

Tessellation: Tiling a plane

- Filling a plane with a shape or image – no gaps
- From Latin "tessella" - a small cubical piece of clay, stone or glass used to make mosaics.
 - The word "tessella" means "small square"
 - ... from "tessera" - square
 - ... which in its turn is from the Greek word for "four"

Wikipedia...

Regular Tessellations

- Only three "regular" tilings – all the same shape
 - Variations on these regular shapes work too...

Semi-Regular Tessellations

- Two or more regular polygons – 8 possibilities

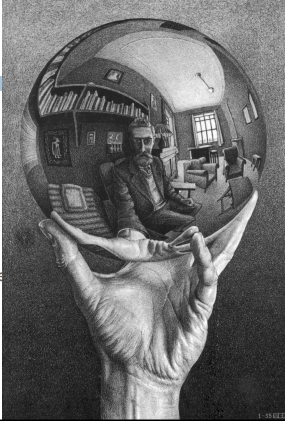
Demiregular Tessellations

- Harder to define...

Tile a plane with images - Escher

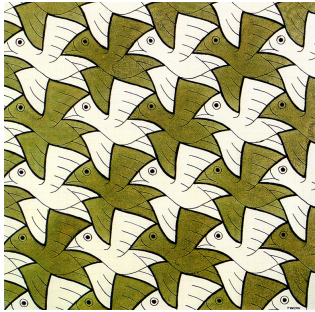
M. C. Escher

- Dutch graphic artist – (1898 – 1972)
 - ▣ known for his often mathematically inspired woodcuts, lithographs, and mezzotints.
 - ▣ These feature impossible constructions, explorations of infinity, architecture, and tessellations.




Wikipedia...

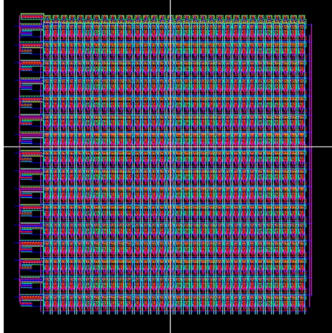
Tile a plane with images - Escher



Tile a plane with images - Escher



VLSI Version?

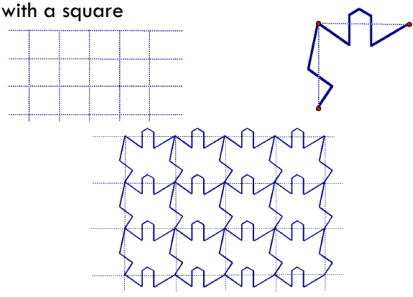


How to do it...

- Exploit symmetries
 - ▣ Translation
 - ▣ Rotation
 - ▣ Reflection
- Start with a regular tessellation
 - ▣ square, triangle, hexagon
- Then apply transformations
 - ▣ and see what they end up looking like

Simple example: Translation

- Start with a square



http://euler.slu.edu/escher/index.php/Tessellations_by_Recognizable_Figures

Simple example: Translation

- Start with a square in this example
- It works for any of the regular tessellations

The Tessellation Project

Simple example: "Glide Reflection"

- Start with a triangle...

The Tessellation Project
 Sarah Pele or Buz Garret Lum, Geometry student

Simple example: Reflection

- Start with a triangle...

Tessellations.org

Simple example: Reflection

- Start with a triangle...

Tessellations.org

Escher Translation

The four steps demonstrate how Escher may have created his "Pegasus-type" tessellation.

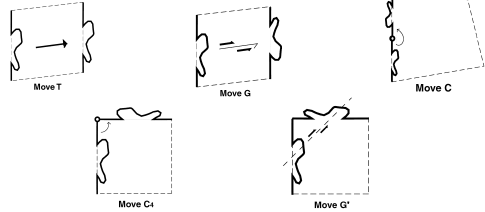
Step 1 Step 2 Step 3 Step 4

Translation

http://euler.slu.edu/escher/index.php/Tessellations_by_Recognizable_Figures

Other Symmetries

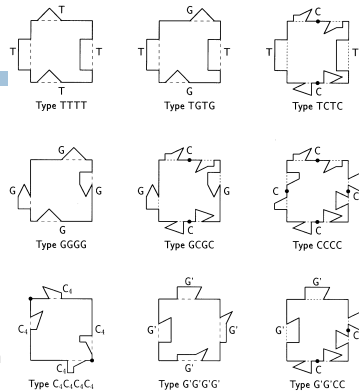
- Translation, Rotation, Reflection
 - Using squares, triangles, and hexagons



<http://www.dustinipurdy.com/Math/PDF/Tessellation.pdf>

The Nine Heesch Types

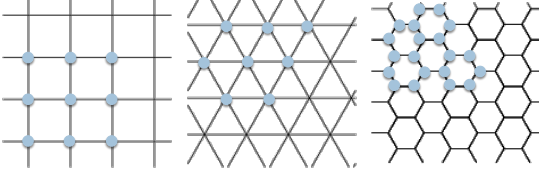
- Tessellation classification code developed by German mathematician Heinrich Heesch



<http://www.dustinipurdy.com/Math/PDF/Tessellation.pdf>

For your Tessellations...

- Start with a regular tessellation grid
 - vertices only can be a good way to go too



- Then start tweaking the lines according to the transformations and see what it looks like

Some Web Sources – good info...

- [http://euler.slu.edu/escher/index.php/Tessellations by Recognizable Figures](http://euler.slu.edu/escher/index.php/Tessellations%20by%20Recognizable%20Figures)
- <http://www.dustinipurdy.com/Math/PDF/Tessellation.pdf>
- <http://library.thinkquest.org/16661/>
- <http://tessellations.org/>
- <http://www.mcescher.com/>

One more – Escher Reptiles

