CS 6620
Shading

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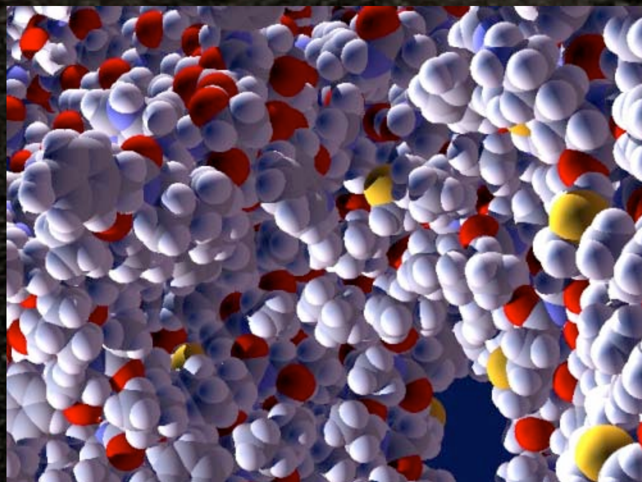
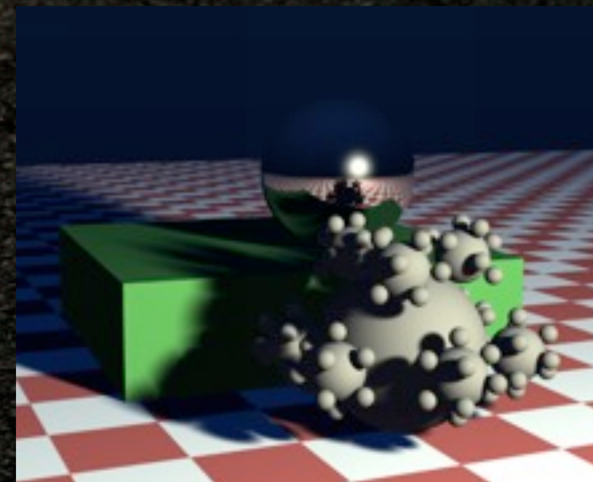
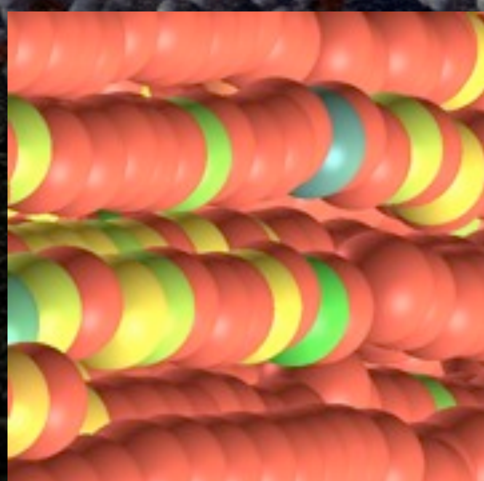
Shading models





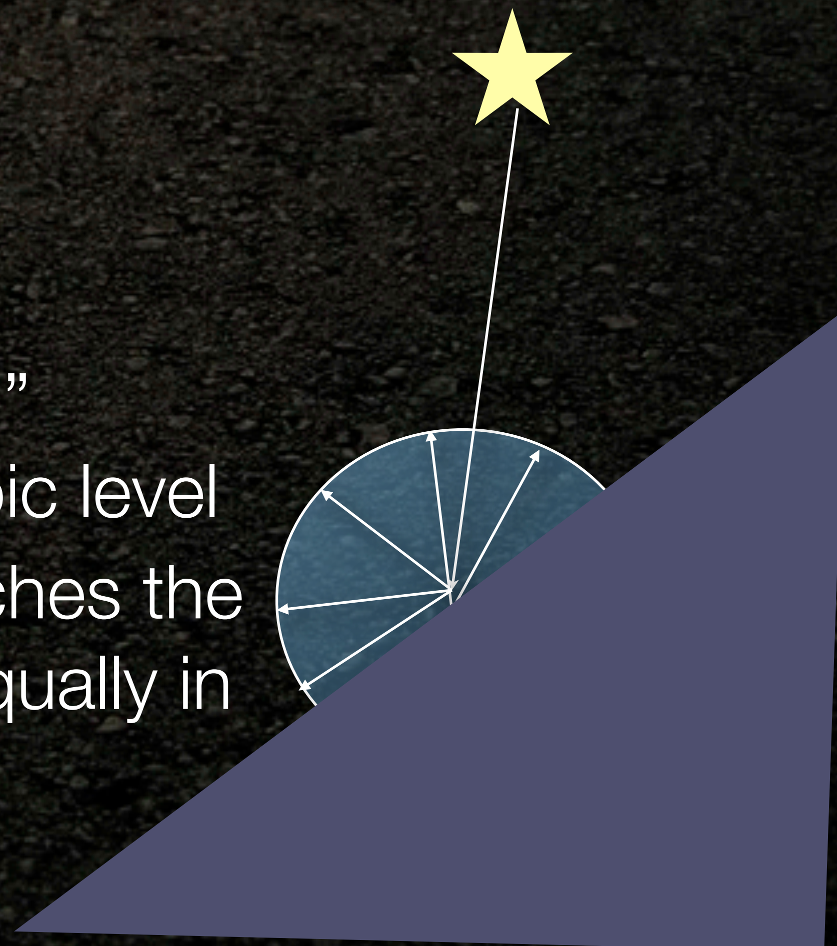


Lambert's cosine law

- Light reaching surface is proportional to projected area: $\cos \theta$

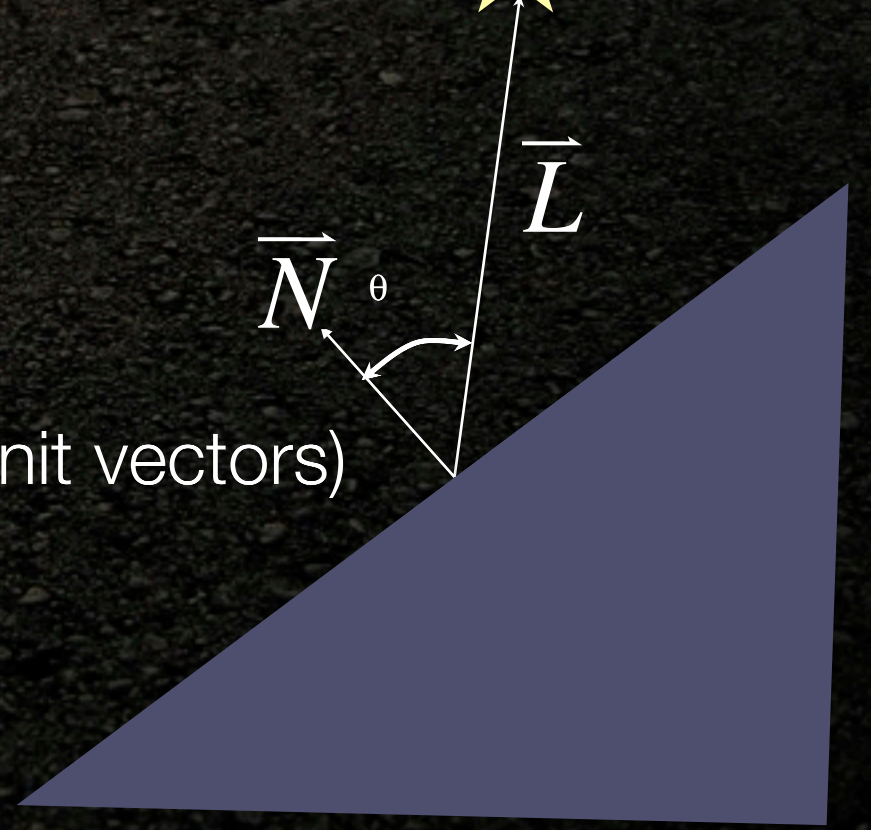
Lambertian shading

- Comes from a “rough” surface (at microscopic level
- Simple: light that reaches the surface is reflected equally in all directions



Lambertian shading

$$(\overrightarrow{N} \cdot \overrightarrow{L})C_L$$



- Color at surface:
- (where N and L are unit vectors)

Ambient light

- With this mechanism, the light in a shadowed region is 0 (black)
- To avoid this, use "ambient" lighting

$$C_{ray} = C_{surface}\left[(\overrightarrow{N} \cdot \overrightarrow{L})C_{Light}K_{d} + C_{ambient}K_{a}\right]$$

Guessing ambient light

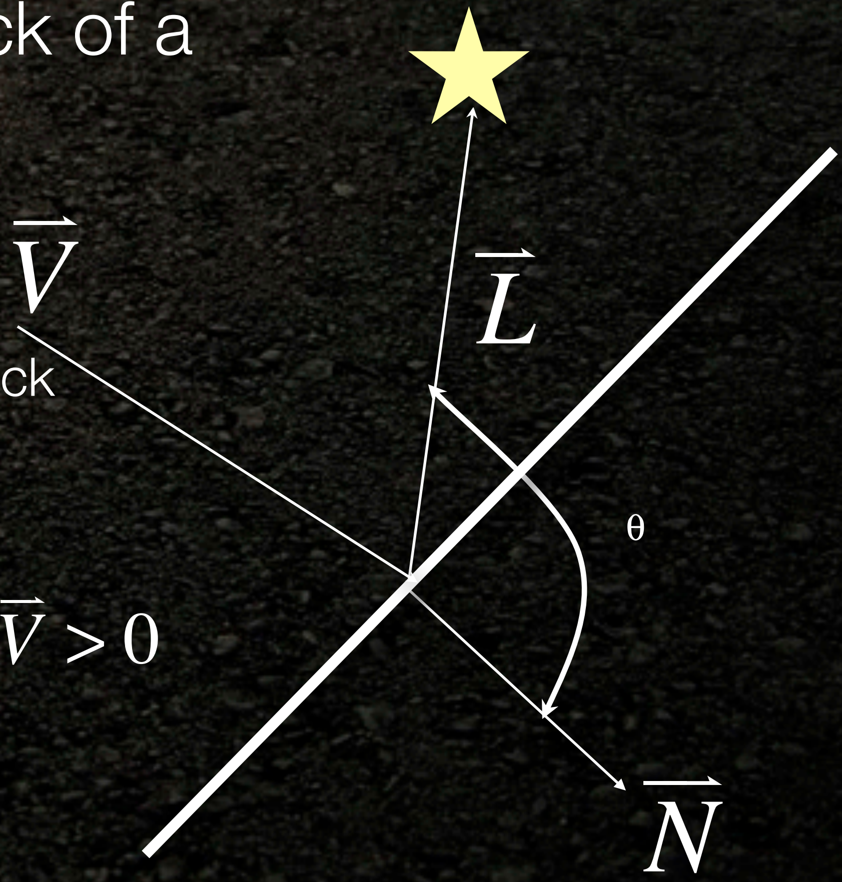
- It should be the average color of surfaces visible to the point being shaded
- What does that imply about outdoors?

Two-sided lighting

- What if light hits the back of a polygon
- Options:
  - Black on back
  - Different materials for front/back
  - Two sided lighting:

okay: $\left|\overrightarrow{N} \cdot \overrightarrow{L}\right|$

better: $\text{Negate } \overrightarrow{N} \text{ if } \overrightarrow{N} \cdot \overrightarrow{V} > 0$



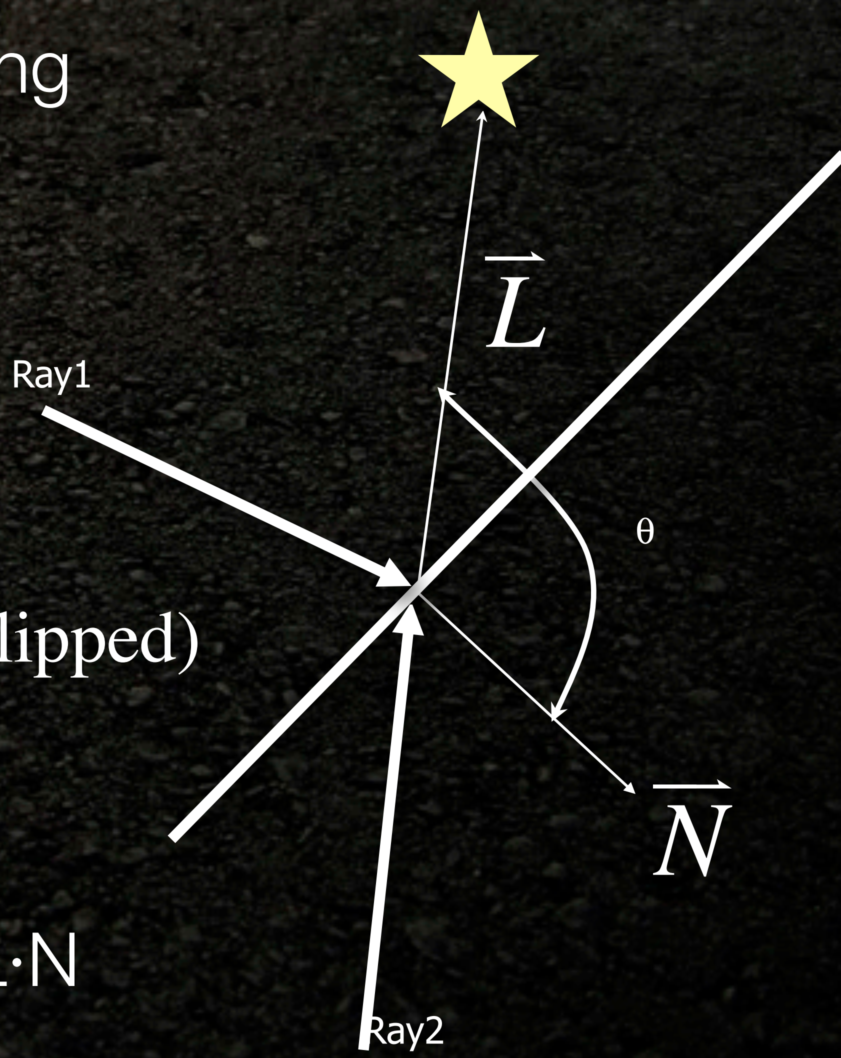
Two-sided lighting

- You might consider checking if the light is on the "right side" of the object

$$\mathrm{sign}(-\vec{V}\cdot\vec{N}) == \mathrm{sign}(\vec{L}\cdot\vec{N})$$

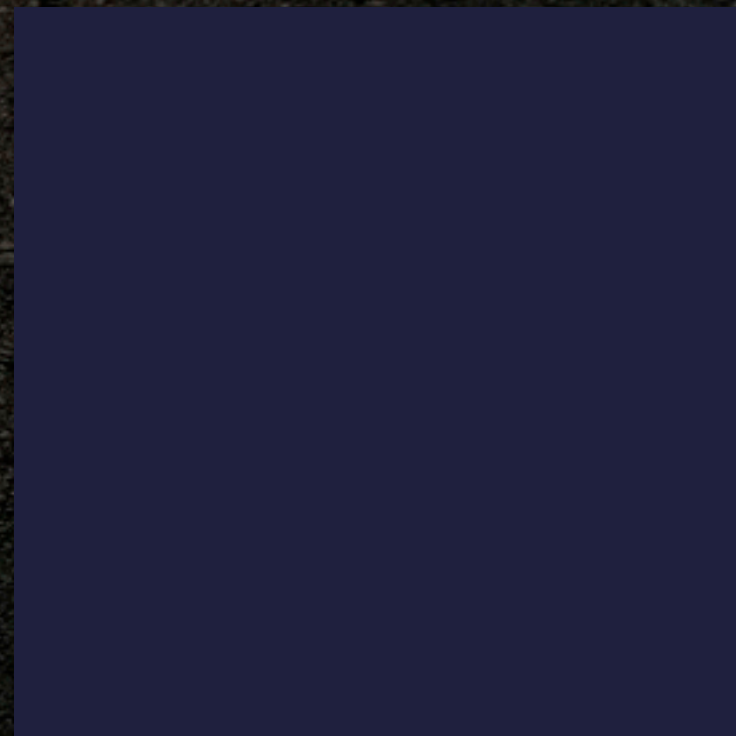
$$\left(\vec{V}\cdot\vec{N}\right)\left(\vec{L}\cdot\vec{N}\right) < 0 : \mathrm{lit}$$

$$\left(\vec{L}\cdot\vec{N}\right) > 0 : \mathrm{lit}\ (\text{when normal flipped})$$



- Can avoid casting shadow rays too
- Or use absolute value for L·N

Two-sided lighting



- Left: Normal two-sided lighting (Abs L·N)
- Right: Comparing signs

Lambertian Shading

Compute hit position ($\vec{P} = \vec{O} + t\vec{V}$)

Call primitive to get normal ($\vec{N}$) (normalized)

$costheta = -\vec{N} \cdot \vec{V}$

$if\,(costheta < 0)$

    normal =-normal

Color light = scene.ambient*Ka

foreach light source

    get C_L and $\vec{L}$

    dist=$\|\vec{L}\|, \vec{L_n} = \frac{\vec{L}}{\|\vec{L}\|}$

    $cosphi = \vec{N} \cdot \vec{L_n}$

    if($cosphi > 0$)

        if(!intersect with $0 < t < dist$)

            light += $C_L * (Kd * cosphi)$

result=light*surface color

Background

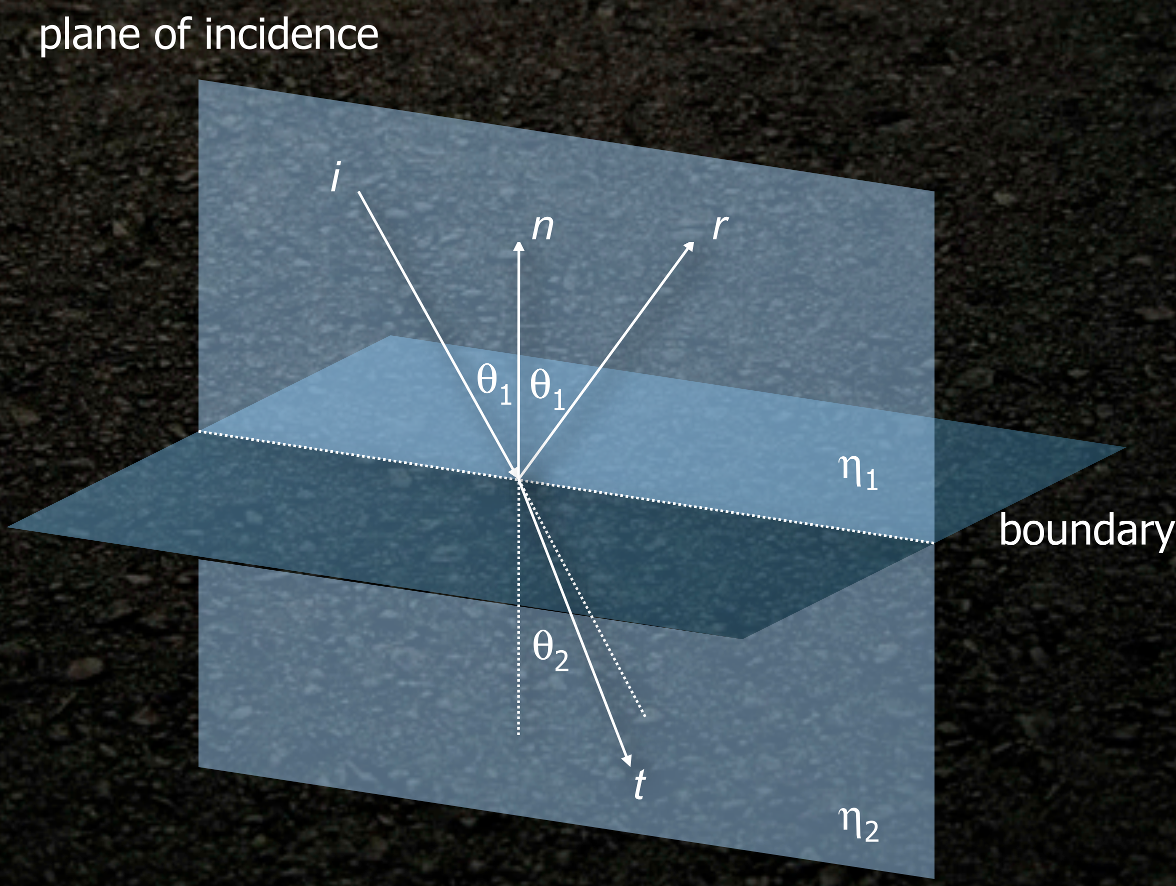
- What do you do when the ray misses all objects?
  - Constant color
    - Assign color to ray
  - Gradient
    - Map -1 to 1: $s = \dfrac{\overrightarrow{V_{ray}} \cdot \overrightarrow{V_{up}}}{\left\|\overrightarrow{V_{ray}}\right\| \left\|\overrightarrow{V_{up}}\right\|}$
    - Map to colors: $C_{down} + (\dfrac{s+1}{2})C_{up}$
  - Star field:
    - Sum up colors: $\sum_{stars} \left(\overrightarrow{V_{ray}} \cdot \overrightarrow{V_{star}}\right)^p C_{star}, p \cong 10000$
  - Environment map

Light transport

There are 4 primary ways that light interacts with a surface (are there hybrids?):

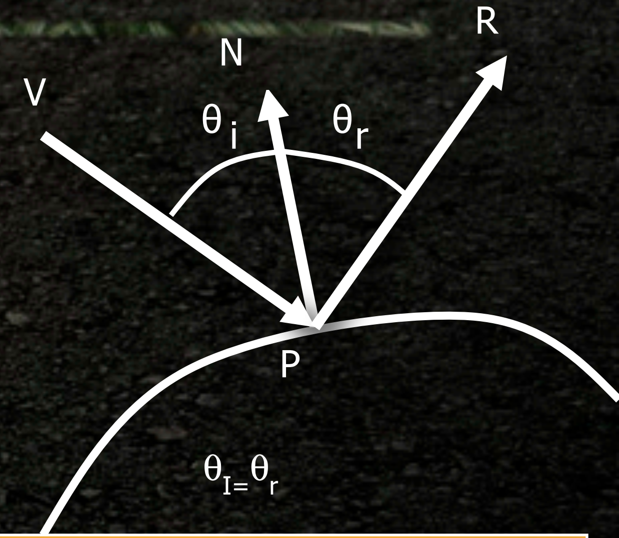
1. Bounces off (perfect specular reflection)
2. Absorbed and retransmitted in an arbitrary direction (perfect diffuse reflection)
3. Travels through surface (perfect specular transmission)
4. Absorbed and retransmitted on other side (perfect diffuse transmission)

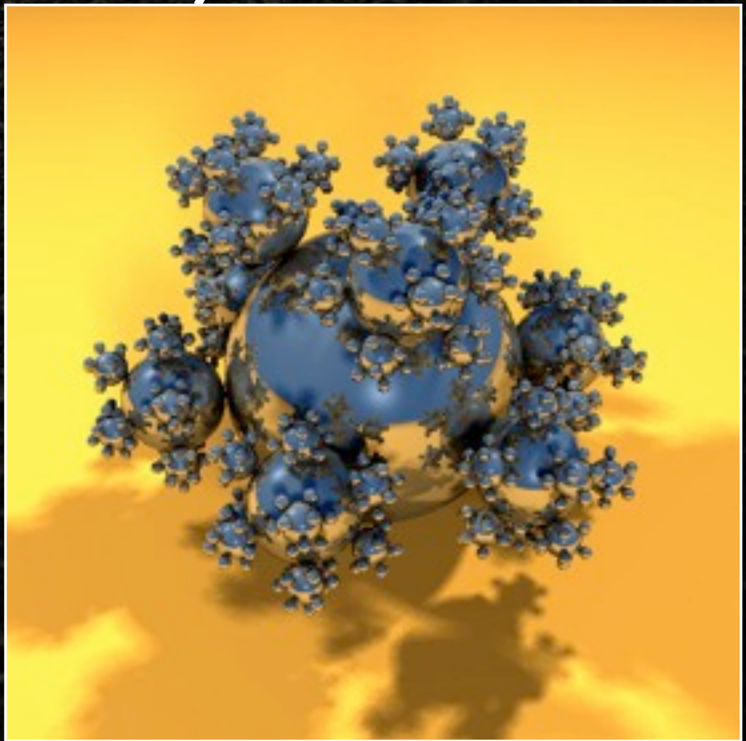
Reflected and transmitted rays



Reflections

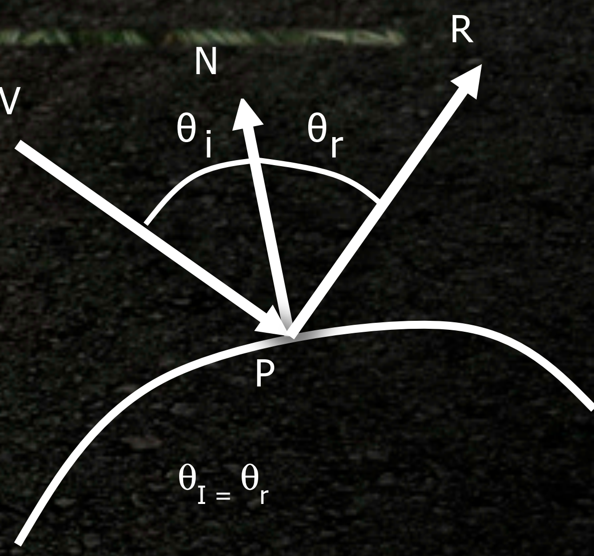
- Reflection can be computed by tracing another ray from the intersection point
- Called perfect specular reflection





Reflections

- Perfect "bounce": incoming angle equal to outgoing angle
- θ_I is called "angle of incidence" or "incident angle"
- V, N and P define a plane
- Trace new ray on the same plane with origin at P and direction R
- Two derivations: algebraic and geometric

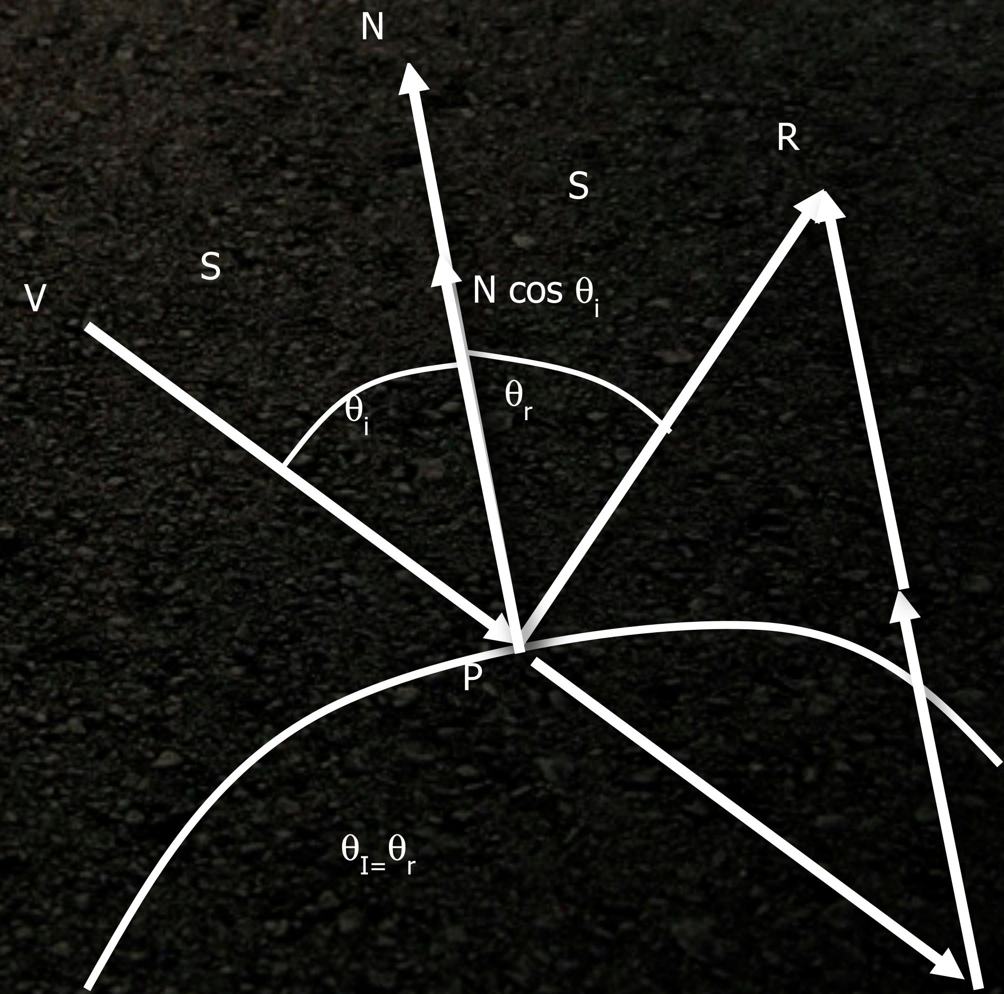


Geometric derivation

$$\vec{R} = 2\vec{N}\cos\theta_i + \vec{V}$$

$$\vec{R} = 2\vec{N}\left(-\vec{N}\cdot\vec{V}\right) + \vec{V}$$

$$\vec{R} = \vec{V} - 2\left(\vec{N}\cdot\vec{V}\right)\vec{N}$$

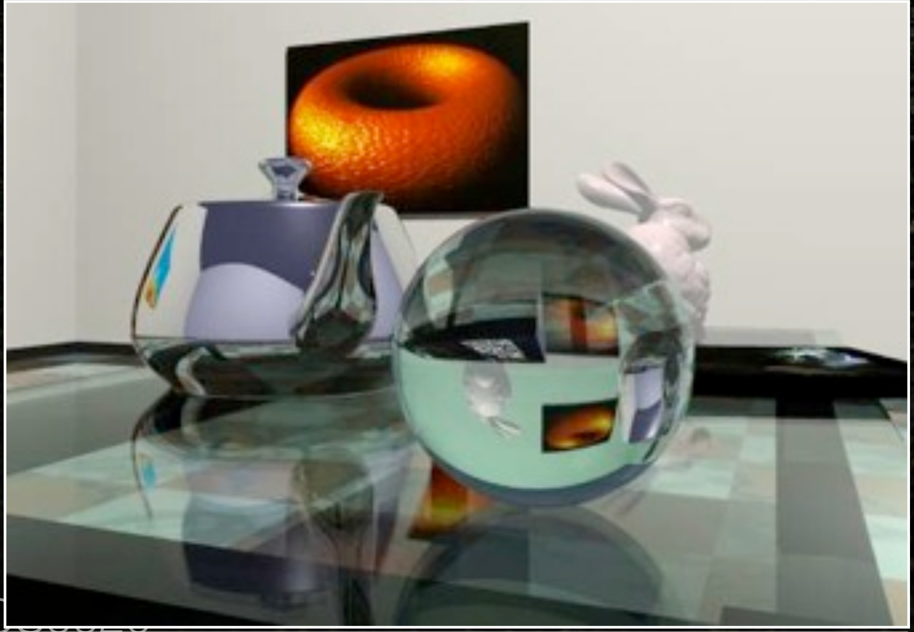


Reflected color

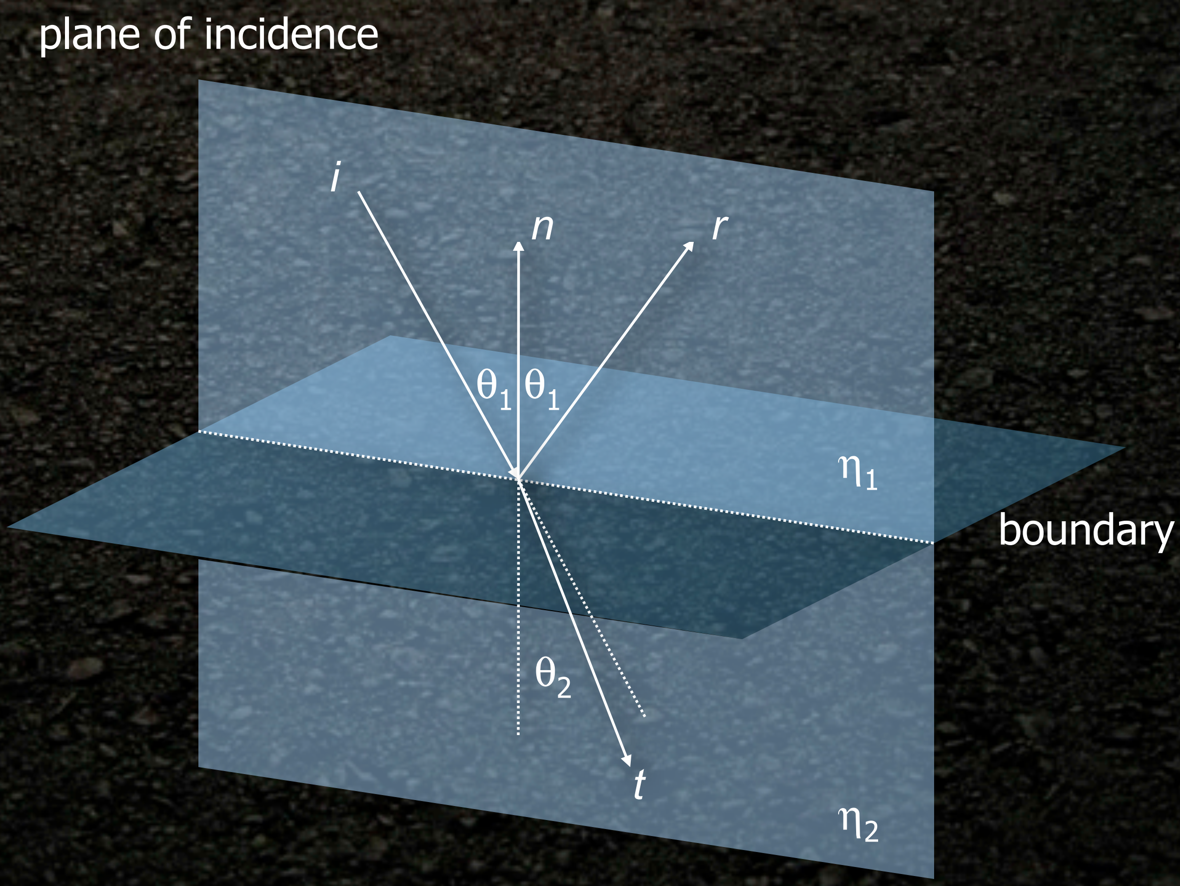
- In Material::shade, trace and shade a ray to get the color along R
- Reflectivity r (range 0-1)
- Result color = r * color along ray
- More about the reflectivity later

Transparency

- Light is an electromagnetic wave
- Some materials conduct energy at visible frequencies



Transmitted rays



Maxwell's equations

- Maxwell's equations relate electricity and magnetism (light) as waves in space
- They predict the speed of light in a vacuum:

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

μ_0 : Electric permittivity of free space

ε_0: Magnetic permeability of free space

Speed of light

- Speed of light changes (slows) in other materials

$$v = \frac{1}{\sqrt{\mu\varepsilon}}$$

μ : Electrical permittivity of material

ε: Magnetic permeability of material

$v \leq c$

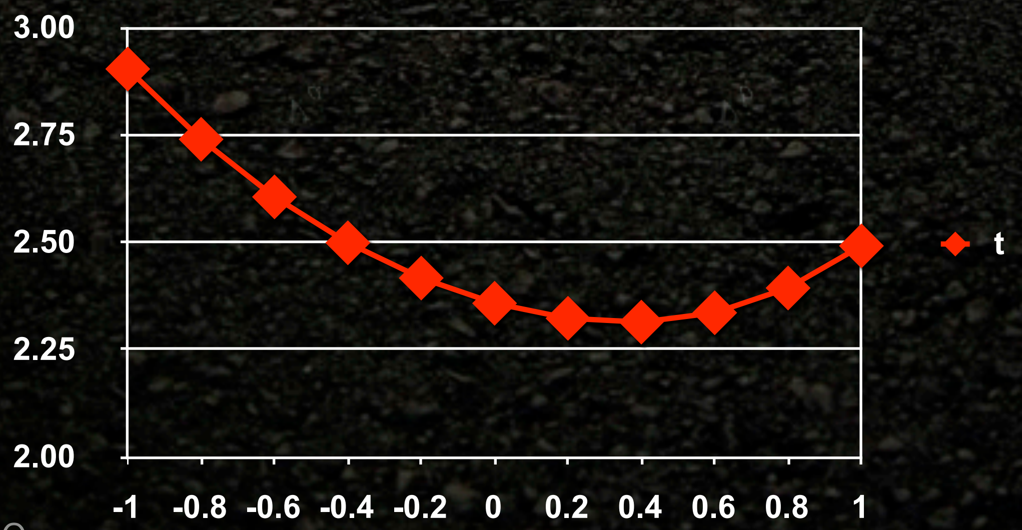
Fermat's principle (1657)

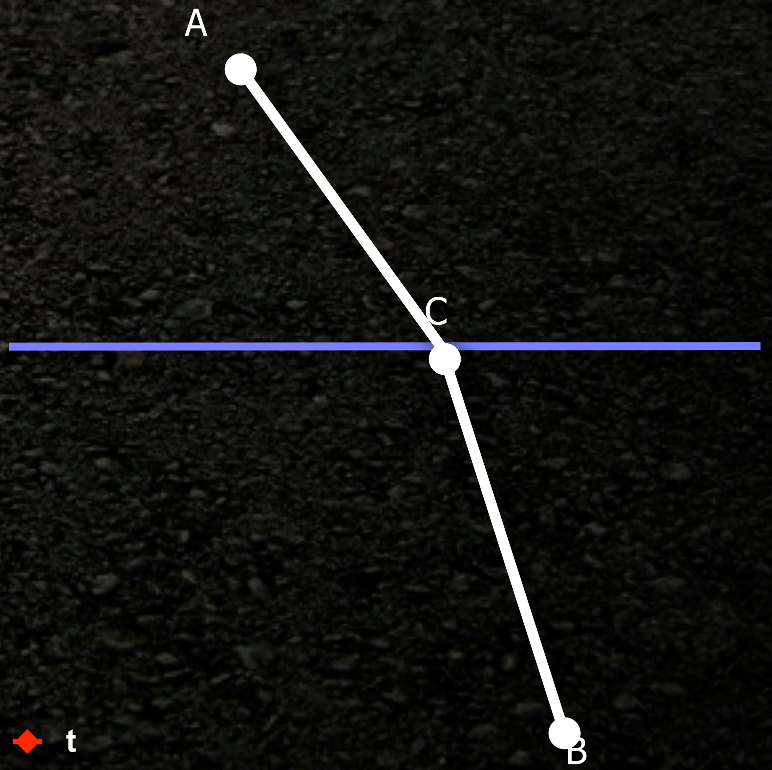
- Light takes the fastest path between two points:

$$t_{AB} = t_{AC} + t_{BC} = \frac{\|C - A\|}{v_a} + \frac{\|B - C\|}{v_b}$$

$$\text{Simplify: } B_y = 0$$

$$\mathrm{t}_{\mathrm{AB}} = \frac{\sqrt{(C_x - A_x)^2 + A_y^2}}{v_a} + \frac{\sqrt{(B_x - C_x)^2 + B_y^2}}{v_b}$$





Refraction

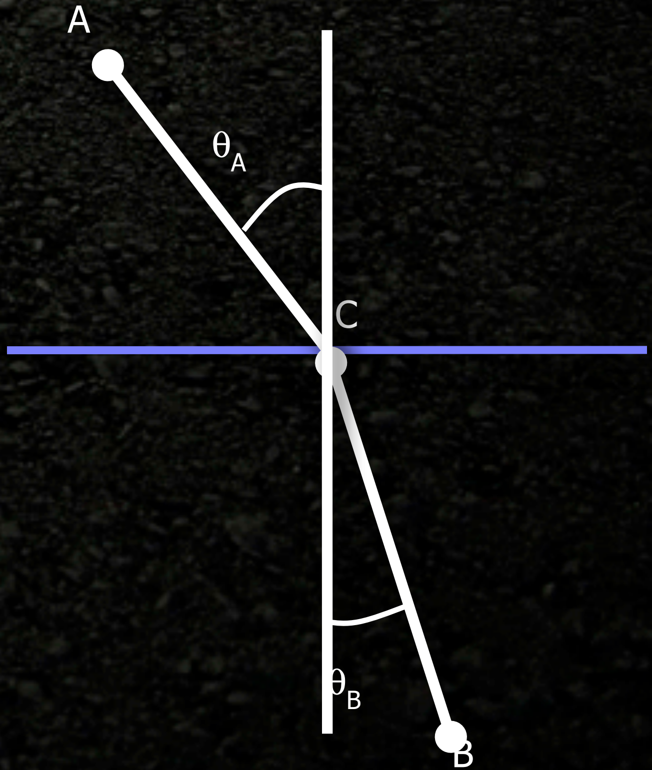
$$t_{AB} = \frac{\sqrt{(C_x - A_x)^2 + A_y^2}}{v_a} + \frac{\sqrt{(B_x - C_x)^2 + B_y^2}}{v_b}$$

$$\frac{d}{dC_x} t_{AB} = 0 = \frac{C_x - A_x}{v_a\sqrt{(C_x - A_x)^2 + A_y^2}} + \frac{C_x - B_x}{v_b\sqrt{(B_x - C_x)^2 + B_y^2}}$$

$$0 = \frac{\sin\theta_a}{v_a} + \frac{-\sin\theta_b}{v_b}$$

$$v_b \sin\theta_a = v_a \sin\theta_b$$

$$\frac{\sin\theta_a}{\sin\theta_b} = \frac{v_a}{v_b}$$



Index of refraction

Absolute index of refraction:

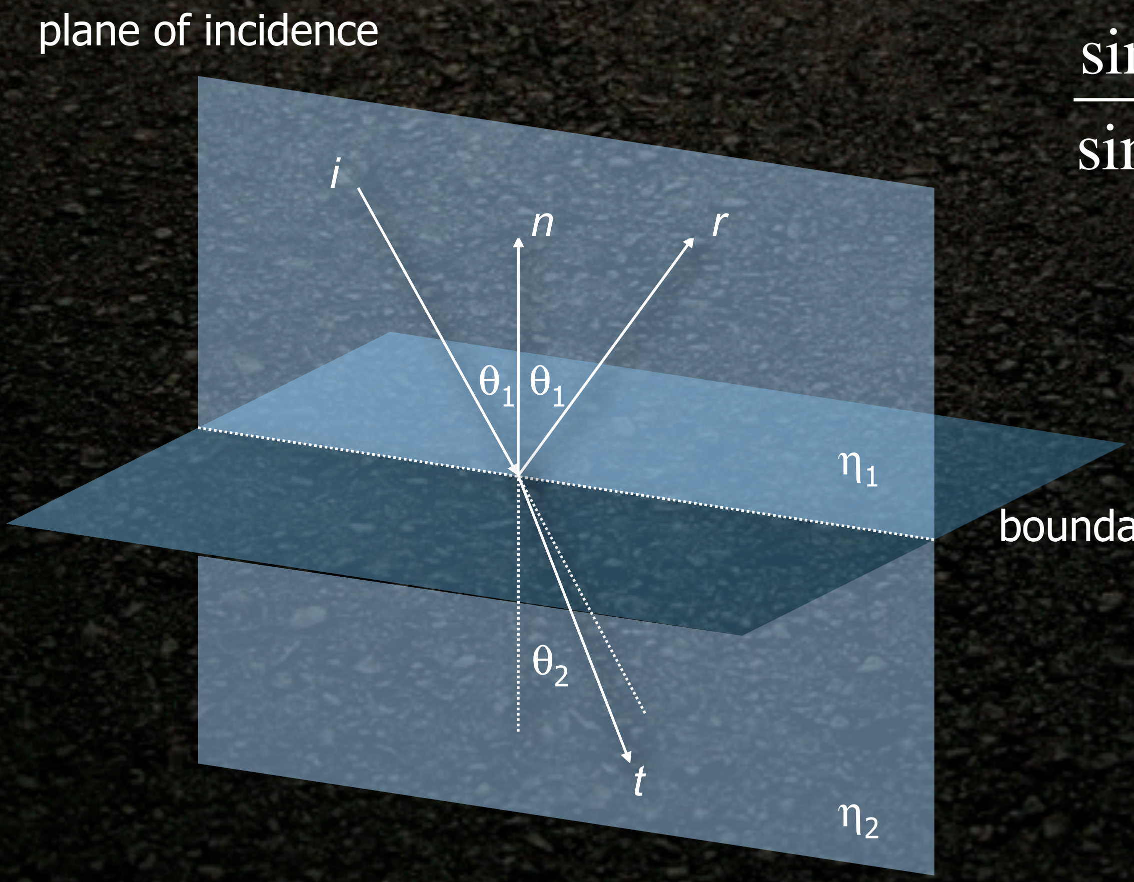
$$\eta_{abs} = \frac{c}{v}$$

where c = 2.99 x 10^8 m/s. Note: $v \leq c$ for all transparent materials, so $\eta \geq 1$.

Relative index of refraction:

$$\eta = \frac{\eta_2}{\eta_1}$$

Snell's Law



$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2} = \frac{\eta_2}{\eta_1} = \eta = \eta_{12}$$

Questions?