O_x}{V_x}$$

Same for y, z planes



Intersection of intervals

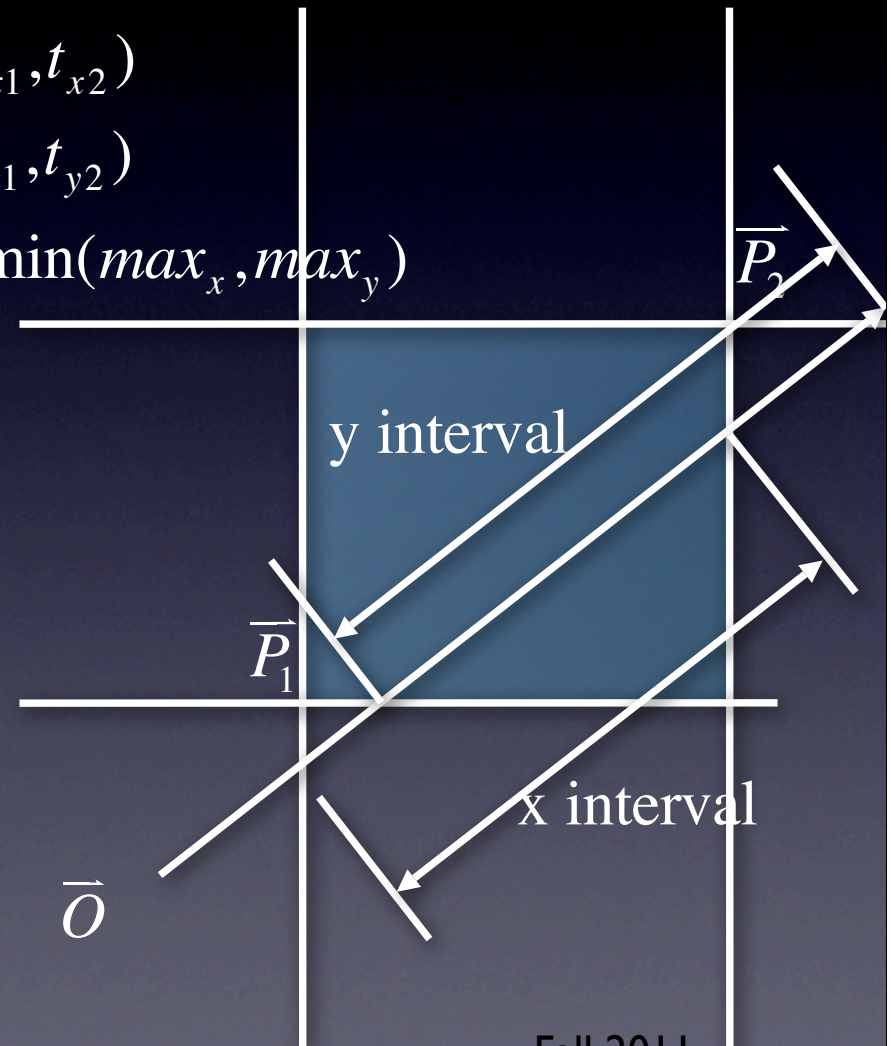
Intersection occurs where x interval overlaps y interval

x interval: $\min(t_{x1}, t_{x2}) \leq t \leq \max(t_{x1}, t_{x2})$

y interval: $\min(t_{y1}, t_{y2}) \leq t \leq \max(t_{y1}, t_{y2})$

intersection: $\max(\min_x, \min_y) \leq t \leq \min(\max_x, \max_y)$

In 3D also check z interval



Improved Box

- <http://www.cs.utah.edu/~awilliam/box/>

Ray tracing optimization

- Faster rays
 - CPU optimization techniques (CS6620 lecture)
- Fewer rays
 - Adaptive supersampling
 - Ray tree pruning
- Faster ray-object intersection tests
 - High-order surfaces
- Fewer ray-object intersection tests
 - Acceleration structures

Adapted from Arvo/Kirk "A survey of ray tracing acceleration techniques"

Acceleration structures

- Current ray tracer is $O(\# \text{ objects})$
- Large # of objects can be SLOW

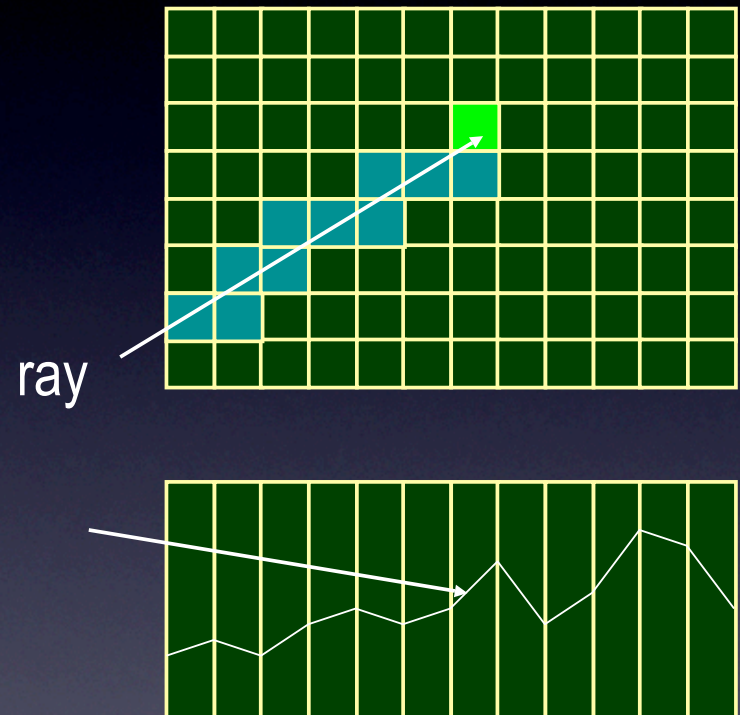
- Solution: intelligently determine which objects to intersect
- Good news: $\ll O(N)$ achievable!
- Real-time ray tracer exploits this to do very large models:
 - 262144x262144 heightfield (10+billion patches)
 - 35 million spheres
 - (1.1 million spheres + volume rendering) * 170 timesteps

Acceleration structures

- Three main types
- Tree-based:
 - Binary Space Partitioning (BSP) tree
 - Bounding Volume Hierarchy
- Grid-based:
 - Uniform grid
 - Hierarchical grid
 - Octree
- Directional

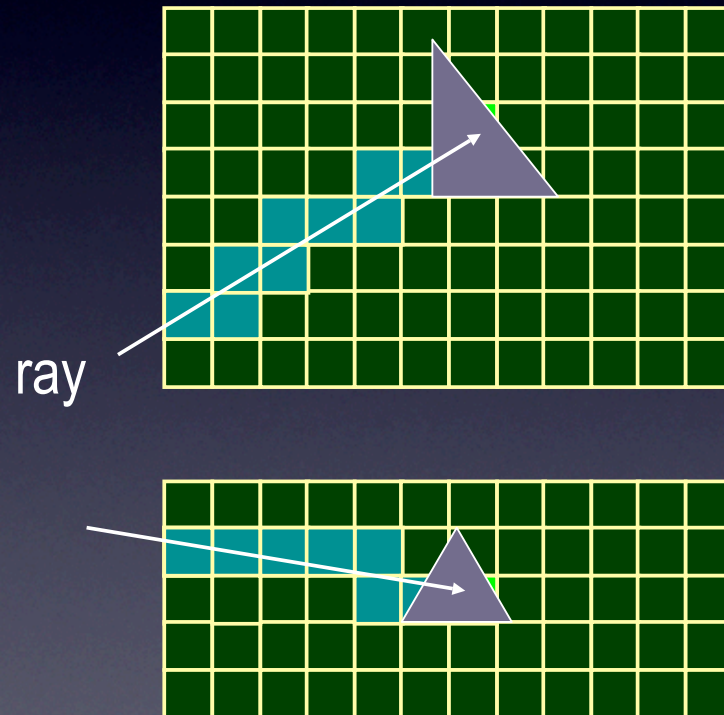
Uniform grid

- Split space up in a grid
- A heightfield is a specialized acceleration structure
- A grid is more general



Grid traversal

- Just like a 2D Grid but in 3D



Heightfield traversal

- Step 1: Compute a few derived values
 - Diagonal: $D = P_{\max} - P_{\min}$
 - Cell Size: $\text{cellsize} = D / (n_x, n_y, 1)$
 - Data min, max: Z_{\min}, Z_{\max}
- Grid:
 - Add n_z
 - Cell size 3D
 - Don't need Z_{\min}/Z_{\max}
 - Data located in cells, not at corners

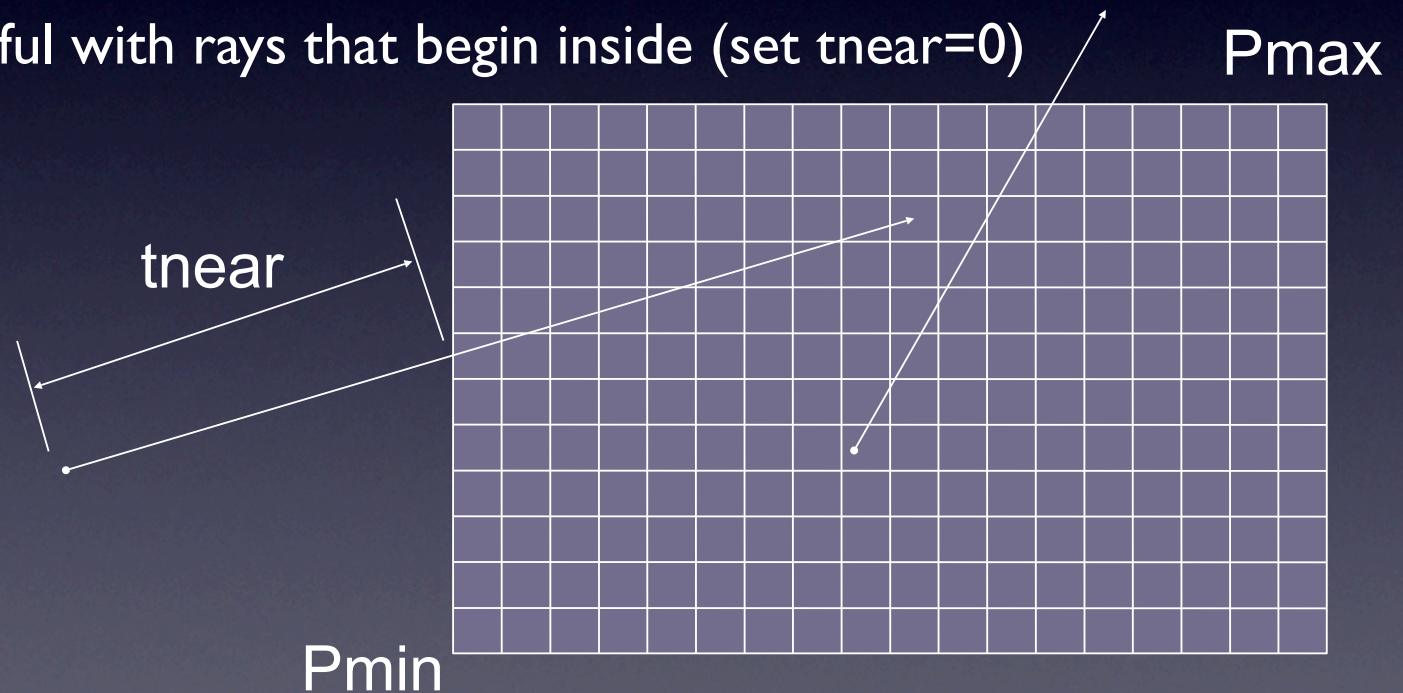


Grid traversal

- Step 2-8: straightforward extension to 3D

Heightfield traversal

- Step 2: Compute t_{near}
 - Use ray-box intersection
 - Be careful with rays that begin inside (set $t_{near}=0$)



Heightfield traversal

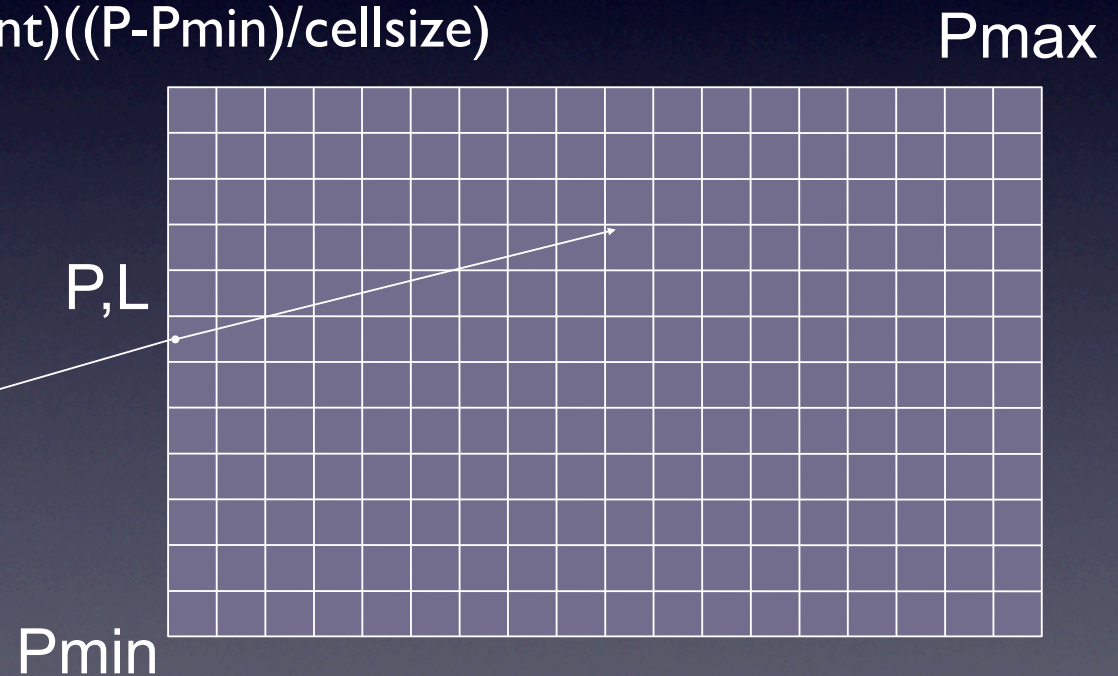
- Step 3: Compute lattice coordinates of near point

- World space: $P = O + t_{near} V$

- Lattice space: $L = (int)((P - P_{min}) / cellsize)$

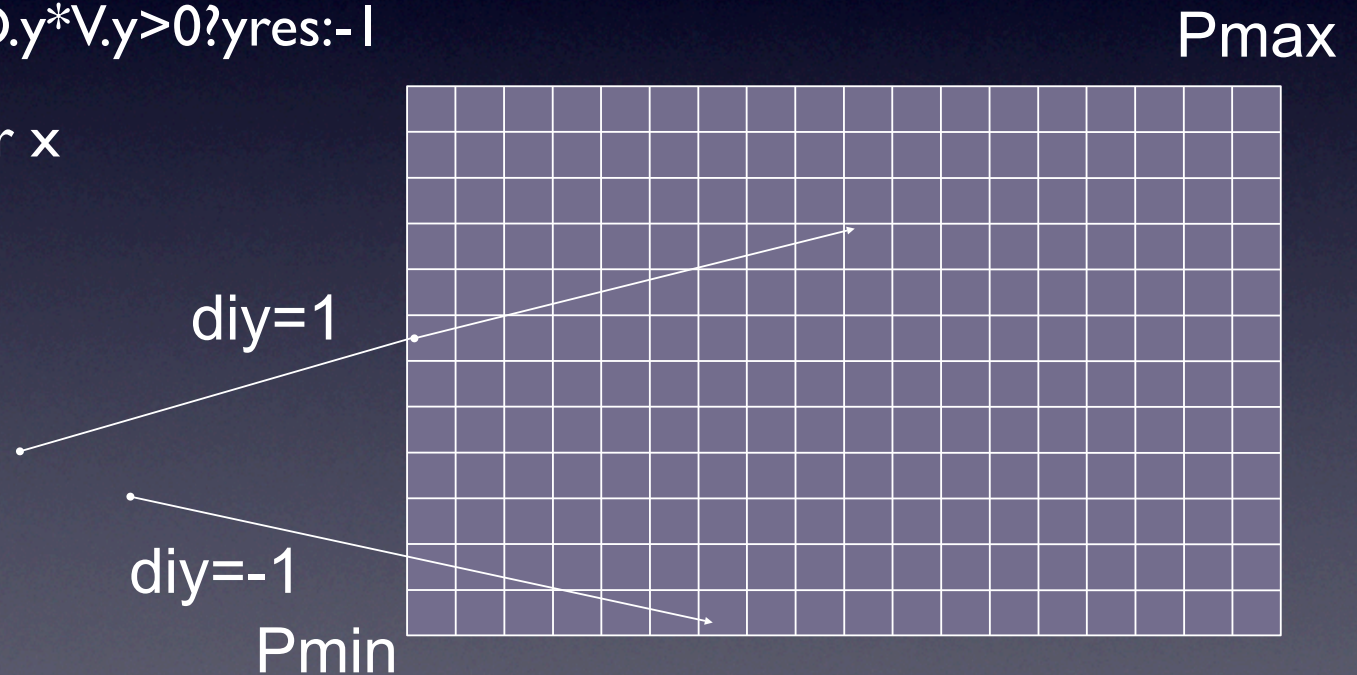
- Be careful of

- roundoff error



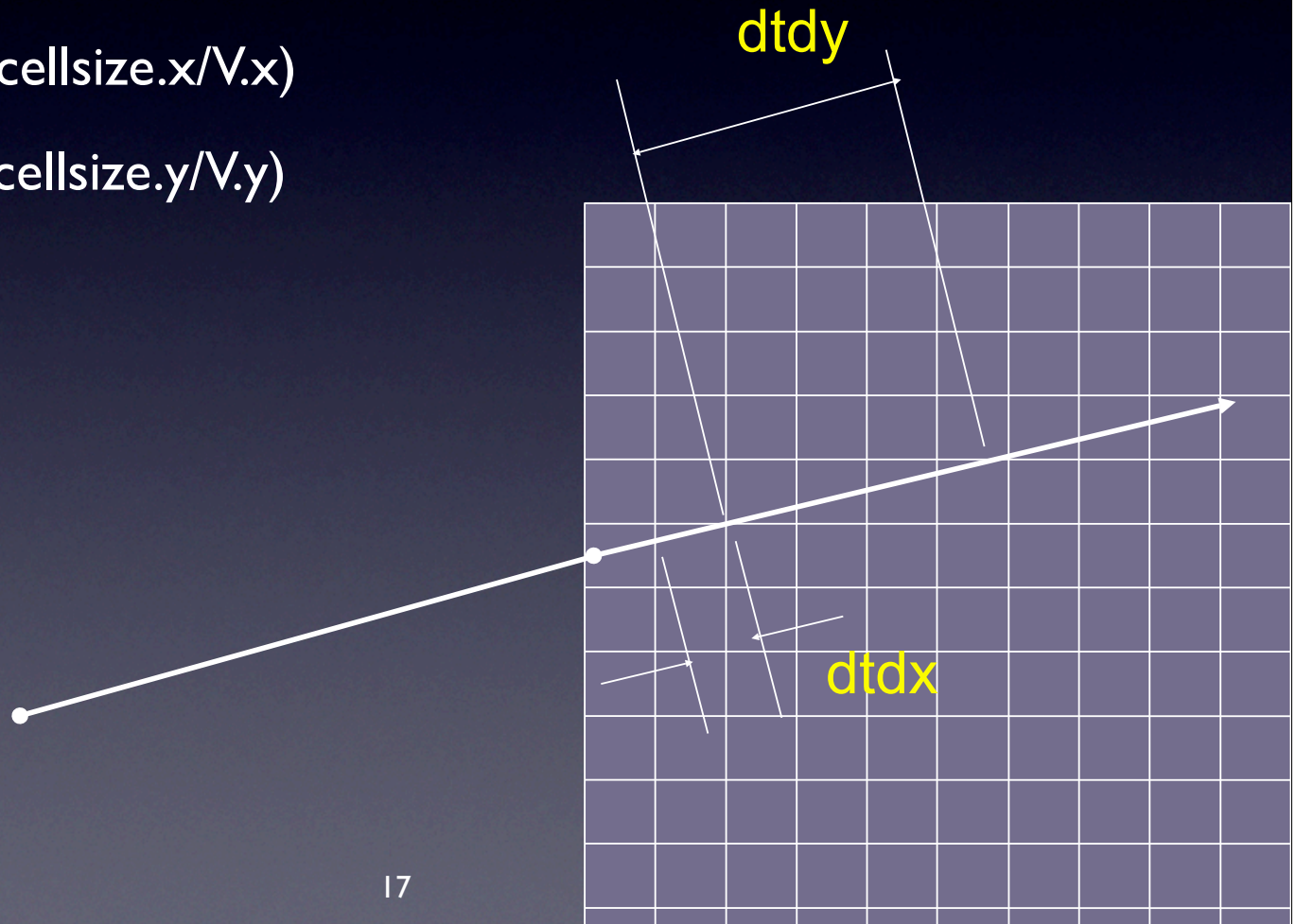
Heightfield traversal

- Step 4: Determine how ray marching changes index
 - $diy = D.y * V.y > 0 ? 1 : -1$
 - $stopy = D.y * V.y > 0 ? yres : -1$
 - similar for x



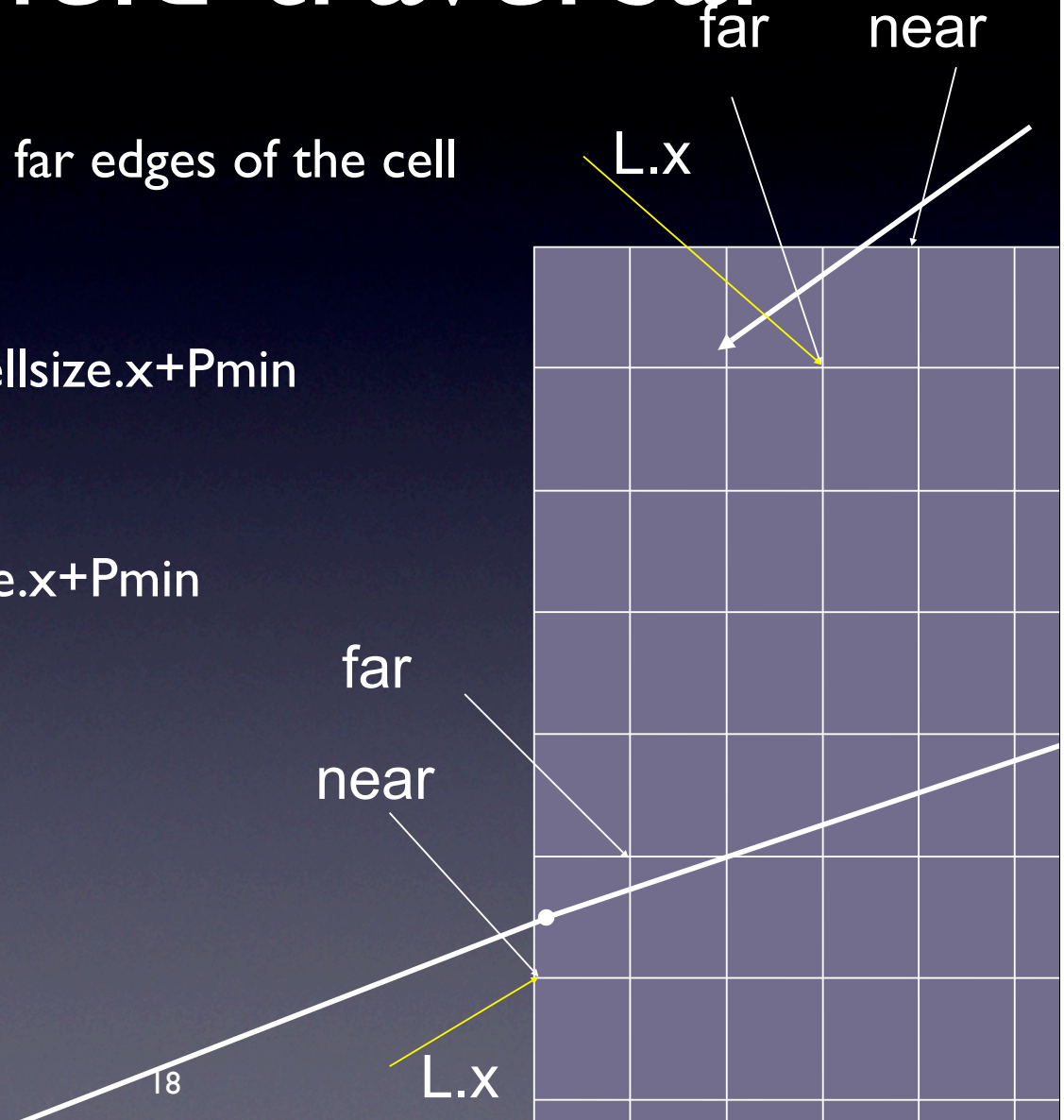
Heightfield traversal

- Step 5: Determine how t value changes with ray marching:
- $dtdx = \text{Abs}(\text{cellsize.x}/V.x)$
- $dtdy = \text{Abs}(\text{cellsize.y}/V.y)$



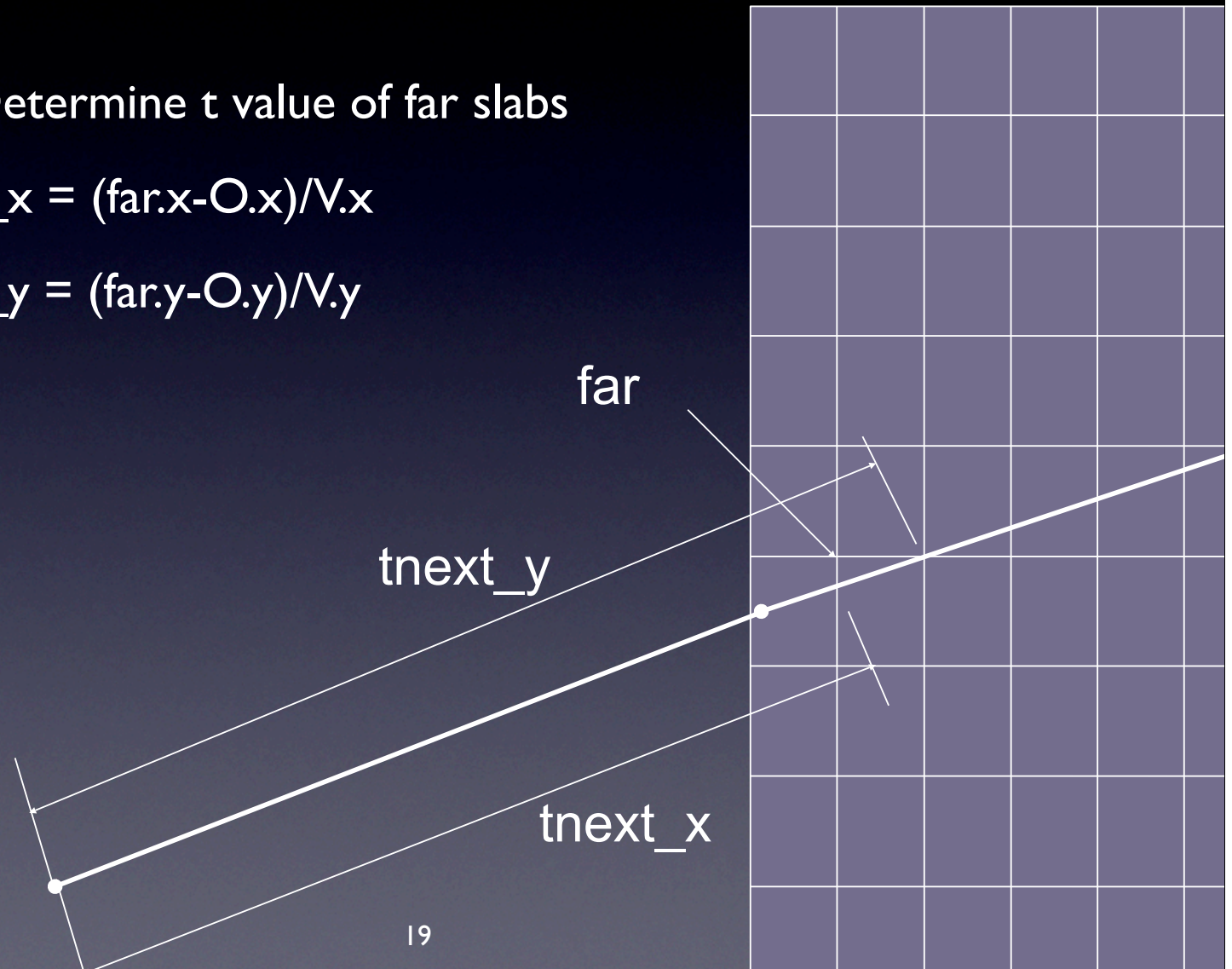
Heightfield traversal

- Step 6: Determine the far edges of the cell
 - if $dix == 1$:
 - $far.x = (L.x + 1) * cellsize.x + Pmin$
 - if $dix == -1$:
 - $far.x = L.x * cellsize.x + Pmin$
- Similar for y



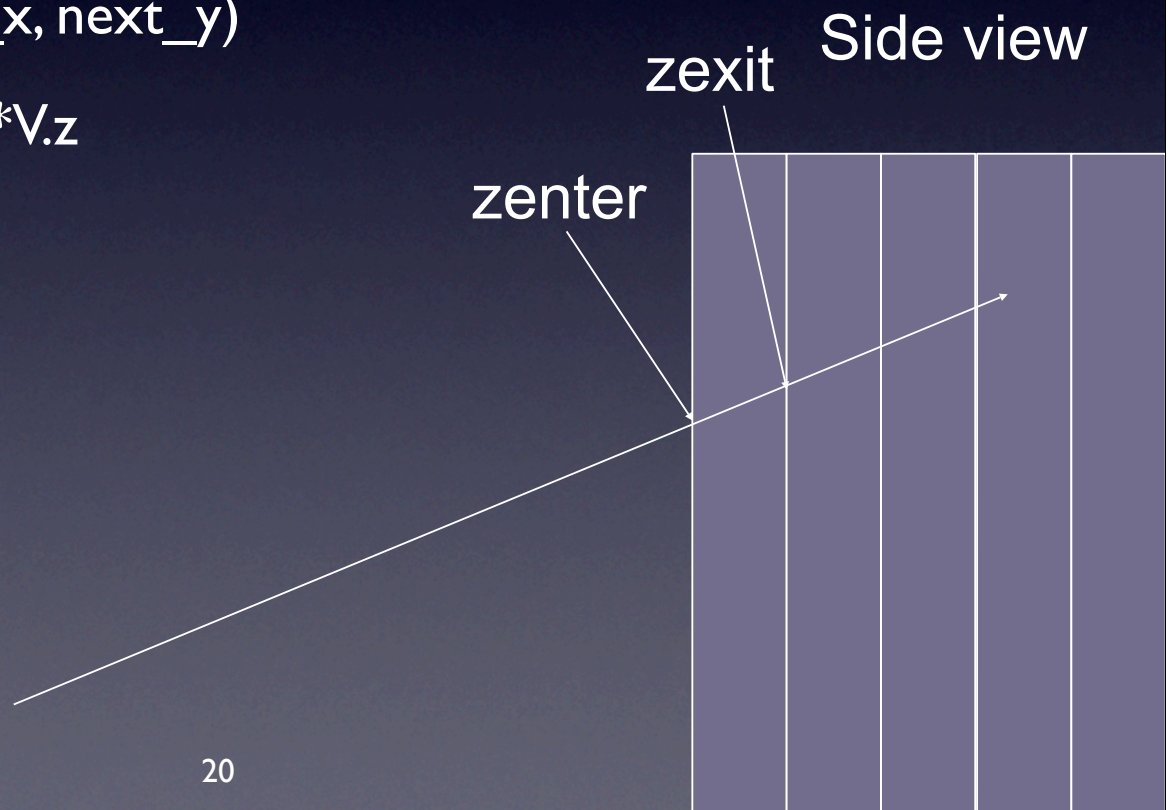
Heightfield traversal

- Step 7: Determine t value of far slabs
 - $t_{next_x} = (far.x - O.x) / V.x$
 - $t_{next_y} = (far.y - O.y) / V.y$



Heightfield traversal

- Step 8: Beginning of loop Compute range of Z values
 - $z_{center} = O.z + t_{near} * V.z$
 - $t_{exit} = \text{Min}(\text{next}_x, \text{next}_y)$
 - $z_{exit} = O.z + t_{exit} * V.z$

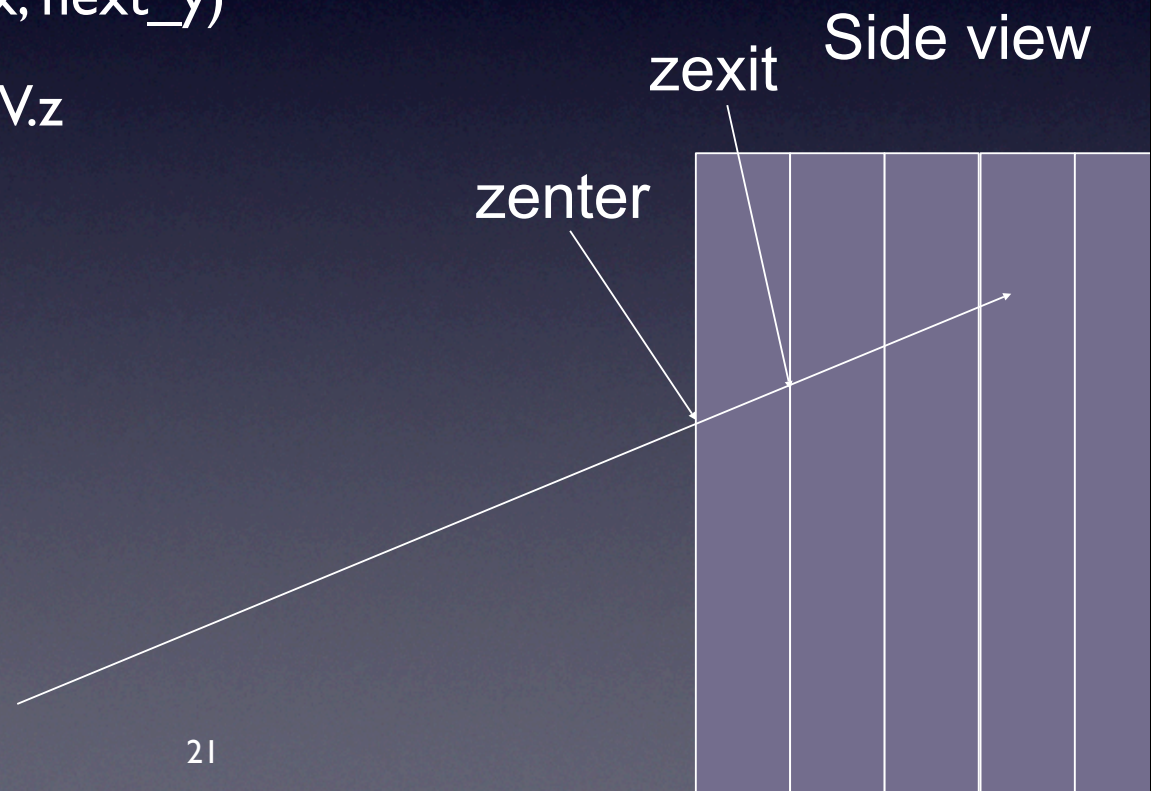


Heightfield traversal

- Step 8: Beginning of loop Compute range of Z values

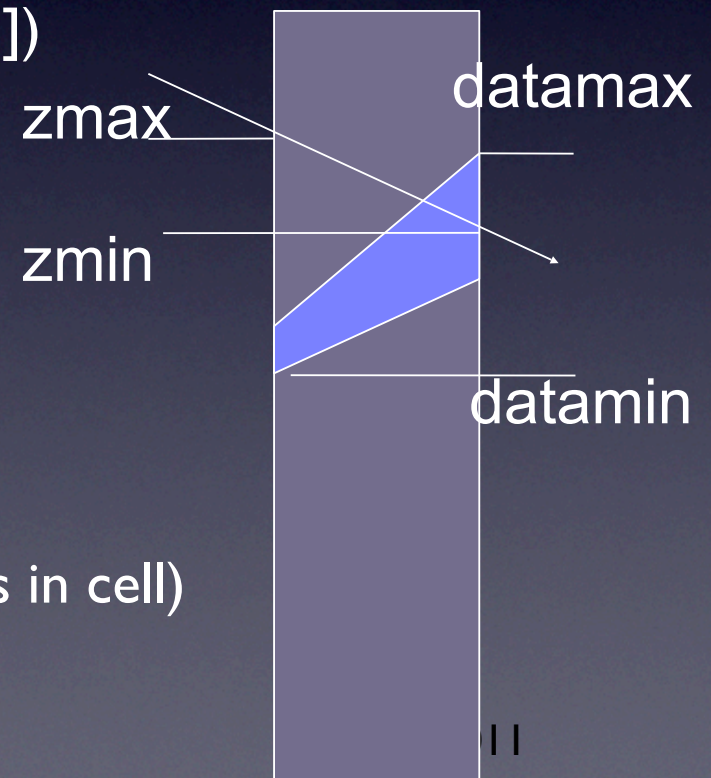
- $z_{enter} = O.z + t_{near} * V.z$
- $t_{exit} = \text{Min}(\text{next}_x, \text{next}_y)$
- $z_{exit} = O.z + t_{exit} * V.z$

- Grid: not needed



Heightfield traversal

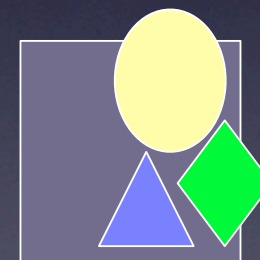
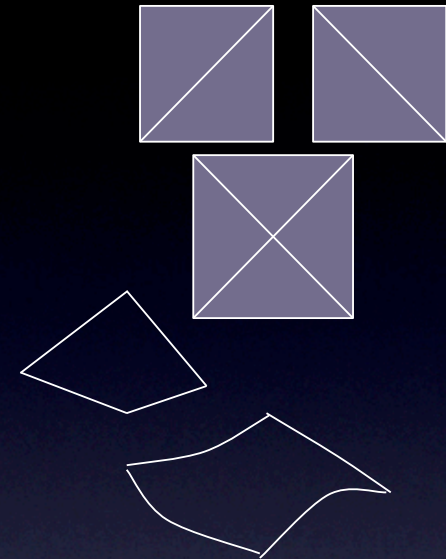
- Step 9: Determine overlap of z range
 - $\text{datamin} = \text{Min}(\text{data}[\text{L.x}][\text{L.y}],$
 - $\text{data}[\text{L.x}+1][\text{L.y}],$
 - $\text{data}[\text{L.x}][\text{L.y}+1],$
 - $\text{data}[\text{L.x}+1][\text{L.y}+1])$
 - $\text{zmin} = \text{Min}(\text{zenter}, \text{zexit})$
 - Similar for max
 - if $\text{zmin} > \text{datamax} \ || \ \text{zmax} < \text{datamin}$
 - skip to step 11
 - Grid: not needed (skip cell if no objects in cell)



Step 10:

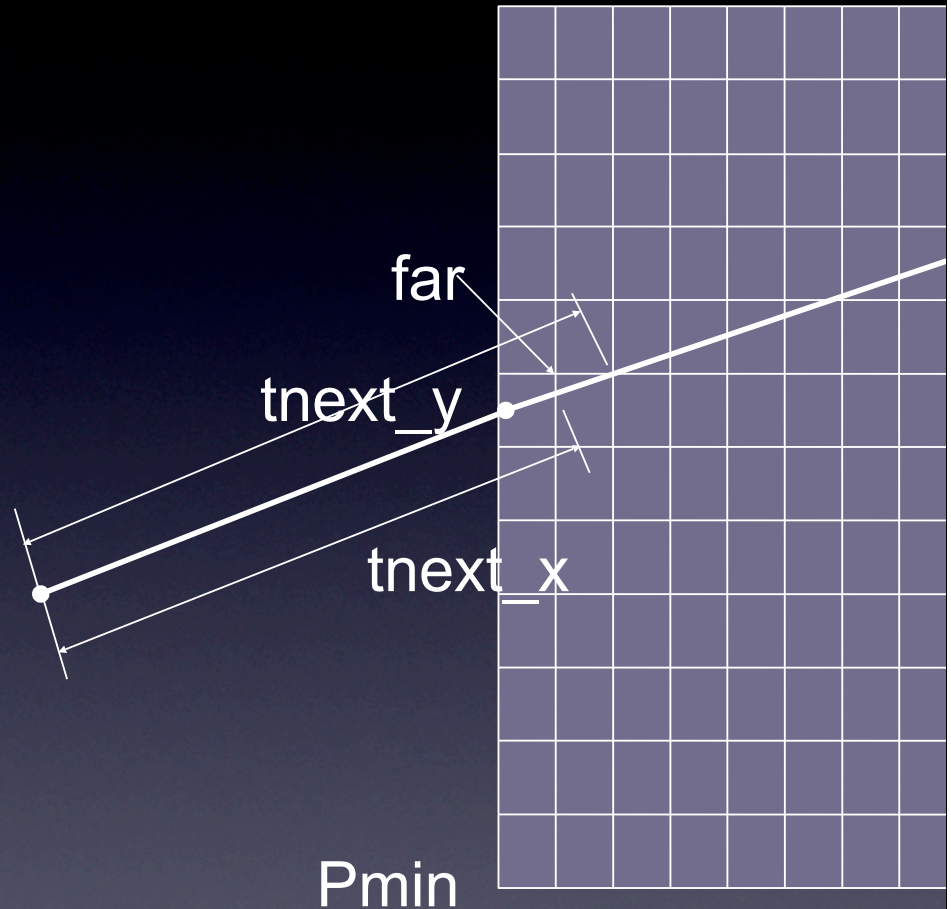
- Intersect ray with cell
 - 2 triangles, 4 triangles, Bilinear patch, Bicubic patch

- Grid: intersect with list of objects that partially overlap cell



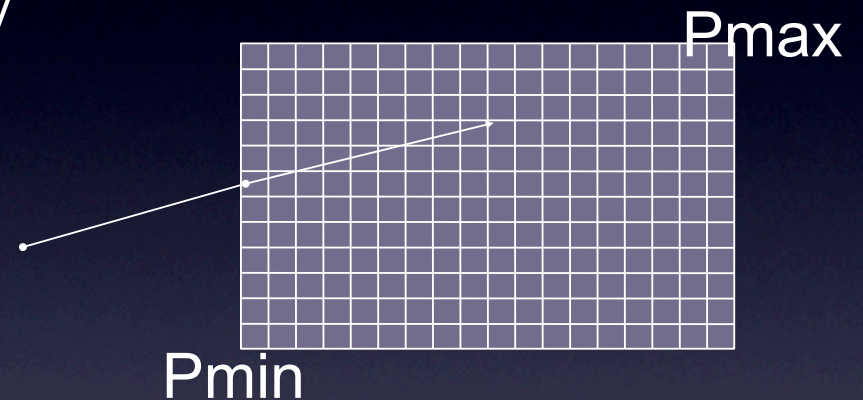
Heightfield traversal

- Step 11: March to next cell
- if $t_{next_x} < t_{next_y}$
- $t_{near} = t_{next_x}$
- $t_{next_x} += dt_{dx}$
- $L.x += dx$
- else
- Similar for y
- Grid: 3 way minimum



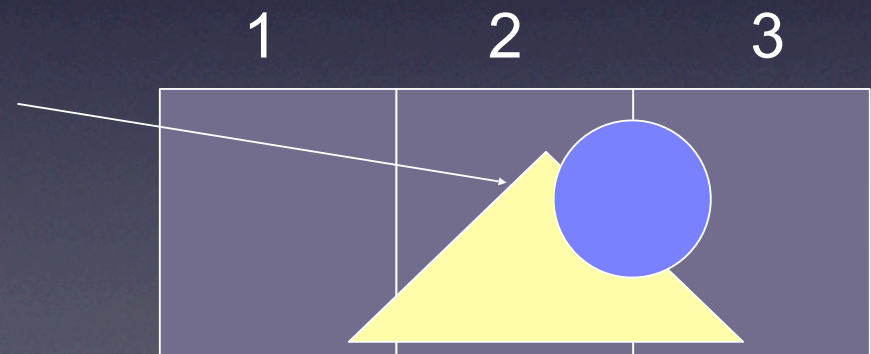
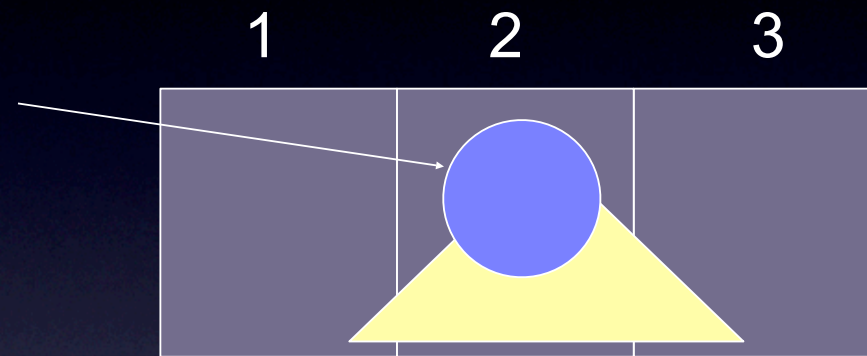
Heightfield traversal

- Step 12: Decide if it is time to stop
 - Stop if hit the surface at $t > \text{epsilon}$
 - Stop if $L.x == \text{stop}_x$ or $L.y == \text{stop}_y$
 - Stop if $t_{\text{near}} > t_{\text{far}}$
- Otherwise, back to step 8
- Grid:
 - Cannot stop if you find a hit!
 - Stop if $\text{hit.minT} < \text{new } t_{\text{near}}$
 - Stop if $L.x == \text{stop}_x$ or $L.y == \text{stop}_y$ or $L.z == \text{stop}_z$
 - $t_{\text{near}} > t_{\text{far}}$ condition is redundant



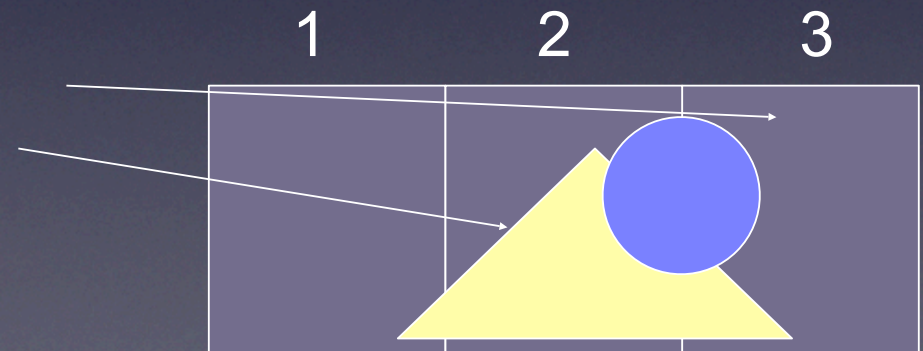
Stopping example

- When intersecting cell 1, ray finds yellow triangle, t outside of cell
- Proceed to next cell and intersect again, ray finds blue circle with smaller t
- Second example shows the opposite



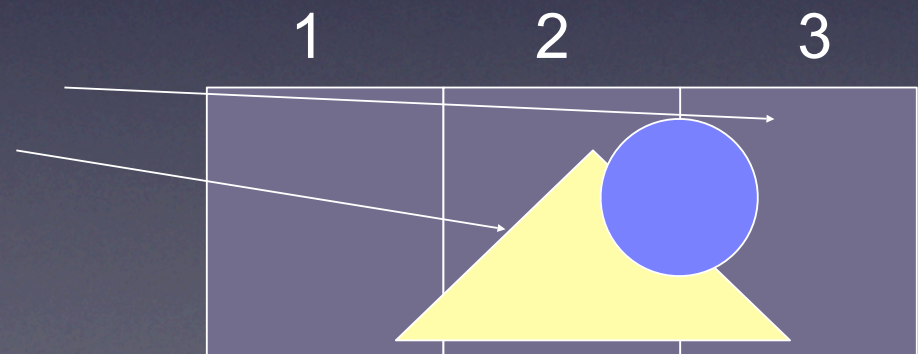
Extra work

- This example highlights a common grid problem: redundant intersections
- Ray can intersect yellow triangle up to three times!
- Worst case: ground polygon
- Gets worse with increased grid size



Avoiding extra work

- Don't worry about it? (not always effective)
- Mailbox: store unique ray ID with each object, don't call intersect if ray ID == last ray ID for object (NOT thread safe!)
- Remember last N intersected objects, don't re-intersect (extra overhead)
- Hierarchical grid



Building a Grid

Building a grid

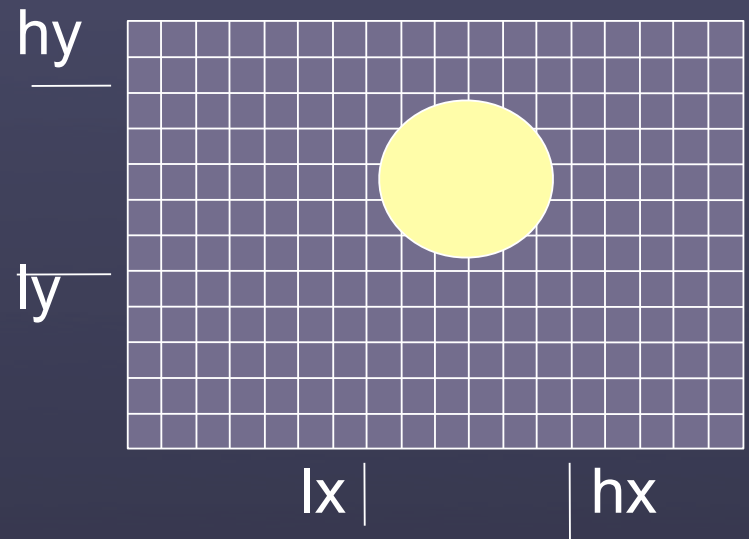
- Add two methods to your object class:
 - Required:
 - // Get the bounding box for this object
 - // Expand the input bounding box
 - void getBounds(BoundingBox& bbox);
 - Optional:
 - // Does the object intersect this box?
 - // always return true if you aren't sure
 - bool intersects(BoundingBox& cell_bbox);
- Methods to find these later

Building a grid

Foreach object:

Compute bounding box

Transform extents to index space (careful with rounding)



Rounding:

Lower: round down

Upper: round up

Building a grid

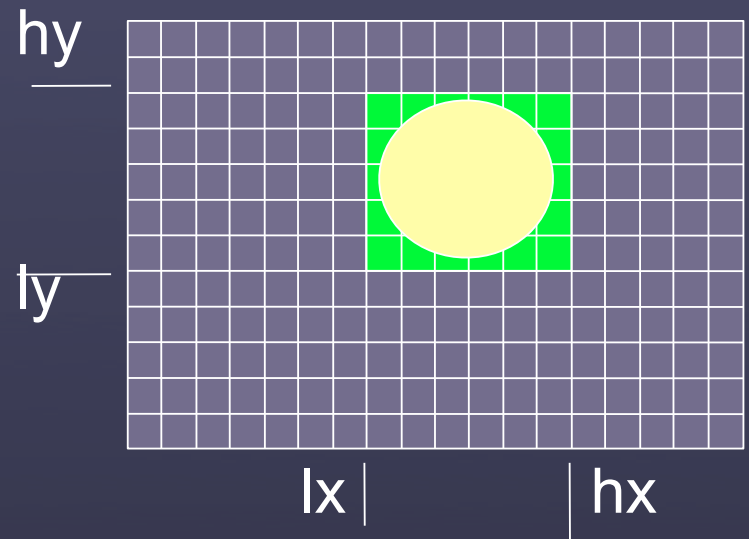
Foreach object:

Compute bounding box

Transform extents to index space (careful with rounding)

3D loop lx, ly, lz to hx, hy, hz :

Add object to each cell



Loop over green area

More efficient

Foreach object:

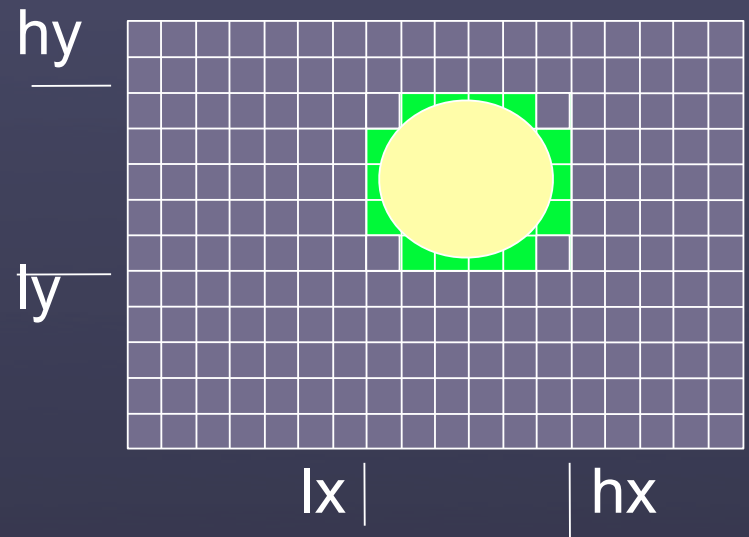
Compute bounding box

Transform extents to index space (careful with rounding)

3D loop lx, ly, lz to hx, hy, hz :

If(object intersects cell boundary)

Add object to each cell



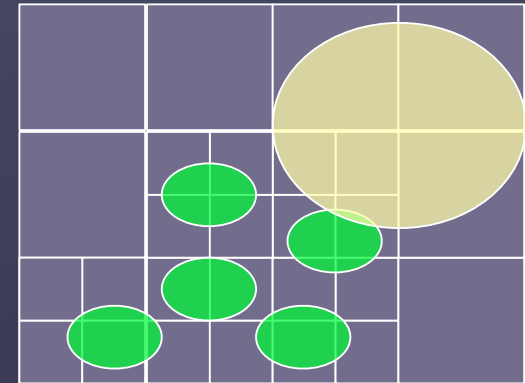
Blue cells won't get added with an additional check

Still more efficient

- Two pass algorithm:
 - First pass: count objects in each cell
 - Allocate memory all at once
 - Second pass: insert objects into list
- Huge memory savings over linked lists (2X to 8X or more)
- Huge memory coherence improvement

Other improvements

- Memory tiling (or bricking in 3D)
- Pseudo-tiling of contiguous object lists
- Hierarchical:
 - Objects only at bottom levels
 - Objects mixed throughout the tree
 - Lots of variations
- Octree:
 - Theoretically optimal
 - BUT traversal across grid is much faster than up/down grid



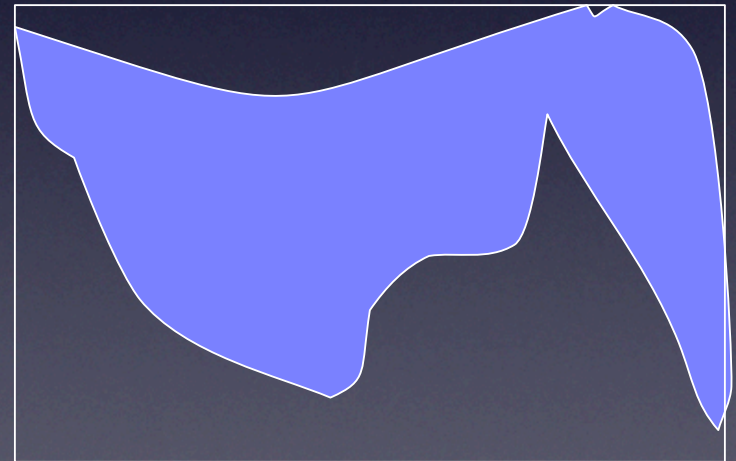
Grid summary

- Grids work very well for objects of uniform size
- Should be easy if you understood the heightfield traversal
- Build is straightforward

- Possibly large memory requirements
- Grid spacing requires tuning (tradeoff memory consumption and redundant intersections vs. efficiency)

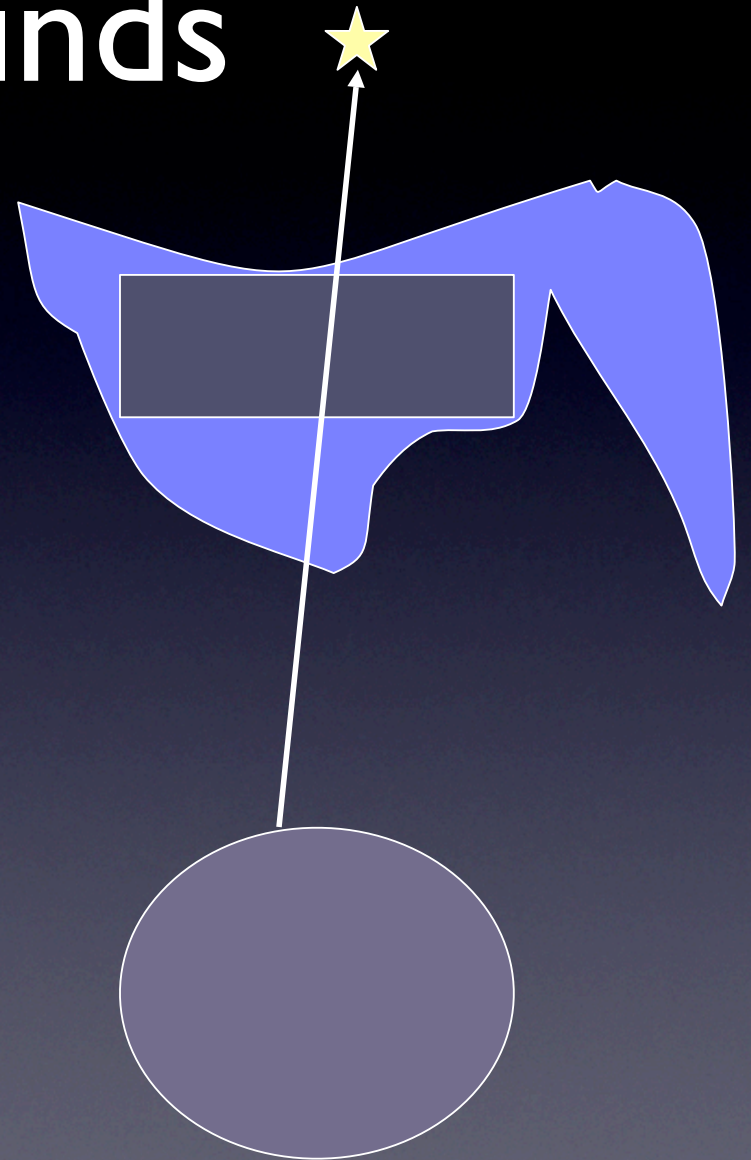
Bounding primitives

- Optimize intersections
- Enclose expensive objects in a simpler primitive
- If a ray misses the bounds, it misses the object



Inner bounds

- Can also be used to know that a ray hits a particular object
- Only good for shadows
- Rarely used



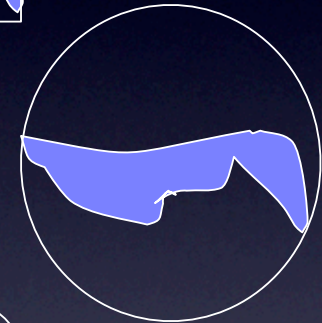
Bounding primitives

- Tradeoff: tight bounds vs. intersection speed

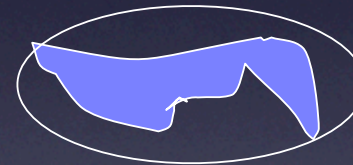
- Box



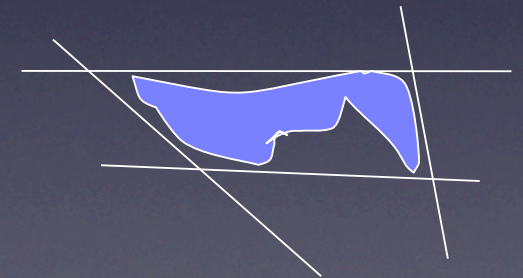
- Spheres



- Ellipsoids



- Slabs

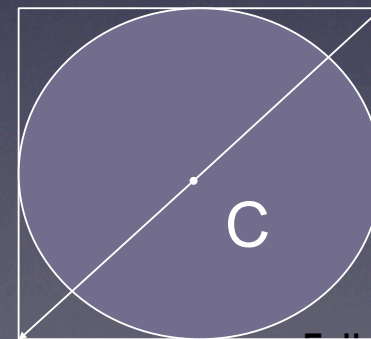
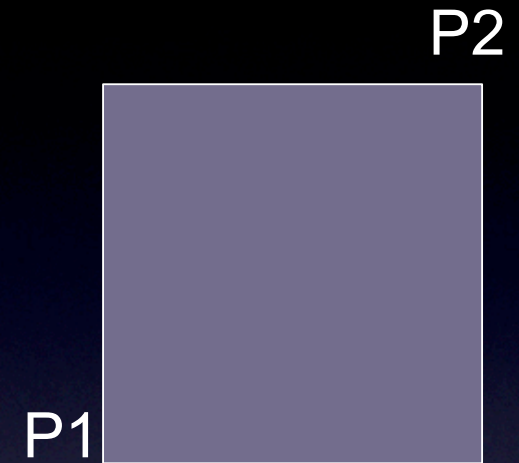


- Box is most common

- Depends on the relative costs of intersection

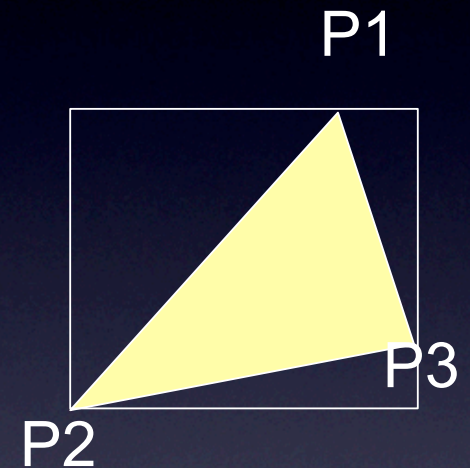
Bounding boxes

- Box: easy
- $\text{bounds} = \text{Min}(p1, p2), \text{Max}(p1, p2)$
- Sphere:
- $\text{bounds} = \text{C-Vector}(\text{radius}, \text{radius}, \text{radius})$
- $\text{C+Vector}(\text{radius}, \text{radius}, \text{radius})$



Bounding Boxes

- Triangle:
 - $\text{bounds} = \text{Min}(p1, p2, p3), \text{Max}(p1, p2, p3)$



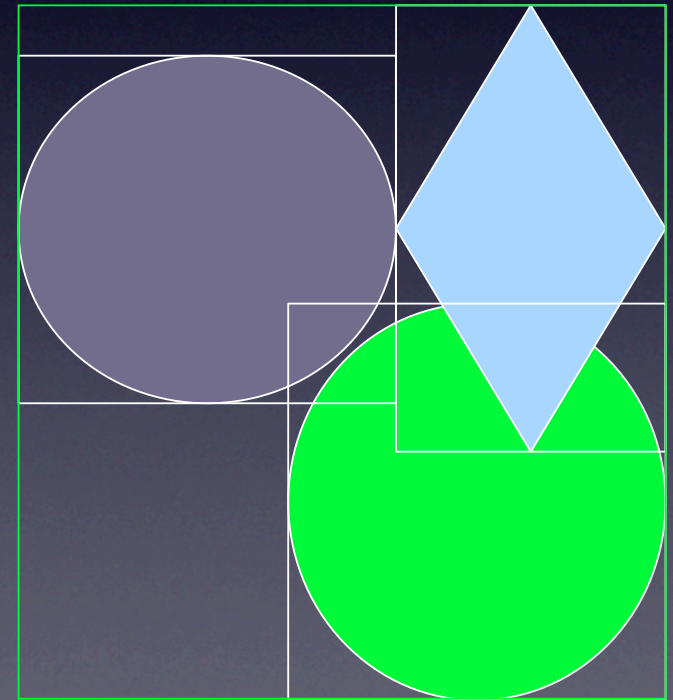
- Plane
 - infinite bounds
 - You may want to consider removing planes from your renderer at this point
 - Or keep them outside of your accel structures

Bounding boxes

- Disc/Ring: $C_x \pm rad \sqrt{N_y^2 + N_z^2}$
 $\|\vec{N}\| = 1$
Similar for y/z



- Group:
- Union of object bounding boxes
- min of mins
- max of maxs

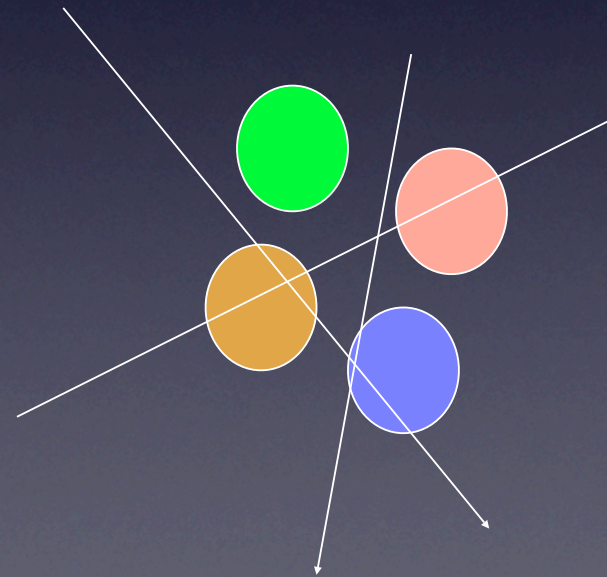


Uses for bounding boxes

- Quick reject for expensive primitives
- To fill in grid cells
- Directly in Bounding Volume Hierarchy or similar

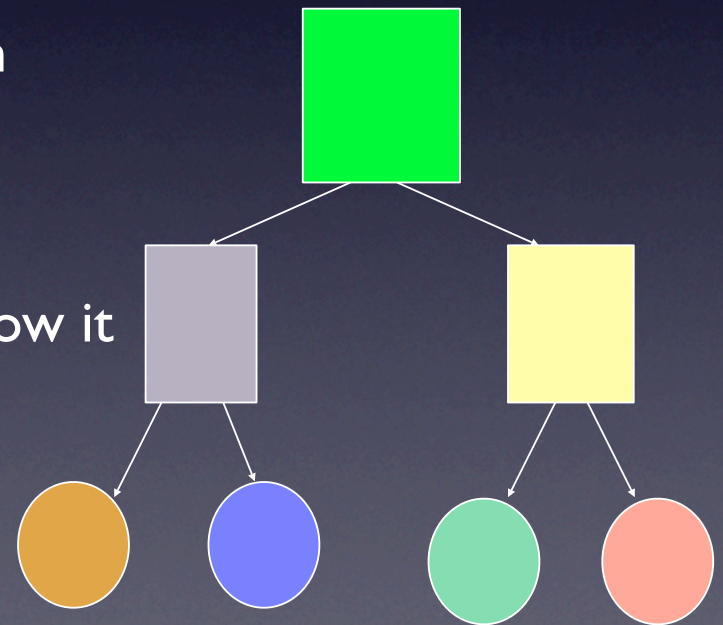
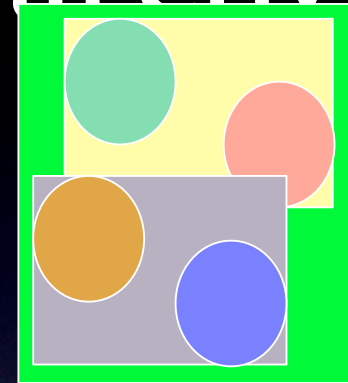
Bounding Volume Hierarchy

- Observe that intersection is like a search
- We know that searching is $O(n)$ for unsorted lists but $O(\log n)$ for sorted lists
- How do we sort objects from all directions simultaneously?



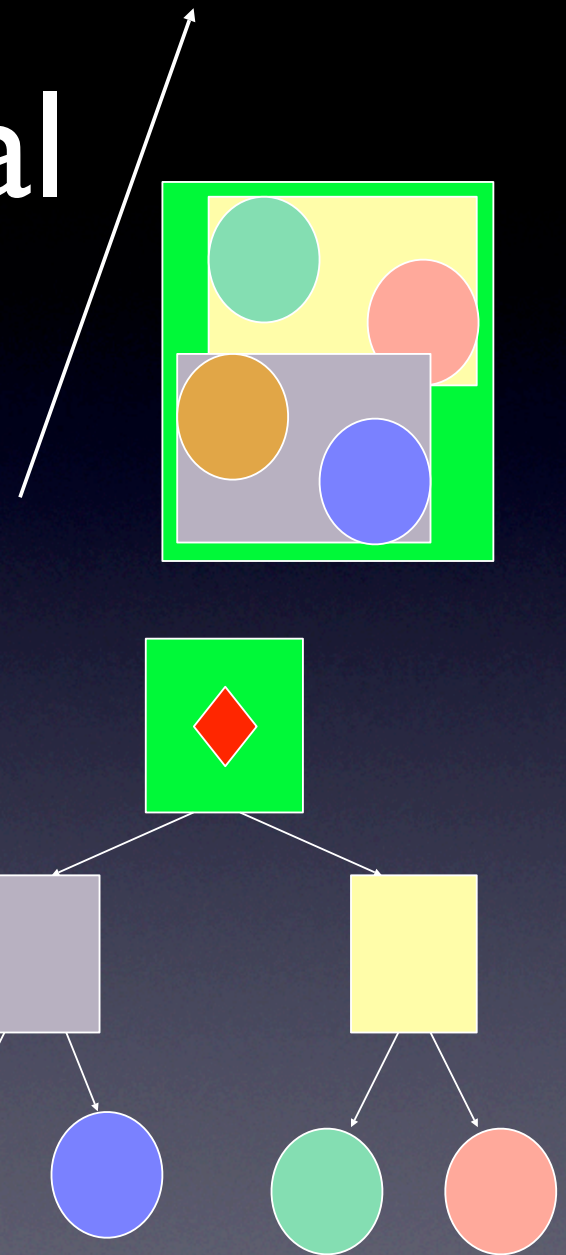
Bounding volume hierarchy

- Organize objects into a tree
- Group objects in the tree based on spatial relationships
- Each node in the tree contains a bounding box of all the objects below it



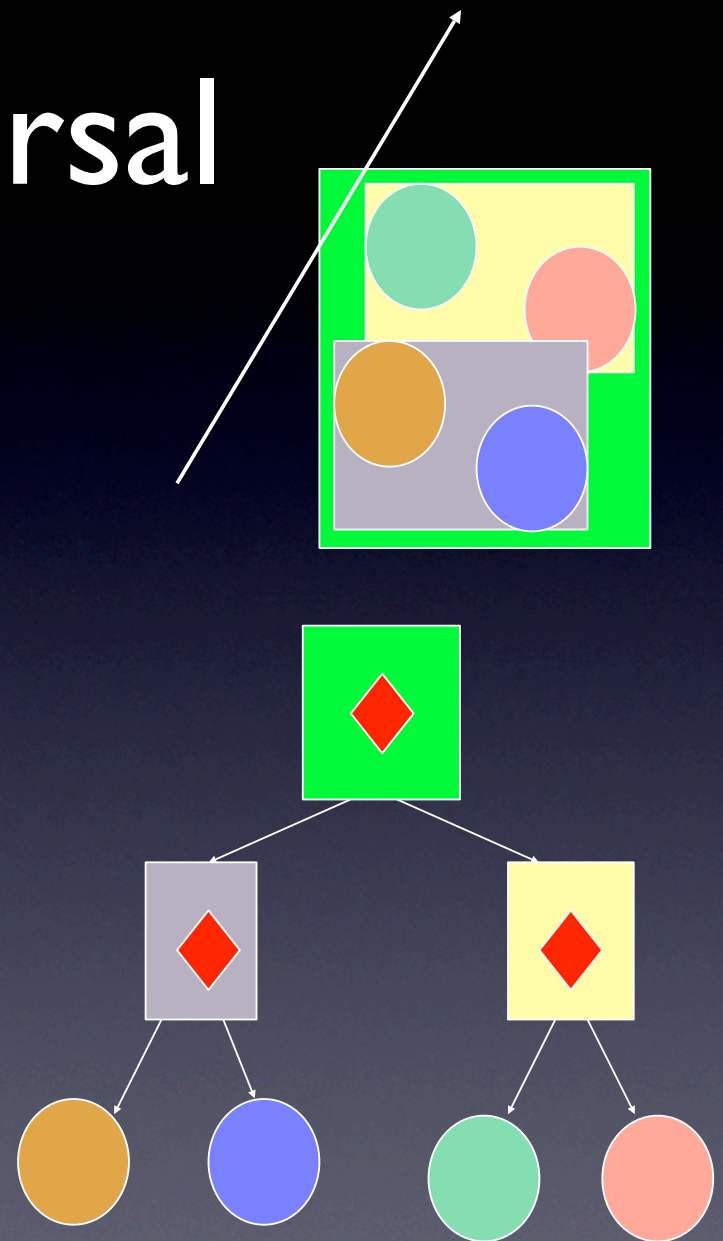
BVH traversal

- At each level of the tree, intersect the ray with the bounding box
- miss: ray misses the entire subtree
- hit: recurse to both children



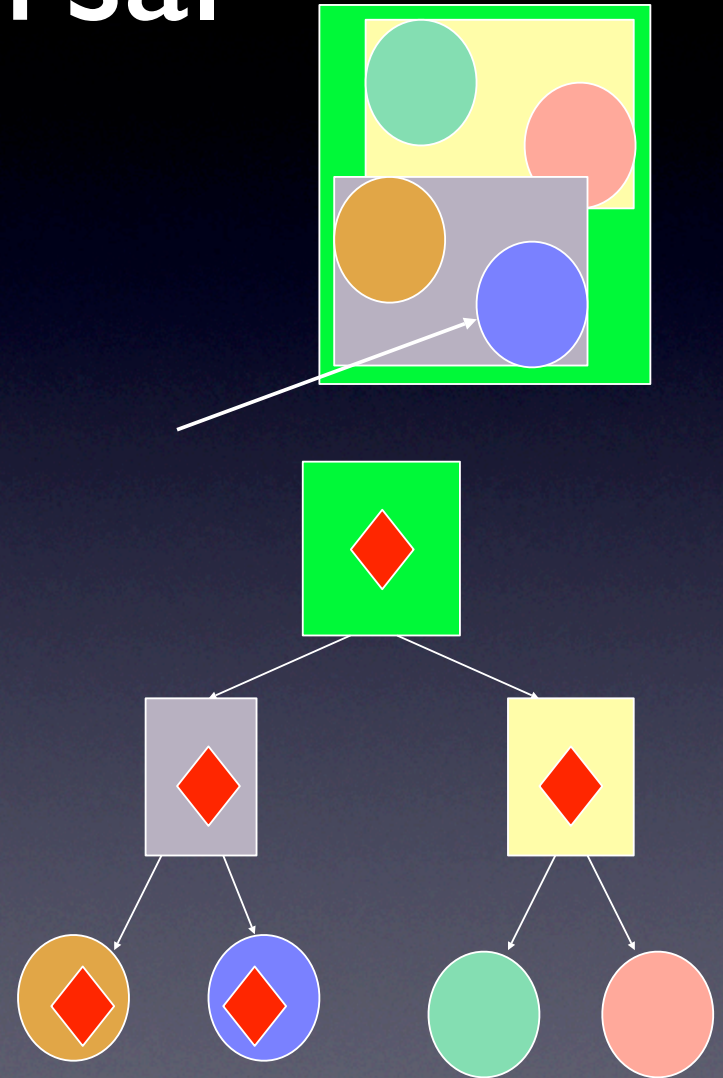
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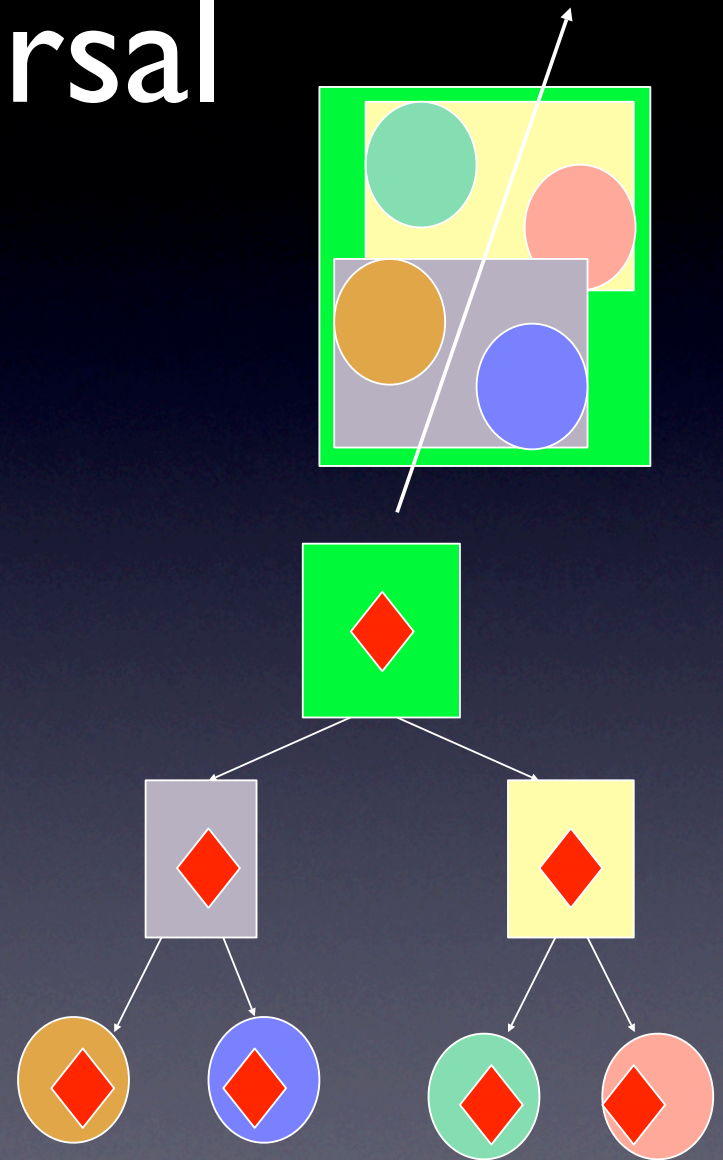
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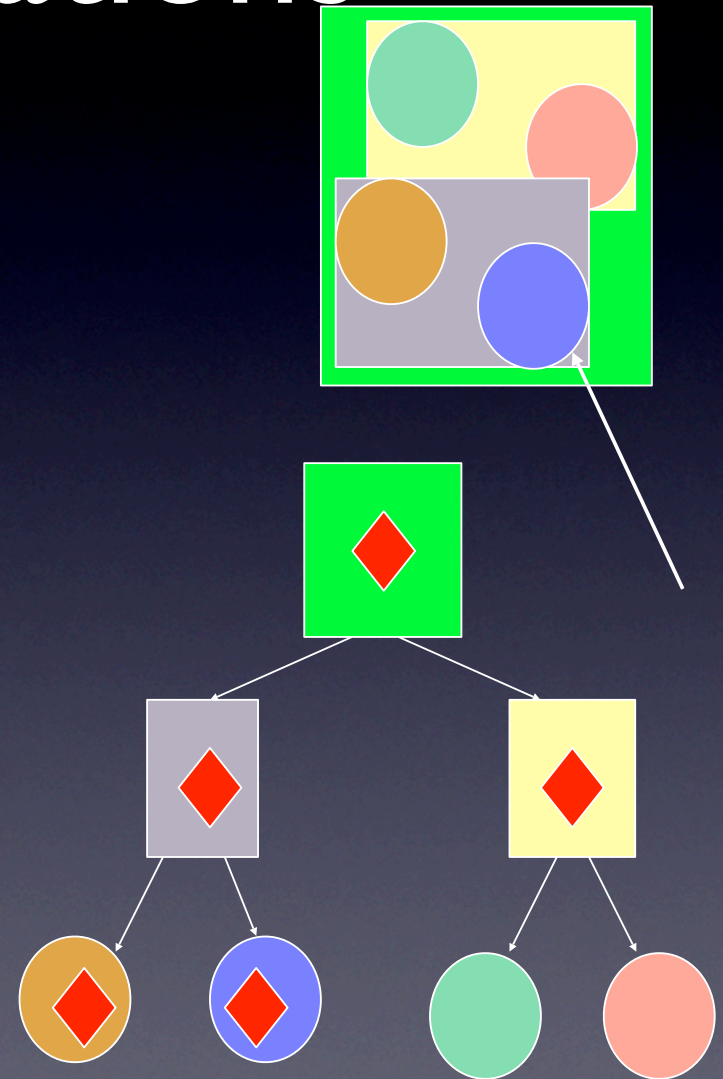
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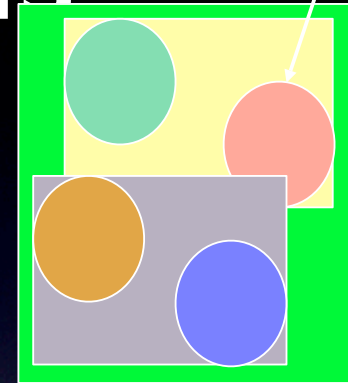


BVH optimizations

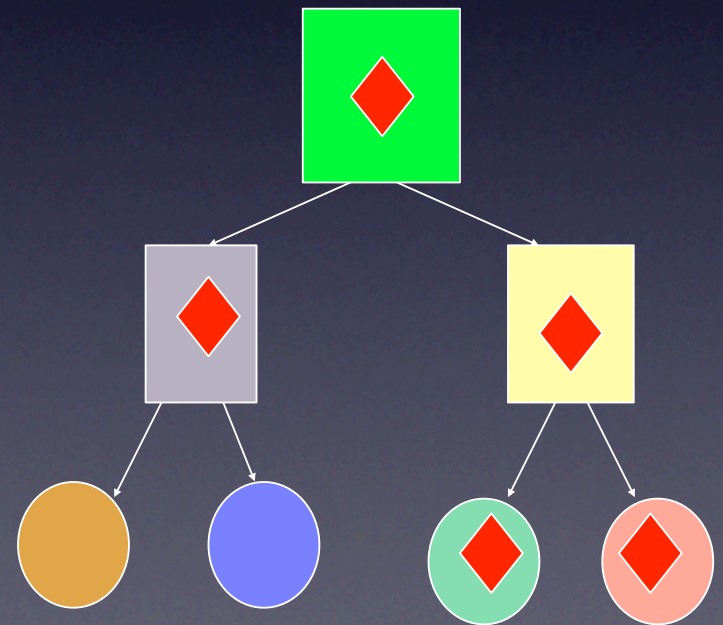
- Stop if the current T value is closer than the BVH node



BVH optimizations

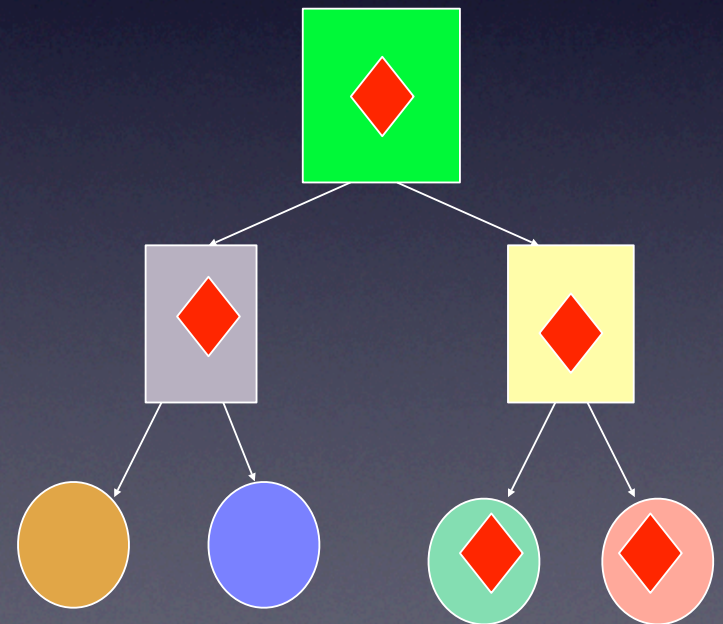
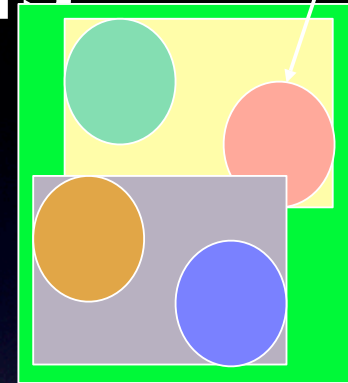


- Stop if the current T value is closer than the BVH node
- Traverse down side of tree that is closer to origin of the ray first



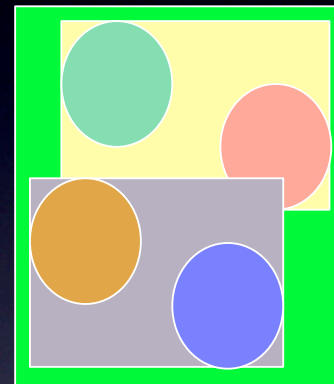
BVH optimizations

- Stop if the current T value is closer than the BVH node
- Traverse down side of tree that is closer to origin of the ray first
- Three or more way split

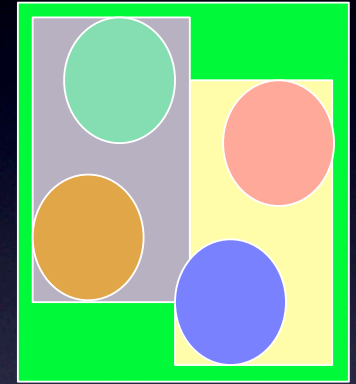


Building a BVH

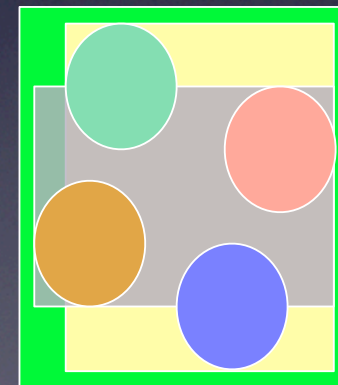
- Determining optimal BVH structure is NP-hard problem
- Heuristic approaches:
 - Cost models (minimize volume or surface area)
 - Spatial models
- Categories of approaches:
 - Top down
 - Bottom up



okay



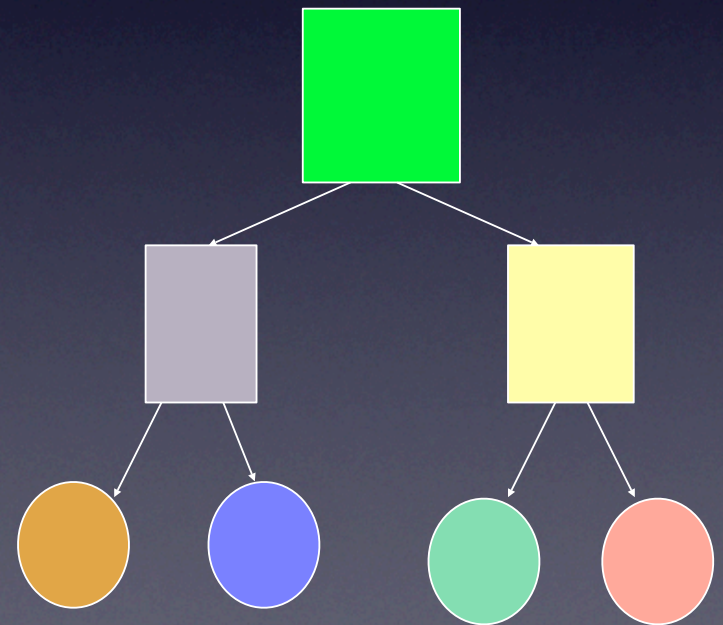
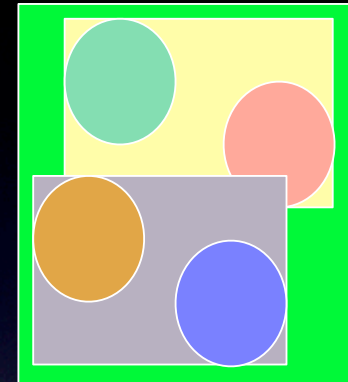
okay



bad

Median cut BVH construction

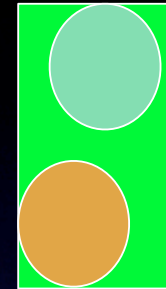
- Top down approach:
- Sort objects by position on axis
 - cycle through x,y,z
 - use center of bounding box
- Insert tree node with half of objects on left and half on right



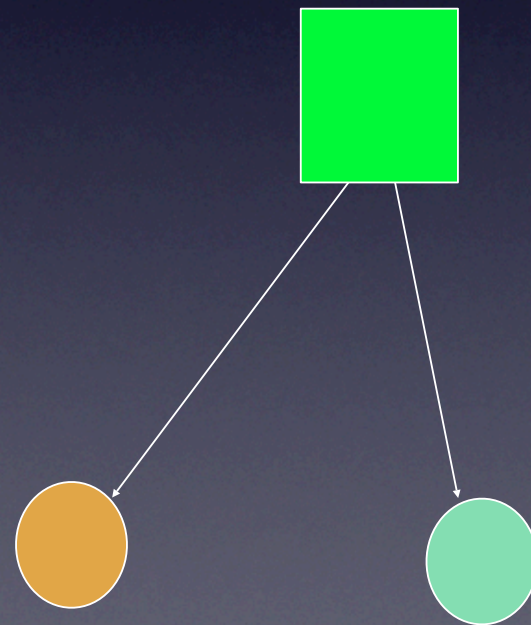
Weghorst BVH construction

- Bottom up construction
- Add objects one at a time to tree
- Insert to subtree that would cause smallest increase to area

Weghorst BVH construction

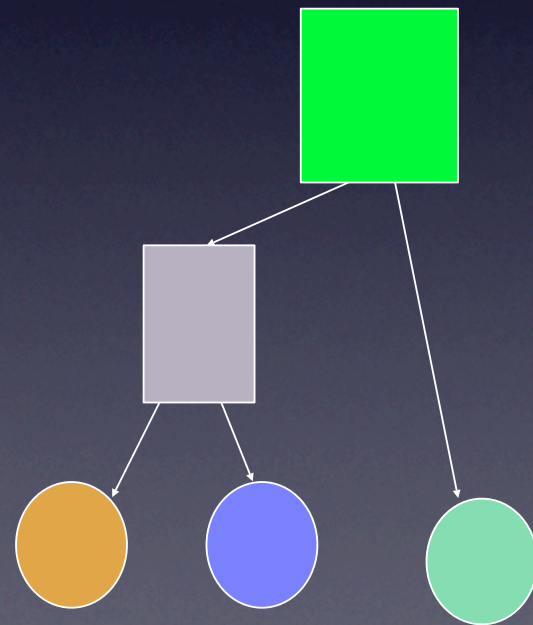
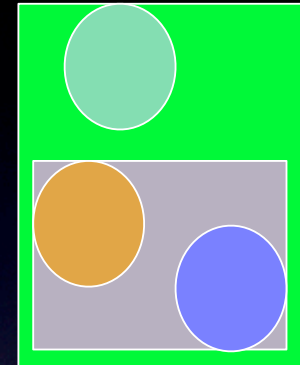


- Bottom up construction
- Add objects one at a time to tree
- Insert to subtree that would cause smallest increase to area



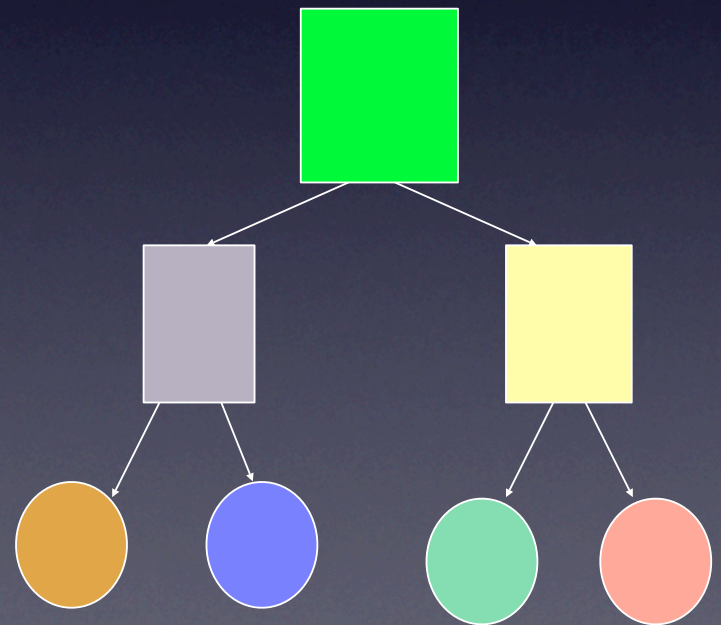
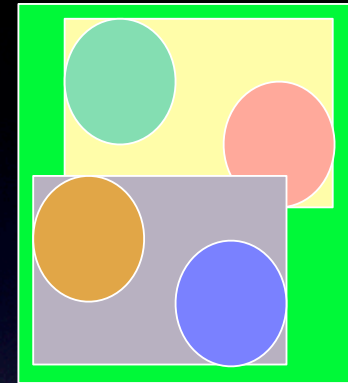
Weghorst BVH construction

- Bottom up construction
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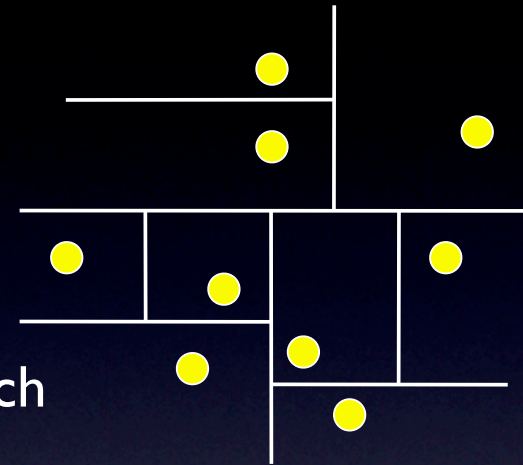
Weghorst BVH construction

- Bottom up construction
- Add objects one at a time to tree
- Insert to subtree that would cause smallest increase to area



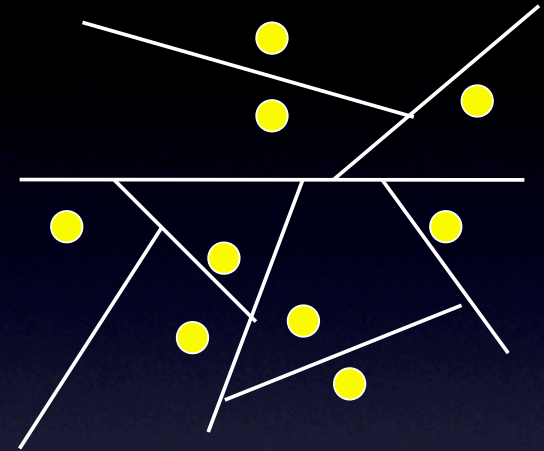
k-d tree

- Recursively divide space in half
 - Alternate coordinate axes
 - Cycle through axes or store axis split in each node
-
- What do you do with objects split by the plane?
 - Hard part: where do you split in each dimension?



BSP tree

- Like a k-d tree where splitting planes can be arbitrarily located



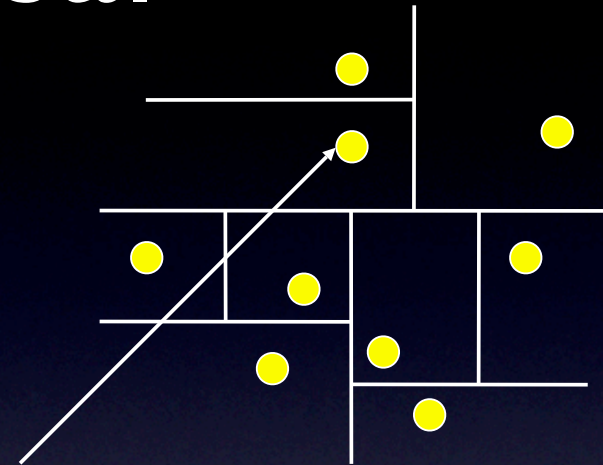
- Hard part: where do you split in each dimension?
- Harder part: how do you orient each plane?
- More storage/computation, tighter bounds

BSP/k-d tree tradeoffs

- Build is NP-hard problem
- Heuristic approaches are always used
- Traversal is quick
- Storage can be lower than BVH or grid

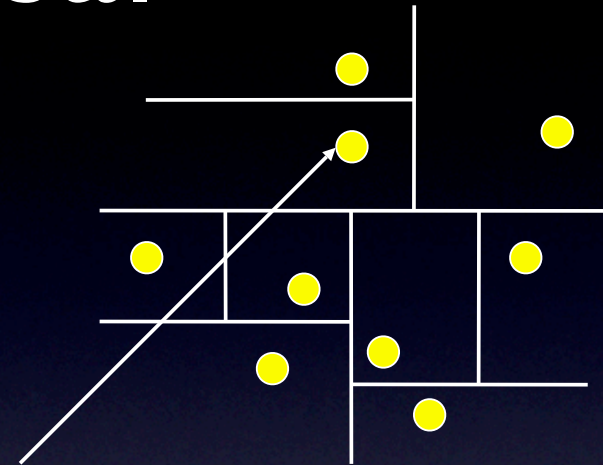
K-d traversal

- Keep track of intervals on ray:
 - min: 0
 - max: ∞
- Compute t of split plane
- If $t > \max$: goto near child
- If $t < \min$: goto far child
- Otherwise: goto both children with updated intervals



K-d traversal

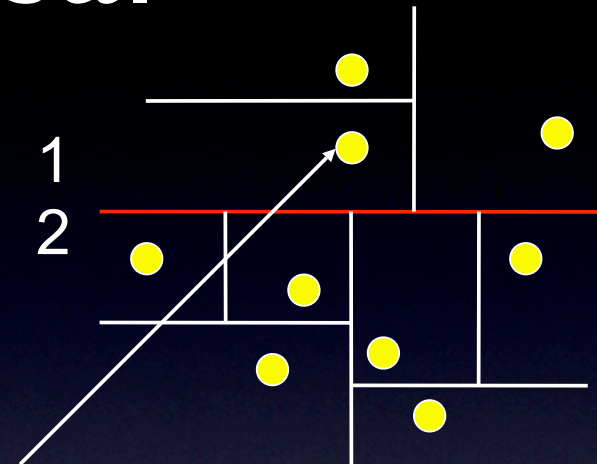
- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \max$: goto near child
 - If $t < \min$: goto far child
 - Otherwise: goto both children with updated intervals



Min: 0
Max: ∞

K-d traversal

- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \max$: goto near child
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 - Otherwise: goto both children with updated intervals



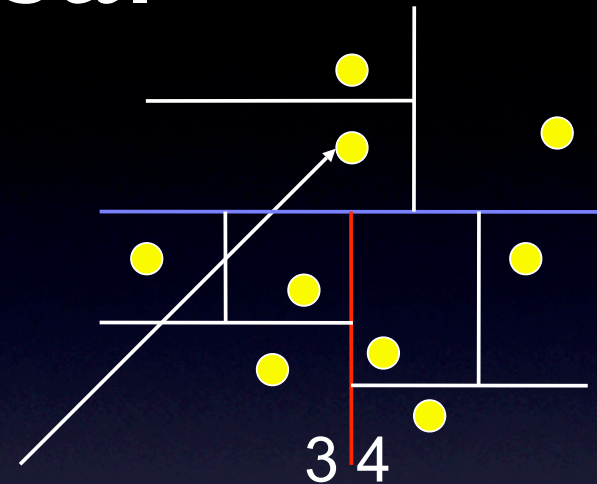
Min: 0
Max: ∞

Split: 1.0
Stack: 1

New min: 0
New max: 1.0

K-d traversal

- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \max$: goto near child
 - If $t < \min$: goto far child
 - Otherwise: goto both children with updated intervals



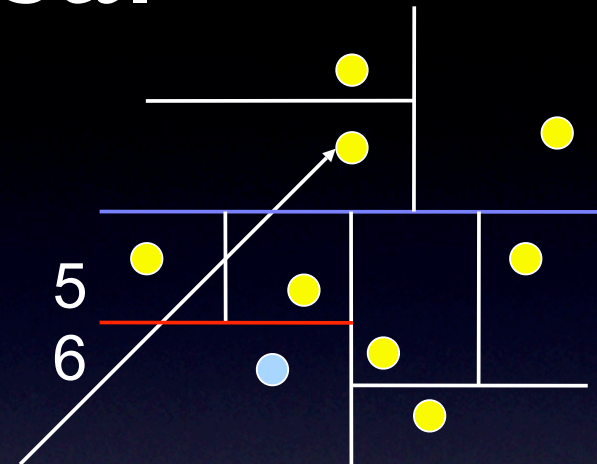
Min: 0
Max: 1.0

Split: 1.3
Stack: 1

New min: 0
New max: 1.0

K-d traversal

- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \text{max}$: goto near child
 - If $t < \text{min}$: goto far child
 - Otherwise: goto both children with updated intervals



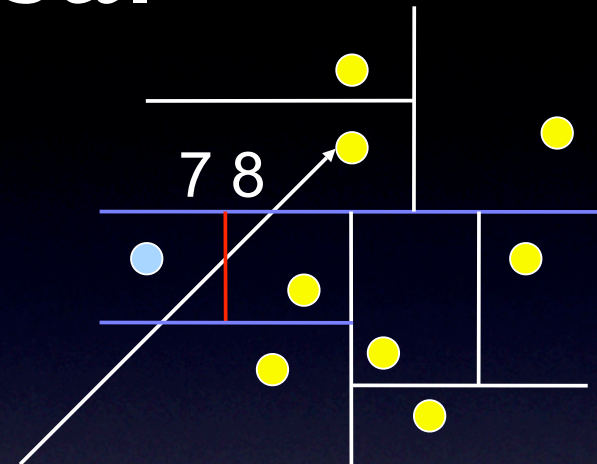
Min: 0
Max: 1.0

Split: 0.4
Stack: 1 5

New min: 0
New max: 0.4

K-d traversal

- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \text{max}$: goto near child
 - If $t < \text{min}$: goto far child
 - Otherwise: goto both children with updated intervals



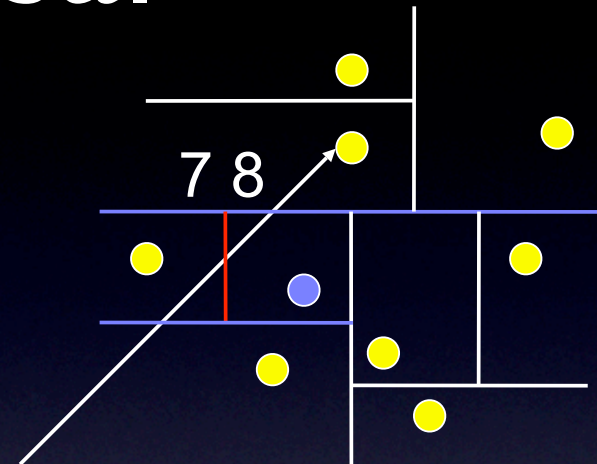
Min: 0.4
Max: 1.0

Split: 0.7
Stack: 1 8

New min: 0.4
New max: 0.6

K-d traversal

- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \max$: goto near child
 - If $t < \min$: goto far child
 - Otherwise: goto both children with updated intervals



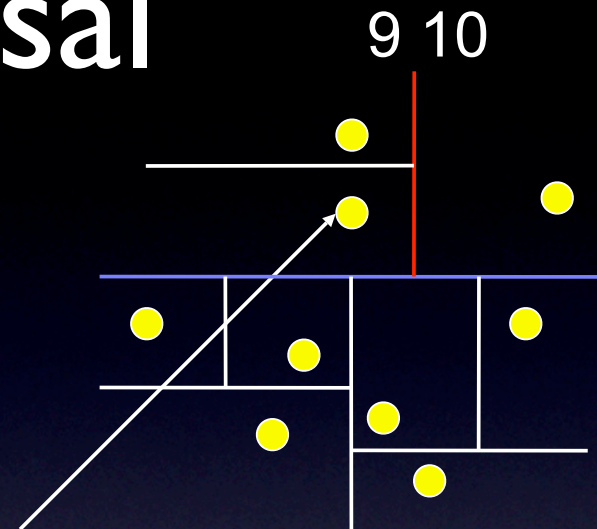
Min: 0.7
Max: 1.0

Split:
Stack: 1

New min: 0.4
New max: 0.6

K-d traversal

- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \max$: goto near child
 - If $t < \min$: goto far child
 - Otherwise: goto both children with updated intervals



Min: 1.0

Max: ∞

Split: 1.5

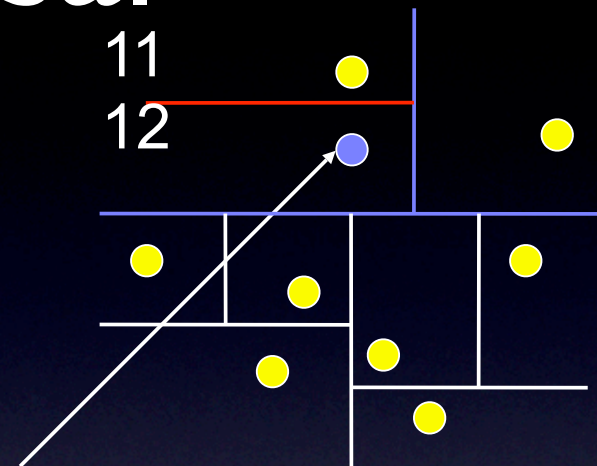
Stack: 10

New min: 1.0

New max: 1.5

K-d traversal

- Keep track of intervals on ray:
 - Compute t of split plane
 - If $t > \max$: goto near child
 - If $t < \min$: goto far child
 - Otherwise: goto both children with updated intervals



Min: 1.0
Max: 1.5

Split: 1.4
Stack: 10 11

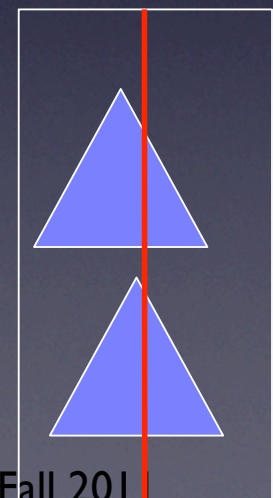
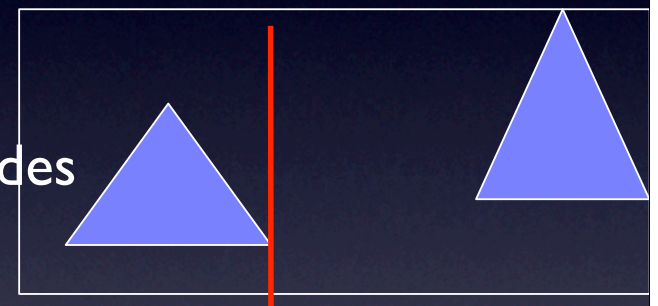
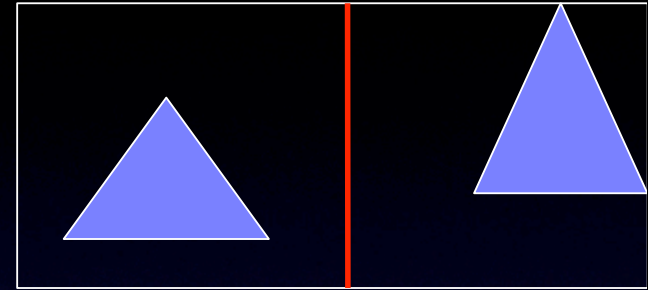
New min: 1.0
New max: 1.4

K-d tree build

- Optimal solution is NP-hard
- Spatial median split (a little like octree)
- Object median split (makes balanced trees)
- Cost model based (better)
 - Cost of traversal
 - Cost of intersection
 - Probability of hit

K-d tree build

- Havran '01:
- Start with bounding box of scene
- Select split plane along each axis
 - Start with spatial median
 - Move toward bounding box of child nodes
- Recurse
 - Stop if node contains 2-3 objects
 - Stop if depth $>$ max (20-30)
- Backtrack
 - Combine children with identical content



Acceleration Structure Summary

- Most common: Grid, Hierarchical Grid, k-d tree, BVH
- Grid based:
 - P time deterministic build
 - Grid resolution tradeoff
- Tree based:
 - NP hard build
 - Heuristic approaches